Reliability Models under the Microscope

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Abstract

Material defects caused by the movement of particles , or collisions between them at a molecular level are investigated. Einstein associated Brownian motions with the second law of thermodynamics. Brownian motion has been shown to cause a Wiener process degradation. Other types of random walks are introduced and the estimates of failure time distribution due to the defects is estimated. The practical application of the findings in reliability engineering practice (prediction, corrective action etc) is surveyed. The failure mechanisms described herein are a Physics of failure approach. Yet it validates some of the basics of Reliability Standards (e.g. the existence of constant failure rates, the Arrhenius model) Prediction standards would overcome some of the weaknesses of current prediction standards if they incorporated some PoF models, which are generally applicable for all materials

keywords : Failure mechanisms, Random walks , thermodynamics, degradation

1 Introduction

Put a piece of solid under a powerful microscope. Continue to zoom in until the continuous surface is replaced by distinct molecules, arranged in different configurations , see Fig 1. Molecules are the smallest particle which retain the chemical properties of a substance.
"Many of the physical characteristics of compounds that are used to identify them like boiling points, melting points, density are due to inter molecular interactions" ([20])

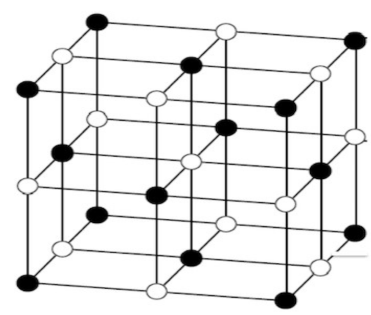

Figure 1: Cristal under Microscope

This paper will focus on characteristics of the device material which induce failures of the device. . Table 1 lists the questions answered by the paper.

SN	Question
1	Is there a common cause failure for all materials composing devices? 1
2	Are the material level failure modes of the degradation type, or of the sudden failure type, or both?
3	Are the classical failure probability distributions (constant, Weibull, Normal, Log-normal etc) encountered ?
5	If degradation processes do occur, are the classical degradation models (Wiener process, Gamma process)applicable?2

1 A precious hint for the answer is provided by [5], which showed that the limit of different random walk processes " are Poisson processes". [5] does not discuss failures, but the fact that the intensities of these Poisson processes have intensity distributions like "constant, Weibull, normal, log-normal" served as a sign-way that the answer lies in this neighbourhood. And in this neighbourhood it was found , see section 3.
2 It is not surprising to find the Wiener process degradation, well explained by material level processes. The applicability of the Gamma process, however will be more of a surprise: Usually the model is based on external shocks: The Essary Marshall Proschan shocks (i.e. [6], [21], [2] or the shot noise model" [12].Cause of failure is associated with shocks applied to the degrading device. The shocks are usually **external**. [12] allows internal shocks, (e.g. component failures), but even these failures are triggered externally. Our investigation excludes the presence of external shocks (mechanical or electrical). We note that Gamma distribution is not mentioned by [5] either. A random walk of particles leading to Gamma process and Transformed Gamma process degradation will be introduced in the paper.

Table 1: Questions answered by paper

A list of notations and symbols is contained in section 2.

Section 3 provides answers to the questions listed in Table 1

The questions posed in Table 1 may appear pure philosophy , yet they have **practical reliability engineering applications**: A failure mechanism at this level is a feature of the material. You choose to use the material, you inevitably get its failure mode. As opposed to failures induced by external factors there are no protection methods (shock absorbents, lightning- rods etc.)against these failures. The methods which can reduce the frequency of the failure, or of its effects, like cooling, redundancy, replacement scheduling, require a reliability model for failure at this level. Section 4 is dedicated to reliability engineering applications of the findings. While exploring this issue, we unexpectedly found ourselves in a controversial region: Classical Reliability Models vs. Physics of Failure (PoF). While a peaceful settlement of this ongoing dispute, is beyond the scope of this paper, the clarification of some of its aspects , relevant to our microscopic examination, will be tackled.

Section 5 will conclude the findings.

Appendix A contains theorems and proofs used in the paper.

2 Notations and Symbols

Table 2 describes the notations used in the paper.

Notation	type	Meaning	Description
a	scalar	throughput of a particle	
$\mathbf{X}$	random variable (rv)		Bold character
$pmf(x)$	probability	probability of $\mathbf{X} = x$	
$Pdf(x)$	probability density	probability of $x < \mathbf{X} < x + dx$	
$Cdf(x)$	probability	probability of $\mathbf{X} < x$	
$\mathbb{E}(\mathbf{X})$ $\mathbb{V}ar(\mathbf{X})$	scalar scalar	Expectation of $\mathbf{X}$ Variance of $\mathbf{X}$	Upper case, black board font letter
$\Phi(x)$	probability	Cdf of the standard normal distribution of $\mathbf{X}$	
IGD	probability distribution	Inverse Gaussian distribution	Distribution of the time, a Brownian motion with positive drift, takes to reach a fixed positive level.
$\mathbb{R}(t)$	Reliability	Probability of no failures until t	Upper case, black board font letter
$f(t)$	pdf	Probability of a failure between (t,t+dt)	
ϵ	small scalar		Defines the acceptable error in a calculated probability
L	Real	The maximum degradation allowed in the device throughput	
η	integer	Limit for the number of degradation causing collisions	$= int(L/a)$
q(t)	probability	probability of collision between particles of a material	
Δt	Time Interval		

Table 2: Notations and symbols

3 Random Walks

3.1 Random walks on a lattice

Definition 1. *Lattice*

An integer lattice , denoted $\mathbb{Z}^n$, is the lattice in the Euclidean space $\mathbb{R}^n$ whose lattice points are n-tuples of integers. A one-dimensional lattice consists of points with integer coordinate. Herein we shall allow lattices consisting of points with coordinate equal to an integer multiplied by a constant scale factor (say a), not necessarily integer.

Definition 2. *Random walk on a lattice.*

A random walk is called "on a lattice " when at any time the walking particles remain on the lattice.

Random walks on a lattice are used to describe diffusion of atoms, ions or molecules inside a crystalline solid.

3.1.1 Brownian Motion

Definition 3. *Standard Wiener or Brownian (motion) process*

"A real-valued stochastic process $Wt :\geq 0$ defined on a probability space (Ω, A, P) is a standard Wiener (or Brownian motion) process if it has the following properties:

1. *The initial value of the stochastic process $W_t \geq 0$ is zero with probability one, meaning $P(W_0 = 0) = 1$.*
2. *The increment $W_t - W_s$ is independent of the past, that is, W_u, where $0 \leq u \leq s$*
3. *The increment $W_t - W_s$ is a normal variable with mean 0 and variance proportional to $t - s$."*

Einstein [4] found that the position of a particle in a fluid moving and colliding randomly with other particles will have a normal distribution with average 0 and standard deviation growing linearly with time. His deduction was based on the second law of thermodynamics and indicated that the growing standard deviation is related to the growing entropy .

The mathematical expression of a Brownian Motion, based on a simple random walk was introduced by Norbert Wiener. A very simple derivation is found in [19], which is summarized as follows:

The position of a particle at $t = 0$ is $\mathbf{X}_0 = 0$. During Δt it will move as defined by:

$$\mathbf{X}_{\Delta t} = \begin{cases} a, & \text{with probability } 1/2 \\ -a, & \text{with probability } 1/2 \end{cases} \tag{1}$$

Obviously:

$$\mathbb{E}(\mathbf{X}_{\Delta t}) = +1/2a - 1/2a = 0$$
$$\mathbb{V}ar(\mathbf{X}_{\Delta t}) = +1/2a^2 + 1/2a^2 = a^2$$

If the process is repeated τ times then, since the movements are independent

$$\begin{cases} \mathbb{E}(\mathbf{X}_{t+\tau}) = & 0 \\ \mathbb{V}ar(\mathbf{X}_{t+\tau}) = & \tau a^2. \end{cases} \tag{2}$$

The movement of N particles will have an average 0 and a standard deviation $N\tau a^2$. If N is large, by the central limit theorem the movement will have a Normal distribution.

Thus the above motion satisfies the criteria of the definition of a Wiener Process.

Failures caused by Wiener process A Wiener process will manifest itself in a stand alone component by continuous degradation of a parameter. When the parameter falls beyond a threshold it is considered failed.The probability distribution of the time to failure will have an **Inverse Gaussian Distribution (IGD)**.

Example 1. *The capacity of an accumulator degrades with every charging. If it drops below a limit it is considered "failed". If it is used in a car, it won't necessarily cause failure even if capacity is below limit. Failure will occur only in long enough trips.*

When N such components are shared to provide a throughput, one component "failure " will not be felt, while other components did not yet reach the limit. The distribution of the parameter being normal, so will be the distribution of the total throughput. The total throughput is still Normally distributed but the variance increases by a factor of N. Time to failure can be calculated using IGD.

3.1.2 Random walk with activation energy

The idea of a "potential hole" retaining a particle, was presented in [11].

Suppose a particle A, in a "potential hole", will move only if the energy transferred to A is greater than a given *activation energy* E_a. Suppose that the probability of the energy transfer being greater than E_a during a collision is q. The motion of a particle A will be defined as follows:

$$\mathbf{X}_{\Delta t} = \begin{cases} 0, & \text{with probability } p = 1 - q \\ a, & \text{with probability q/2} \\ -a, & \text{with probability q/2} \end{cases} \tag{3}$$

Let A be a particle in a component; its displacement causes a failure of the component. Let there be a number, N, of such particles in the component. If particles of type B flow in a Poisson process with average rate ν then within a short interval dt the probability for a collision will be $dp = N\nu dt$, and the probability for a failure of the component will be $N\nu q dt$. The failure rate of the component will be $\lambda = N\nu q$, that is this random walk will develop into failures at a **constant rate**.

The above explains why is the Arrhenius model applicable for failure rate models. The Arrhenius model was deducted for the speed of chemical reactions. Component failures can be caused by a physical dislocation, not necessarily by a chemical reaction. But the start of both processes requires to move a particle from a stable position: In one case the movement causes a failure, in the other it allows the forming of a chemical bound.

For the source of free moving particles B in a Poisson process with a constant rate see 3.2.1.

3.1.3 An asymmetric random walk

[10] presents a rather general example of asymmetric random walk on one dimensional lattice.

$$\mathbf{X}_{\Delta t} = \begin{cases} a, & \text{with probability } q_+ \\ -a, & \text{with probability } q_- \\ 0 & \text{with probability } p = 1 - q_+ - q_- \end{cases} \tag{4}$$

Observe that there is a probability that there is no movement at all. Why? The movement takes its energy from the surrounding. It comes in packets in a Poisson process. The level of energy is a r.v. with a distribution. When the level is below the activation energy required to move the particle from its position, it won't move.

Why is $q_+ \neq q_-$? In a diffusion process which has a directional field (gravitational, electrical, magnetic, chemical etc.) there is a preference to movement in one direction. A particular case for (4) will be considered, namely the case when $q_- = 0$ and $q_+ \ll p$. Since p_- will not be required, we denote $q \triangleq q_+$

$$\mathbf{x}_{k\Delta t} = \begin{cases} x_{(k-1)\Delta t} + a, & \text{with probability q} \\ x_{(k-1)}\Delta t & \text{with probability p=1-q} \end{cases} \tag{5}$$

Obviously:

$$\mathbb{E}(\mathbf{X}_{\Delta t}) = +kqa + p * 0 = kqa$$
$$\mathbb{V}ar(\mathbf{X}_{\Delta t}) = +kqa^2 + p0 = kqa^2$$

Theorem 3.1. *Asymmetric random movements of a particle*

If:

1. *the position of the particle at $t = 0$ is $x = 0$*
2. *a particle moves each Δt according to (5)*

3. $q \ll 1$

4. when x, the position of the particle exceeds a threshold L, occurs a failure of the component, which contains it

5. $\eta \triangleq int(L/a) < \epsilon/(2q^2)$

then:
at $N\Delta t$ $(N \to \infty; Nq < \infty)$, *the degradation (increase of x) will be :*

1. a Gamma degradation process , if q is constant

2. a Transformed Gamma degradation process [8] if q is monotonically increasing with time

Proof.
$\mathbf{J}(N\Delta t) = \mathbf{X}_{N\Delta t}/a$ is the r.v. of the number of times the particle moved until $N\Delta t$. The reliability of the component at $t = N\Delta t$, i.e., the probability of $\mathbf{J} < L/a$ is

$$\mathbb{R}(N\Delta t) = \Sigma_{j=0}^{\eta-1}\binom{N}{j} q^j p^{N-j} \tag{6}$$

If $\eta \triangleq int(L/a) < \epsilon/(2q^2)$ (see [22]), then $\mathbb{R}(t)$ can be approximated by Poisson distribution, allowing an error ϵ for the probability..

$$\mathbb{R}(N\Delta t) = \Sigma_{j=0}^{\eta}\frac{(Nq)^j}{j!}e^{-Nq} \tag{7}$$

By theorem A.1 (see appendix):

$$R(t|\eta) = \Sigma_{j=0}^{\eta}\frac{(q'(t)t)^j}{j!}e^{-q'(t)t}$$

$$pdf(t|\eta) = \begin{cases} a)\frac{[q't]^{\eta-1}}{(\Gamma(\eta))}q'e^{-q't}; \text{ If q constant} \\ \\ b)\frac{[tq'(t)]^{\eta-1}}{(\Gamma(\eta))}[tq'(t)]'e^{-tq'(t)}; \text{ If q increases with t} \end{cases} \tag{8}$$

(8) a) describes a random walk causing a Gamma degradation process
(8) b) describes a random walk causing a Transformed Gamma degradation process

□

In the above a was described as a movement of a particle due to a collision. It also could represent a **particle throughput decrease** of amount a .

In this case the Gamma process is involved with the increase of degradation of a throughput until it reaches a certain allowed limit L

Theorem A.2 proves, (see Appendix A) that if motion is bi-directional, see (4), but $P_- \ll P_+ \ll 1$ the pdf of failure will be identified as a gamma process, by a limited accuracy experiment.

3.2 Random displacement random walk

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3.2.1 Simple linear random walk

Eliazar [5] defines a "simple linear random walk" as follows:

$$\begin{cases} \mathbf{X}_0 = 0 \\ \mathbf{X}_t = \Sigma_{I=0}^{t}\Delta_i \end{cases} \tag{9}$$

where Δ_i is the i^{th} IID sample drawn from $\mathbf{\Delta}$, a r.v. with a PDF given as $f_\Delta(x); -\infty < x < \infty$.

What will be $f(\mathbf{X})$? Following [5]

if $\mathbf{X}_{t-1} = x - u$ and $\Delta_{t-1} = u$ then: $x_{t+1} = x$. The above will hold for any $\infty < u < \infty$:

$$f(x_{t+1}) = \int_{-\infty}^{\infty} f(x-u)f_\Delta(u) \tag{10}$$

This is a functional equation with $f(\mathbf{X})$ as unknown.[5] finds two functions which can satisfy (10). The first one is a constant function $f(x) = c$. The other one is an exponential function which will not be treated in this paper. It is easy to verify that $f(x) = c$ satisfies (10):

$$c = c\int_{-\infty}^{\infty} f_\Delta(u) = c$$

Failures caused by simple linear random walk Suppose a failure occurs in a component if the position of one of its particles exceeds a limit ($|\mathbf{X}| > L$).

Assume the position of the particle at t was at $\mathbf{X}_t = u - x; -L < u - x < L$

-A failure of a component occurs between $(t, t + dt)$, given that it was operational at t if $|\mathbf{X}_{t+1}| > L$. The probability of such an event is usually called hazard function and is denoted by h(t). Following the reasoning of [5]:

$$h(t) \triangleq f(|x_{t+1}| > L \Big| (|x_t| \le L) =$$

$$= \frac{\int_{-\infty}^{-L} f(x-u) f_\Delta(u) du + \int_L^\infty f(x-u) f_\Delta(u)}{\int_{-\infty}^{\infty} f(x-u) f_\Delta(u)} du$$

Using $f(x) = c; \forall x$

$$h(t) = \frac{c \int_{-\infty}^{-L} f_\Delta(u) du + c \int_L^\infty f_\Delta(u) du}{c} = \lambda$$

where $\lambda \triangleq \int_{-\infty}^{-L} f_\Delta(u) du + \int_L^\infty f_\Delta(u) du$ is a constant in t.

Reliability engineers call components with constant hazard function **"constant failure rate components"**.

3.2.2 Simple geometric random walks

Definition 4. [5] defines a simple geometric walk as follows *:*

" The walk begins, at time $t = 0$, from an initial position $Y0$ which is a general positive-valued random variable. At times $t = 1, 2, 3,...$ the walk makes multiplicative steps of magnitudes $m1, m2, m3,...$, respectively. The steps are independent of the initial position y_0, and their magnitudes are IID copies of a general positive-valued random variable $\mathbf{M}$. The temporal positions of the simple geometric random walk are thus given by: "

$$\mathbf{Y} = y_0 \Pi_{i=1}^t \mathbf{M}_i \tag{11}$$

Example 2. *An example for such a walk: Let $\mathbf{Y}$ be the number of free electrons in an insulating material. At $t = 0$ there are y_0 free electrons, During dt a free electron i collides with another stable electron j and frees it with probability r. As the number of free electrons increases the insulator becomes more and more conductive.*

$$\mathbf{Y}_{\Delta t} = y_0(1 + r_1)$$

$$\mathbf{Y}_{t\Delta t} = y_0 \Pi_{i=1}^t (1 + r_i)$$

Denote $\mathbf{M}_i \triangleq 1 + r_i$, you get (11). $\mathbf{M}_i$ is a r.v. with a PDF $g_M(x)$.

Following the reasoning of [5] the pdf of $\mathbf{Y}$ is deduced as follows: if $\mathbf{Y}_t = y/u$ and $M_t = u$ then $y_{t+1} = y$. The probability for $\mathbf{Y} = y$ for all $0 \le u < \infty$:

$$g_{t+1}(y) = \int_0^\infty \frac{dF_t(y/u)}{dy} g_M(u) du =$$

$$= \int_0^\infty \frac{dF_t(y/u)}{d(y/u)} \frac{d(y/u)}{dy} g_M(u) du$$

$$g_{t+1}(y) = \int_0^\infty g(y/u) \frac{1}{u} g_M(u) du \tag{12}$$

By [5] there are two solutions for g(t), to this functional equation, for all $t > 0$:

1. $g_{har}(y) = c/y$
2. $g_{pow}(y) = cy^{\beta-1}$, where β is a constant which satisfies

$$\int_0^\infty u^{-\beta} g_m(u) = 1 \tag{13}$$

We are interested in g_{pow}. It is easy to verify that it satisfies (12):

$$cy^{\beta-1} = \int_0^\infty c(\frac{y}{u})^{\beta-1} \frac{1}{u} g_M(u) du$$

$$cy^{\beta-1} = cy^{\beta-1} \int_0^\infty c(\frac{1}{u^\beta} u g_M(u) du$$

Using (13)

$$cy^{\beta-1} = cy^{\beta-1} \tag{14}$$

Failures caused by simple geometric random walks.

Assume that collision with a free particle evolving by a geometric random walk causes failures of a component. The probability of a failure at t growth linearly with the probability density of $\mathbf{Y}$ at t. Denote by η the growth factor

$$h(y) = \eta g(y) = \eta c y^{\beta-1}$$

Denote $\alpha^\beta \triangleq \beta / c\eta$

$$h(y) = (\beta/\alpha)(y/\alpha)^{\beta-1}$$

The above is recognized as the hazard rate of a **Weibull distribution**. (e.g [17] eq (5.5) pge 64).

Table 3: Pdf of hazard rate.

Random walk	k = 1[a]	k > 1		
		Symmetric		**Asymmetric**
		No Ea [b]	with Ea	(stress) with Ea
Linear (on a lattice)	const	Wiener (IGD)	Γ	Γ^c
Linear (random displacement)	const	const	Γ	Γ
Geometric	Weibull	Tr Γ^d	Tr Γ	Tr Γ

a [a] $k = L/a$ is the number of collisions until the threshold L is exceeded , where a is the throughput loss by a collision .
b [b] Ea stands for activation energy .
c [c] Γ symbolizes a Gamma Process , see (8).
d [d] Tr Γ is a symbol for Transformed Gamma Process , see (8)

3.3 Summary of random Walks

Table 3 lists the random walk types described in the paper

Many failures are the result of collisions between moving and stable particles of the material. Table 3 summarizes the resulting failure rate distributions

3.3.1 Constant failure rate

> [16] *In a metal line carrying significant current density, the free electrons push and move the metal atoms in the direction of the electron wind, i.e., towards the anode end of the line; hence the name electromigration (EM) for this effect. The resulting atomic flow increases compressive stress at the anode and tensile stress at the cathode, which creates a stress gradient that presents an opposing driving force that retards EM [1]. If the levels of stress become high enough, a void may be created due to high tensile stress near the cathode, or a hillock (extrusion of metal through cracks in the dielectric) may form due to high compressive stress near the anode, which can either way result in circuit failure.*

A few failue mechanism examples can be deducted from the above :

1. let x of (9) represent the momentum of an electron in the *electron wind*. If it will exceed a certain limit L its high momentum will be transmitted to its surrounding , so that the *electron wind* becomes an *electron storm*: The many voids will accumulate fast and cause a cutoff of the conductor. By 3.2.1 such a failure will occur at a **constant rate**.
2. Electromigration : Is the results of accumulated number of voids , or of hillocks . If x in (9) is the accumulated count of voids /hillocks then the failure will occur at a constant failure rate. This matches Blacks Law (see equation (5) of [14]) . The constant rate increases in accordance with Arrhenius equation. This is consistent with the fact that the creation of void/hillock requires the movement of a stable atom from a "potential trap".
3. The very existence of the electron wind , with randomly distributed momentum, explains the Johnson-Nyquist Noise (see 4.2.4.1 [14])

3.3.2 Weibull failure rate

Example 2 was given for failures of an insulating material with an increasing failure rate with a Weibull distribution. Indeed insulating materials have been found having a Weibull failure rate (e.g. [17] paragraph 10.3)

4 Reliability Engineering Application

Reliability engineering is about designing , verifying and correcting products to assure a desired reliability level. The differentiation between failures , inherent to the material of a component , and those caused by the design (e.g. stress due to interaction with other devices), or by the environment is obviously mandatory.

4.1 Reliability prediction

Reliability prediction is a basic tool for reliability design. The various methods are discussed in current literature [23]. The findings of this paper are relevant to the " merits and demerits "of two "bottom up " approaches widely utilized (see [7], [3]) : the black box (a.k.a prediction standards) and the white box (a.k.a PoF) approaches.

1. The "black box" approaches (see [18]) defines formulas for a closed set of components. A typical form is :

   $$\lambda = \lambda_b * \Pi_t, *\Pi_s * \Pi_a$$

   where λ_b is a basic failure rate of the components while the Π coefficients describe environmental constraints : temperaure, operational stress, on-off activations etc. While formulas for λ_b might depend on some design elements : material (ceramic vs. tantalum, silicon vs. germanium), structure : (number of gates) , the term "black box " is justified since these design data are usually part of the specification of the components and they don't depend on the mechanism which makes the component work.

2. White box approach (see [14]) , a.k.a Physics of failure (PoF) concentrates on mechanisms of failures : electrical isolation or connection failures, errosion/corrosion/deformation due to the stresses during operation. The progress of the state of the art allows identification of new signatures or failure mechanisms of operating systems . Each failure mode has a "non-exhaustive list" of failure modes.

Table 4 lists a short list of comparing items (most relevant to the discussion herein).

Table 4: Black box vs White box.

	Merits	**Drawbacks**
Black box (Standards)	Closed list of methods Generic Simple System level calculations	constant failure rates continuous update required industry averages provided inaccurate Π factors
White box (Pof)	Accuracy Sensitivity analysis available	Costly tests Need for manufacturing design data Complex system level models

The following can be concluded from the comparison:

1. **Constant failure rates**. The available standards deal only with constant failure rate components (mostly for electronics). **Non constant failure rate** components must utilize some form of Pof/Accelerated Life tests/Accelerated degradation tests. A few number of non-constant rate components can be integrated in mission reliability models based on Markov [9], or Monte Carlo techniques [13]. They also can be integrated in Logistic and LCC costs predictions [15].

   For a complex electronic equipment with a large number of components a specific formula for each component would be a tremendous chalenge. The use of constant failure rate models makes this models practicable. There has been a lot of misuse of constant failure rate models and as a consequence a "No MTBF" movement was established. Some of its devotes went as far as denying the existence of constant failure rate. This paper however states that as long as simple linear random walks exist , so do constant failure rates.

2. **The need for design data** on the structure and processes inside a component will make some POF techniques possible only if the manufacturer cooperates, or provides his test data.

3. **Continuous update requirement**. MIL-HDBK-217 hasn't been updated for quite a long time. It ceased to be applicable. The continuous effort to maintain more recent standards (Telcordia, IEC 62380 etc.) is a demanding one. This paper shows that this efforts can be diminished significantly if the basic failure rate models had a parameter proportional to the number of particles in the device (e.g. weight, volume) . If such a parameter was incorporated in the model, it could have forseen the dramatic improvement of reliability of semiconductor devices, due to its miniaturization. As a consequence fewer update would be necessary.

4. **Inaccurate Π factors**. The Π_t factor accounting for temperatures were based on the Arrhenius model. This is consistent with the model presented herein. Some other Π factors could be developped, based on PoF and incorporated in the standards , similarly to Π_t

   (a) mechanical shocks and vibration , based on finite element models could replace the environmental parameters (GM, GF, AIF etc.). Such a model would differentiate between a good shock absorbing design and a poor one, between vehicles moving on well maintained higways and degraded rural roads.

   (b) thermal shocks based on finite element models could provide parameters affecting failure rates for mission profiles with sudden temperature changes

   (c) Peck's temperature humidity model , could allow the provision of adequate Π_h instead of the currently used , too general, Naval shelterd/ unsheltered coefficients

4.1.1 Reliability improvement

Inprovements as a consequence of Reliability growth program, FRACAS, Customer complaints can benefit from the findings of this paper by:

1. Differentiating between failure modes attributed to the material and those attributed to external factors (environment, interaction with other devices)

2. Provision of natural limits to reliability improvements at material level.

5 Conclusions

The second law of thermodynamics (random walks of particles) was identified as a source of failure mode at material level. The universality of this law implies that these failures appear in all materials.

The Wiener degradation process has been developed based on random walks of particles , right from the beginning. The (Transformed) Gamma process was found as a general mfailure mechanisms [1] . It is shown that particle random movements can be the mechanisms of such a degradation. These movements can cause a variety of failures of the device : T degradation failures (e.g. Gamma process) and sudden failure models (constant failure rate, Weibull, Normal). The distinction between sudden failures and degradation failures is often a matter of definition (e.g. if a car's velocity became less than 1 km/hour is the car degraded , or faulty?)

This paper can be classified as a Physics of failure approach. Yet it validates some of the basics of Reliability Standards (e.g. the existence of constant failure rates, the Arrhenius model). The advantage of prediction standards is, that they allow prediction of large and complex system. Its drawbacks (frequent updates, coarse estimates for environmental factors) could be diminished by incorporating parameters developed by PoF approaches, including this work , like in the case of Π_t factor. The number of non standard units in the system which require POF ,Accelerated life/degradation tests can be reduced.

Appendix

A pdf of failure time

A1 Asymmetric random walk

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Theorem A.1. *Uni- directional asymmetric random walk .*
Given that the reliability of the particle is calculated by:

$$\mathbb{R}(N\Delta t) = \Sigma_{j=0}^{\eta-1}\frac{(Nq(t)\Delta t)^j}{j!}e^{-Nq(N\Delta t)}$$

Case 1. *accelerating q*

$$pdf(t)_H = \frac{[tq'(t)]^{H-1}}{(\Gamma(H))}[tq'(t)]'e^{-tq'(t)}$$

Case 2. *Constant q*
the pdf of the time to failure is given by

$$pdf(t)_H = \frac{[tq]^{H-1}}{(\Gamma(H))}qe^{-tq}$$

Proof.

Note A.1. *The commonly encountered collision time between collisions is a fraction of a second. Typical times to failures in components is expessed in million hours. N the number of collisions required for a failure $N > 10^6$. Since Δt is fraction of a second and the degradation processes happen in million hours it is reasonable to consider $N\Delta t$ as a continuous time : Taylor expansion of reliability functions with $\Delta t < 10^{-5}$ will allow to neglect terms in $\Delta t^j, j > 1$. So $\Delta y/\Delta t \approx dy/dt$*

Replacing $N\Delta t$ by t the reliability of the particle becomes

$$\mathbb{R}(t) = \Sigma_{j=0}^{C-1}\frac{(tq'(t)^j}{j!}e^{-tq'(t)}$$

$$pdf(t) = \frac{d}{dt}\Sigma_{j=0}^{C-1}\frac{(tq'(t)^j}{j!}e^{-tq'(t)} \tag{A1}$$

The proof is by induction.

I. for $C = 1$ (A1) becomes

$$f(t) = q'(t)e^{-q(t)(x)}$$

which is true, since the above is $-\frac{d}{dt}\rho(t)$ (for no failures allowed the system is a series system. Remember that $q'(t) = -\rho'(t)$)

II. Suppose the theorem is true for $C = H$ from (A1)"

$$pdf(t)_{H+1} = [\sum_{j=1}^{H}\frac{d}{dt}\frac{tq'(t)}{j!}] + pdf(H+1) =$$

$$= pdf(t)_H - \frac{d}{dt}\frac{[tq'(t)]^H}{(H)!}e^{-tq'(t)}$$

$$= [\frac{[tq'(t)^{H-1}}{(H-1)!}[tq(t)]'e^{-tq'(t))} -$$

$$-\frac{H}{H!}[tq'(t)^{H-1}[tq'(t)]'e^{-tq'(t)(x)}] +$$

$$+\frac{[tq'(t)]^H}{(H)!}[tq'(t)]')e^{-tq'(t)}$$

$$pdf(t)_{H+1} = \frac{[tq'(t)]^H}{(\Gamma(H+1))}[tq'(t)]'e^{-tq'(t)}$$

□

Theorem A.2. *(Bidirectional asymmetric random walk)*
If

1. *an asymmetric random walk on a lattice of a particle is defined by (4)*
2. *the component containing the particle fails if the particle's movement exceeds L as $p_{\to}0$*

then
the limit of pdf of failure time, as $p_{\to}0$ r will have a gamma distribution

Proof. [10] shows that the probability of finding the RW on the lattice site m after N steps is

$$G_m(N) = \Sigma_n g_m(n) W(n|N) \tag{A2}$$

where

$$W(n|N) = \frac{N!}{n!(N-n)!}(P_+ + P_-)^n (1 - P_- - P_+)^{N-n}$$

$$g_m(n) = \frac{n!}{n_+! n_-!} p_+^{n_+} p_-^{n_-} \tag{A3}$$

$$p_+ = \frac{P_+}{P_+ + P_-} \qquad p_- = \frac{P_-}{P_+ + P_-}$$

$$n_+ = (n+m)/2 \qquad n_- = (n-m)/2 \qquad m = n_+ - n_-$$

Introducing the following into (A3)

$$\lim_{P_- \to 0} p_+ = 1 \qquad \lim_{P_- \to 0} p_- = 0 \qquad \lim_{P_- \to 0} n_- = \lim_{P_- \to 0} NP_- = 0$$

$$\lim_{P_- \to 0} n_+ = m + \lim_{P_- \to 0} n_- = m = n$$

$$\lim_{P_- \to 0} p_-^{n_-} = \lim_{P_- \to 0} (\frac{P_-}{P_+ + P_-})^{NP_-} = \lim_{P_- \to 0} \frac{(P_-^{P_-})^N}{(P_+ + P_-)^{NP_-}} = 1$$

Results

$$\lim_{P_- \to 0} g_m(n) = 1$$

Introduce this into (A2)

$$G_m(N) = \Sigma_{n=m}^{m}) W(n|N) = \frac{N!}{m!(N-m)!}(P_+)^m (1-$$

$$-P_+)^{N-m}$$

The reliability is the probability of $m < L$

$$R(N\Delta t) = \Sigma_{m=0}^{L-1} \frac{N!}{m!(N-m)!}(P_+)^m (1 - P_+)^{N-m}$$

The proof follows that of Theorem 3.1 from (6). □

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