



# Fuzzy Conditional Inference and Reasoning for Fuzzy Granular Propositions using Two Fold Fuzzy Logic

---

Venkata Subba Reddy Poli

EasyChair preprints are intended for rapid dissemination of research results and are integrated with the rest of EasyChair.

January 11, 2023

# Fuzzy Conditional Inference and Reasoning for Fuzzy Granular Propositions using Two Fold Fuzzy Logic

*Poli Venkata Subba Reddy*

---

## Abstract

We consider fuzzy inference of the form “if  $\dots$  then  $\dots$  else  $\dots$ ” and “and/or”. We developed logical constructs based on logical intuitions developed by Fukami. With the proposed method of fuzzy inference and causal logic, we apply on logical constructs. We try to show the fuzzy inference satisfy all intuitions under several criteria.

*Keywords:* fuzzy twofold logic, fuzzy granular propositions, fuzzy inference, fuzzy reasoning, fuzzy conditional inference, fuzzy intuitions,

---

## 1. Introduction

Zade [9] proposed fuzzy logic. Zadeh [8] and Mamdani [1] proposed fuzzy conditional inference. Fukami [2] studied fuzzy intuitions and shown that Zadeh fuzzy conditional inference is not suitable for these intuitions. Fukami adapting the Godel and Standard sequence methods and proved not all fuzzy intuitions. We try to prove all fuzzy intuitions using our fuzzy conditional inference method [6].

Zadeh defined fuzzy set with a single membership function. The fuzzy logic with a two membership functions [7] will give more evidence than a single membership function. We studied fuzzy conditional inference using twofold fuzzy sets. We try to studied fuzzy conditional inference for granular fuzzy propositions.

The two fold fuzzy set  $\tilde{A}=\{Z_1, Z_2\}$ , Where  $Z_1$  support the information and  $Z_2$  is against the information.

$\tilde{A}=\{true, false\}, \{belied, bisbelief\}, \{known, unknown\}, \{likely, unlikely\}$

etc

the fuzzy proposition “ $x$  is  $\tilde{A}$ ”.

**Type-1**

If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{Q}$  else  $y$  is  $\tilde{R}$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

If  $x$  is *Demãnd* then  $y$  is *Prõfit* else  $y$  is *Lõss*  
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

**Type-2**

If  $x$  is  $\tilde{P}$  and  $x$  is  $\tilde{Q}$  or  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$   
 $x$  is  $\tilde{P}_1$  and  $x$  is  $\tilde{Q}_1$  or  $x$  is  $\tilde{R}_1$

---

$y$  is ?

If  $x$  is *Demãnd* and  $x$  is *Supplý* or  $x$  is *Príce* then  $y$  is *Prõfit*  
 $x$  is *verydemãnd* and  $x$  is *more-supplý* or  $x$  is *more-príce*

---

$y$  is ?

The evidence is granular if it consists of collection of propositions,

$$E = \{g_1, g_2, \dots, g_n\}$$

$$g_1 = x_1 \text{ is } \tilde{A}_1 \text{ is } \lambda_1$$

$$g_2 = x_2 \text{ is } \tilde{A}_2 \text{ is } \lambda_2$$

...

$$g_n = x_n \text{ is } \tilde{A}_n \text{ is } \lambda_n$$

Suppose we have granular propositions

$$g_1 = x \text{ is very demand is likely}$$

$$g_2 = x \text{ is sales is very true}$$

$$g_3 = x \text{ is profit is very true}$$

**Type-21**

If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{Q}$  else  $y$  is  $\tilde{R}$  is  $\lambda$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

If  $x$  is  $\tilde{Demand}$  then  $y$  is  $\tilde{Profit}$  else  $y$  is  $\tilde{Loss}$  is likely  
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

### **Type-22**

If  $x$  is  $\tilde{P}$  and  $x$  is  $\tilde{Q}_1$  or  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$  is  $\lambda_1$   
If  $x$  is  $\tilde{P}$  and  $x$  is  $\tilde{Q}$  or  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$  is  $\lambda_2$   
 $x$  is  $\tilde{P}_1$  and  $x$  is  $\tilde{Q}_1$  or  $x$  is  $\tilde{R}_1$  is  $\lambda_2$

---

$y$  is ?

If  $x$  is  $\tilde{Demand}$  and  $x$  is  $\tilde{Supply}$  or  $x$  is  $\tilde{Price}$  then  $y$  is  $\tilde{Profit}$  is likely  
 $x$  is  $\tilde{verydemand}$  and  $x$  is  $\tilde{moresupply}$  or  $x$  is  $\tilde{moreprice}$

---

$y$  is ?

## **2. Fuzzy Logic Based on Two Fold Fuzzy Sets**

Zadeh [11] defined fuzzy set with a single membership function [20]. The fuzzy set with two fuzzy member functions “true” and “false” will give more evidence than the single fuzzy membership function to deal with incomplete information. In the following “two fold fuzzy set” is defined with “true” and “false” fuzzy membership functions. The fuzzy logic and fuzzy reasoning of single membership function is extended to fuzzy logic with two membership functions “true” and “false”.

### **2.1. The Two Fold Fuzzy Sets**

“A two fold fuzzy set” may be defined with two membership functions “true” and “false” for the proposition of type “ $x$  is  $A$ ”. The fuzzy set with two membership functions “true” and “false” will give more evidence than the single membership function.

For instance “Rama has Headache”.

In this fuzzy proposition, the fuzzy set “Headache” may be defined with

“true” and “false”.

**Definition 2.1** The “a two fold fuzzy set”  $\tilde{A}$  in a universe of discourse  $X$  is defined by its membership function  $\mu_{\tilde{A}}(x) \rightarrow [0, 1]$ , where  $\tilde{A} = \{\mu_{\tilde{A}}^{true}(x), \mu_{\tilde{A}}^{false}(x)\}$  and  $x \in X$

$\mu_{\tilde{A}}^{true}(x)$  and  $\mu_{\tilde{A}}^{false}(x)$  are the fuzzy membership functions of the “a two fold fuzzy set”  $\tilde{A}$ ,

$$\mu_{\tilde{A}}^{true}(x) = \int \mu_{\tilde{A}}^{true}/x(x) = \mu_{\tilde{A}}^{true}(x_1)/x_1 + \cdots + \mu_{\tilde{A}}^{true}(x_n)/x_n,$$

$\mu_{\tilde{A}}^{false}(x) = \int \mu_{\tilde{A}}^{false}(x)/x = \mu_{\tilde{A}}^{false}(x_1)/x_1 + \cdots + \mu_{\tilde{A}}^{false}(x_n)/x_n$ , where “+” is union,

For example, “young” may be given for the fuzzy proposition “ $x$  is young”

$$\text{young} = \{\mu_{\text{young}}^{true}(x), \mu_{\text{young}}^{false}(x)\},$$

$$\mu_{\text{young}}^{true}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30, \\ + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\},$$

$$\mu_{\text{young}}^{false}(x) = \{0.36/10 + 0.31/15 + 0.26/20 + 0.23/25 + 0.2/30 + 0.18/35 \\ + 0.16/40 + 0.14/45 + 0.12/50\}.$$

For instance. “Rama is young” with fuzziness  $\{0.8, 0.2\}$ , where 0.8 is “true” and 0.2 is “false”.

The Graphical representation of “true” and “false” of “young” is shown in Fig.1.

## 2.2. The Two Fold Fuzzy Logic

The fuzzy logic is combination of fuzzy sets using logical operators. The fuzzy logic with “two fold fuzzy sets” is combination of “two fold fuzzy sets” using logical operators. The fuzzy logic bases on “two fold fuzzy sets” can be studied similar lines of Zadeh’s fuzzy logic.

Some of the logical operations are given below for fuzzy sets with two fold fuzzy membership functions.

$\tilde{A}$ ,  $\tilde{B}$  and  $\tilde{C}$  are fuzzy sets with two fold fuzzy membership functions.

Let tall, weight and more or less weight are two fold fuzzy sets.

$$\begin{aligned}
\tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\
&\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \\
\tilde{\text{weight}} &= \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.3/x_4 + 0.2/x_5, \\
&\quad 0.2/x_1 + 0.2/x_2 + 0.1/x_3 + 0.1/x_4 + 1/x_5\} \\
\text{more or less weight} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.5/x_4 + 0.4/x_5, \\
&\quad 0.4/x_1 + 0.4/x_2 + .3/x_3 + .3/x_4 + 0.3/x_5\}.
\end{aligned}$$

### Negation

$x$  is not  $\tilde{A}$

$$\tilde{A}'(x) = \{1 - \mu_A^{\text{true}}(x), 1 - \mu_A^{\text{false}}(x)\}/x$$

$x$  is not tall

$$\begin{aligned}
\tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\
&\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \\
1 - \tilde{\text{tall}} &= \{0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.6/x_4 + 0.8/x_5, \\
&\quad 0.5/x_1 + 0.6/x_2 + 0.7/x_3 + 0.8/x_4 + 0.9/x_5\}.
\end{aligned}$$

### Disjunction

$x$  is  $\tilde{A}$  or  $y$  is  $\tilde{B}$

$$\tilde{A} \vee \tilde{B} = \{\max(\mu_A^{\text{true}}(x), \mu_B^{\text{true}}(y)), \max(\mu_A^{\text{false}}(x), \mu_B^{\text{false}}(y))\}/(x, y),$$

$$\begin{aligned}
\tilde{\text{tall}} \vee \tilde{\text{weight}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5, \\
&\quad 0.4/x_1 + 0.3/x_2 + 0.2/x_3 + .1/x_4 + .1/x_5\}.
\end{aligned}$$

### Conjunction

$x$  is  $\tilde{A}$  and  $y$  is  $\tilde{B}$

$$\tilde{A} \wedge \tilde{B} = \{\min(\mu_A^{\text{true}}(x), \mu_B^{\text{true}}(y)), \min(\mu_A^{\text{false}}(x), \mu_B^{\text{false}}(y))\}/(x, y),$$

$$\begin{aligned}
\tilde{\text{tall}} \wedge \tilde{\text{weight}} &= \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.2/x_4 + 0.2/x_5, \\
&\quad 0.1/x_1 + 0.1/x_2 + 0.1/x_3 + 0.1/x_4 + 0.1/x_5\}.
\end{aligned}$$

### Composition

if  $x$  is  $\tilde{A}$  then  $y$  is  $\tilde{B}$

$x$  is  $\tilde{A}_1$

$$\frac{}{y \text{ is } \tilde{A}_1 \text{ o } (\tilde{A} \rightarrow \tilde{B})}$$

$$\begin{aligned}
\tilde{A} \text{ o } (\tilde{A} \rightarrow \tilde{B}) &= \{\min\{\mu_A^{\text{true}}(x), \min(1, 1 - \mu_A^{\text{true}}(x) + \mu_B^{\text{true}}(y))\}, \\
&\quad \min\{\mu_A^{\text{Disbelief}}(x), \min(1, 1 - \mu_A^{\text{false}}(x) + \mu_B^{\text{false}}(y))\}\}/y
\end{aligned}$$

if  $x = y$

$$= \{\min\{\mu_A^{\text{true}}(x), \min(1, 1 - \mu_A^{\text{true}}(x) + \mu_B^{\text{true}}(x))\},$$

$$\min\{\mu_A^{Diselief}(x), \min(1, 1 - \mu_A^{false}(x) + \mu_B^{false}(x))\}$$

if  $x$  is tall then  $x$  is weight  
 $x$  is very tall

---

$x$  is very tall o ( $tall \rightarrow weight$ )

### Fuzzy quantifiers

The fuzzy propositions may contain quantifiers like “very” , “more or less” etc. These fuzzy quantifiers may be eliminated as

#### Concentration

$x$  is very  $\tilde{A}$

$$\mu_{very \tilde{A}}(x) = \{\mu_{very \tilde{A}}^{true}(x)^2, \mu_{very \tilde{A}}^{false}(x)^2\}$$

$x$  is very tall

$$\mu_{very \tilde{tall}}(x) = \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, \\ 0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}$$

$$\mu_{very \tilde{A}}(x) \subseteq \mu_{\tilde{A}}(x)$$

i.e.,

$$\mu_{very \tilde{A}}(x) \leq \mu_{\tilde{A}}(x)$$

$$\mu_{very \tilde{A}}(x) \text{ contains } \mu_{\tilde{A}}(x)$$

#### Diffusion

if  $x$  is more or less  $\tilde{A}$

$$\mu_{more \ or \ less \ \tilde{A}}(x) = \{\mu_{more \ or \ less \ \tilde{A}}^{true}(x)^2, \mu_{more \ or \ less \ \tilde{A}}^{false}(x)^{0.5}\}$$

if  $x$  is more or less tall

$$\mu_{more \ or \ less \ \tilde{tall}}(x) = \{0.95/x_1 + 0.89/x_2 + 0.84/x_3 + 0.63/x_4 + 0.45/x_5, \\ 0.70/x_1 + 0.63/x_2 + .054/x_3 + 0.44/x_4 + 0.31/x_5\}.$$

### 3. Fuzzy Conditional Inference

Zadeh [10] fuzzy conditional inference is given as

$$\text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \tilde{A} \rightarrow \tilde{B} = \min\{1, 1 - \tilde{A} + \tilde{B}\}, \quad (3.1)$$

$$= \{\min(1, 1 - \mu_A^{true}(x) + \mu_B^{true}(y)), \min(1, 1 - \mu_A^{false}(x) + \mu_B^{false}(y))\} / (x, y),$$

Mamdani [2] fuzzy conditional inference is given by

$$\text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \tilde{A} \rightarrow \tilde{B} = \tilde{A} \times \tilde{B} \quad (3.2)$$

Reddy[10] fuzzy conditional inference is given by

$$\text{if } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{A}_2 \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_n \text{ then } y \text{ is } \tilde{B} = \min\{\tilde{A}_1, \tilde{A}_2, \dots\}$$

$$\begin{aligned}
& \tilde{A}_n, \tilde{B}\} \\
& \text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \tilde{A} \\
& \text{i.e } \tilde{B} = \tilde{A} \tag{3.3} \\
& \text{if } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{A}_2 \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_n \text{ then } y \text{ is } \tilde{B} = \tilde{A}_1 \text{ and } \tilde{A}_2, \dots, \tilde{A}_n \\
& = \min\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n, \tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\} \\
& = \min\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\} \\
& \tilde{B} = \tilde{A}_1 \text{ and } \tilde{A}_2, \dots, \tilde{A}_n
\end{aligned}$$

The fuzzy conditional inference is given as using Mamdani fuzzy conditional inference,

$$\begin{aligned}
& \text{if } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{A}_2 \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_n \text{ then } y \text{ is } \tilde{B} = \{\{\min(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n), B\}\} \\
& = \{\{\min(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n), B\}\} \\
& \text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \{\tilde{A}\} \tag{3.4}
\end{aligned}$$

Fuzzy conditional inference is given for “if  $x$  is  $\tilde{A}$  then  $y$  is  $\tilde{B}$  else  $y$  is  $\tilde{C}$ ” as,

$$\begin{aligned}
& \text{if } x \text{ is } \tilde{P} \text{ then } y \text{ is } \tilde{Q} \text{ else } y \text{ is } \tilde{R} = (\tilde{P} \times \tilde{Q} \vee \tilde{P}' \times \tilde{R},) \text{ where “} + \text{” is union} \\
& \text{if } x \text{ is } \tilde{P} \text{ then } y \text{ is } \tilde{Q} \text{ else } y \text{ is } \tilde{R} = \text{if } x \text{ is } \tilde{P} \text{ then } y \text{ is } \tilde{Q} = \tilde{P} \rightarrow \tilde{Q}, \text{ for } \tilde{P} \\
& \text{if } x \text{ is } \tilde{P} \text{ then } y \text{ is } \tilde{Q} \text{ else } y \text{ is } \tilde{R} = \text{if } x \text{ is not } \tilde{P} \text{ then } y \text{ is } \tilde{R} = \tilde{P}' \rightarrow \tilde{R}, \text{ for} \\
& \text{not } \tilde{P}
\end{aligned}$$

### 3. Fuzzy plausibility

Plausibility theory will perform inconsistent information into consistent.

#### Generalization

$$\begin{aligned}
& p \vee q, \mu = p, \mu \\
& p \vee q, \mu = q, \mu
\end{aligned}$$

#### Specialization

$$\begin{aligned}
& p \wedge q, \mu = p, \mu \\
& p \wedge q = q, \mu
\end{aligned}$$

The inference is given using generalization and specialization

$$\begin{aligned}
& p \wedge q \vee r, \mu = p \vee q, \mu = p, \mu \\
& p \wedge q \vee r, \mu = q \vee r, \mu = q, \mu \\
& p \wedge q \vee r, \mu = r \vee p, \mu = p, \mu
\end{aligned}$$

Consider fuzzy inference Type-1



The fuzzy inference is given for Type-1 using generalization and specialization

Confider fuzzy inference Type-11

**Type-1**    If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{Q}$  else  $y$  is  $\tilde{R}$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

if  $x$  is  $\tilde{P}$  then  $x$  is  $\tilde{Q}$   
 $x$  is  $\tilde{P}_1$

---

$y$  is  $\tilde{Q}_1$

if  $x$  is  $P'$  then  $x$  is  $\tilde{R}$   
 $x$  is  $\tilde{P}_1$

---

$y$  is  $\tilde{R}_1$

The fuzzy inference is given for Type-2 using generalization and specialization

**Type-2** indent If  $x$  is  $\tilde{P}$  and  $x$  is  $\tilde{Q}$  or  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$   
 $x$  is  $\tilde{P}_1$  and  $x$  is  $\tilde{Q}_1$  or  $x$  is  $\tilde{R}_1$

---

$y$  is ?

If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{S}$   
 $x$  is  $\tilde{P}_1$

---

$y$  is  $\tilde{S}_1$

If  $x$  is  $\tilde{Q}$  then  $y$  is  $\tilde{S}$   
 $x$  is  $\tilde{Q}_1$

---

$y$  is  $\tilde{S}_1$

If  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$   
 $x$  is  $\tilde{R}_1$

---

$y$  is  $\tilde{S}$

From fuzzy conditional inference Type-1 and Type-2, the two criterions may be given as

**Criteria-1**

If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{S}$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

**Criteria-2**

(if  $x$  is  $P'$  then  $x$  is  $\tilde{R}$ )  
 $x$  is  $\tilde{P}'_1$

---

$y$  is ?

The fuzzy inference is drawing a conclusion from fuzzy propositions.

The fuzzy inference is given for Criteria-1 according to fuzzy intuitions.

Table 1: Fuzzy inference for Criteria-1.

Intuition	Proposition	Inference
I-1	$x$ is $\tilde{P}$	$y$ is $\tilde{S}$
I-2	$y$ is $\tilde{S}$	$x$ is $\tilde{P}$
II-1	$x$ is very $\tilde{P}$	$y$ is very $\tilde{S}$
II-2	$y$ is very $\tilde{S}$	$x$ is very $\tilde{P}$
III-1	$x$ is more or less $\tilde{P}$	$y$ is more or less $\tilde{S}$
III-2	$y$ is More or less $\tilde{S}$	$x$ is more or less $\tilde{P}$
IV-1	$x$ is not $\tilde{P}$	$y$ is not $\tilde{S}$
IV-2	$y$ is not $\tilde{S}$	$x$ is not $\tilde{P}$

## 4. Verification of fuzzy intuition using Fuzzy Conditional Inference

Verification of fuzzy intuitions for Criteria-1

### 4.1.1 In the case of intuition I-1

$$\begin{aligned} & \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) \\ &= \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y)) \end{aligned}$$

Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$

Considering  $\tilde{S} = \tilde{P}$

$$\begin{aligned} &= \int \mu_{\tilde{S}}(y) \circ (\int \mu_{\tilde{S}}(y)) \\ &= \int \mu_{\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y)) \end{aligned}$$

Using specialization

$$\begin{aligned} &= \int \mu_{\tilde{S}}(y) \\ &= y \text{ is } \tilde{S} \end{aligned}$$

intuition I-1 satisfied.

### 4.1.2 In the case of intuition I-2

$$\begin{aligned} & (\tilde{P} \rightarrow \tilde{S}) \circ \tilde{S} \\ &= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\tilde{S}}(y) \end{aligned}$$

Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$

Considering  $\tilde{S} = \tilde{P}$

$$\begin{aligned} &= \int \mu_{\tilde{P}}(x) \circ \int \mu_{\tilde{P}}(x) \\ &= \int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{P}}(x) \end{aligned}$$

Using specialization

$$\begin{aligned} &= \int \mu_{\tilde{P}}(x) \\ &= x \text{ is } \tilde{P} \end{aligned}$$

intuition I-2 satisfied.

### 4.1.3 In the case of intuition II-1

$$\begin{aligned} & \text{very} \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) \\ &= \int \mu_{\text{very} \tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y)) \end{aligned}$$

Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$

Considering  $\tilde{S} = \tilde{P}$

$$\begin{aligned} &= \int \mu_{\text{very} \tilde{S}}(y) \circ (\int \mu_{\tilde{S}}(x)) \\ &= \int \mu_{\text{very} \tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y)) \end{aligned}$$

Using specialization

$$= \int \mu_{\text{very}\tilde{S}}(y)$$

$$= y \text{ is } \text{very}\tilde{S}$$

intuition II-1 satisfied.

#### 4.1.4 In the case of intuition II-2

$$(\tilde{P} \rightarrow \tilde{S}) \circ \text{very } \tilde{S}$$

$$= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\text{very}\tilde{S}}(y)) \circ \int \mu_{\tilde{S}}(y)$$

Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$

Considering  $\tilde{S} = \tilde{P}$

$$= \int \mu_{\tilde{P}}(x) \circ \int \mu_{\text{very}\tilde{P}}(x)$$

$$= \int \mu_{\tilde{P}}(x) \wedge \int \mu_{\text{very}\tilde{P}}(x)$$

Using specialization

$$= \int \mu_{\text{very}\tilde{P}}(x)$$

$$= x \text{ is } \text{very}\tilde{P}$$

intuition II-2 satisfied.

#### 4.1.5 In the case of intuition III-1

$$\text{indent } \text{moreorless}\tilde{P} \circ (\tilde{P} \rightarrow \tilde{S})$$

$$= \int \mu_{\text{moreorless}\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$$

Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$

Considering  $\tilde{S} = \tilde{P}$

$$= \int \mu_{\text{moreorless}\tilde{S}}(y) \circ (\int \mu_{\tilde{S}}(x))$$

$$= \int \mu_{\text{moreorless}\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$$

Using specialization

$$= \int \mu_{\text{moreorless}\tilde{S}}(y)$$

$$= y \text{ is } \text{very}\tilde{S}$$

intuition III-1 satisfied.

#### 4.1.6 In the case of intuition III-2

$$(\tilde{P} \rightarrow \tilde{S}) \circ \text{more or less } \tilde{S}$$

$$= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\text{moreorless}\tilde{S}}(y)) \circ \int \mu_{\tilde{S}}(y)$$

Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$

Considering  $\tilde{S} = \tilde{P}$

$$= \int \mu_{\tilde{P}}(x) \circ \int \mu_{\text{moreorless}\tilde{P}}(x)$$

$$= \int \mu_{\tilde{P}}(x) \wedge \int \mu_{\text{moreorless}\tilde{P}}(x)$$

Using specialization

$= \int \mu_{moreorless\tilde{P}}(x)$   
 $= x$  is *very* $\tilde{P}$   
 intuition III-2 satisfied.

**4.1.7 In the case of intuition IV-1**

$\text{not } \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S})$   
 $= \int \mu_{\text{not}\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\tilde{S}}(y))$   
 Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$   
 Considering  $\tilde{S} = \tilde{P}$   
 $= \int \mu_{\text{not}\tilde{S}}(y) \circ (\int \mu_{\tilde{S}}(x))$   
 $= \int \mu_{\text{not}\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$   
 Using specialization  
 $= \int \mu_{\text{not}\tilde{S}}(y)$   
 $= y$  is *not* $\tilde{S}$   
 intuition IV-1 satisfied.

**4.1.8 In the case of intuition IV-2**

$(\tilde{P} \rightarrow \tilde{S}) \circ \text{very } \tilde{S}$   
 $= (\int \mu_{\tilde{P}}(x) \rightarrow \int \mu_{\text{very}\tilde{S}}(y)) \circ \int \mu_{\tilde{S}}(y)$   
 Considering  $\tilde{P} \rightarrow \tilde{S} = \tilde{P}$   
 Considering  $\tilde{S} = \tilde{P}$   
 $= \int \mu_{\tilde{P}}(x) \circ \int \mu_{\text{very}\tilde{P}}(x)$   
 $= \int \mu_{\tilde{P}}(x) \wedge \int \mu_{\text{very}\tilde{P}}(x)$   
 Using specialization  
 $= \int \mu_{\text{very}\tilde{P}}(x)$   
 $= x$  is *very* $\tilde{P}$   
 intuition IV-2 satisfied.

Criteria-1 is suitable for I-1,I-2, II-1, II-2, III-1, III-2, IV-1 and IV-2.

The fuzzy intuitions are give based on Fukami for Criteria-2 .

**I'-1**

if  $x$  is  $\tilde{P}'$  then  $y$  is  $\tilde{R}$   
 $x$  is  $\tilde{P}'$

---

$y$  is  $\tilde{R}$

**I'-2**

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$   
 $y$  is  $\tilde{R}$

---

$y$  is  $\tilde{P}'$

**II'-1**

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$   
 $x$  is very  $\tilde{P}'$

---

$y$  is very  $\tilde{R}$

**II'-2**

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$   
 $y$  is very  $\tilde{R}$

---

$y$  is very  $\tilde{P}'$

**III'-1**

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$   
 $x$  is more or less  $\tilde{P}'$

---

$y$  is more or less  $\tilde{R}$

**III'-2**

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$   
 $y$  is more or less  $\tilde{R}$

---

$y$  is more or less  $\tilde{P}'$ ;

**IV'-1**

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$   
 $x$  is not  $\tilde{P}'$

---

$y$  is not  $\tilde{R}$

#### IV'-2

if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$

$y$  is not  $\tilde{R}$

---

$x$  is not  $\tilde{P}'$

The inference is given for Criteria-1 according to intuitions.

Table 2: Fuzzy inference for Criteria-2.

Intuition	Proposition	Inference
I'-1	$x$ is $\tilde{P}'$	$y$ is $\tilde{R}$
I'-2	$y$ is $\tilde{R}$	$x$ is $\tilde{P}'$
II'-1	$x$ is very $\tilde{P}'$	$y$ is very $\tilde{R}$
II'-2	$y$ is very $\tilde{R}$	$x$ is very $\tilde{P}'$
III'-1	$x$ is more or less $\tilde{P}'$	$y$ is more or less $\tilde{R}$
III'-2	$y$ is more or less $\tilde{R}$	$x$ is more or less $\tilde{P}'$
IV'-1	$x$ is not $\tilde{P}'$	$y$ is not $\tilde{R}$
IV'-2	$y$ is not $\tilde{R}$	$x$ is not $\tilde{P}'$

Verification of fuzzy intuitions for Criteria-2

#### 4.2.1 In the case of intuition I'-1

$$\tilde{P}' \circ (\tilde{P}' \rightarrow \tilde{R})$$

$$= \int \mu_{\tilde{P}'}(x) \circ (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y))$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$= \int \mu_{\tilde{R}}(y) \circ (\int \mu_{\tilde{R}}(y))$$

$$= \int \mu_{\tilde{R}}(y) \wedge (\int \mu_{\tilde{R}}(y))$$

Using specialization

$$= \int \mu_{\tilde{R}}(y)$$

$$= y \text{ is } \tilde{R}$$

intuition I'-1 satisfied.

#### 4.2.2 In the case of intuition I'-2

$$(\tilde{P}' \rightarrow \tilde{R}) \circ \tilde{R}'$$

$$= (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\tilde{R}'}(y)$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$= \int \mu_{\tilde{P}'}(x) \circ \int \mu_{\tilde{P}'}(x)$$

$$= \int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\tilde{P}'}(x)$$

Using specialization

$$= \int \mu_{\tilde{P}'}(x)$$

$$= x \text{ is } \tilde{P}'$$

intuition I'-2 satisfied.

#### 4.2.3 In the case of intuition II'-1

$$\text{very } \tilde{P}' \circ (\tilde{P}' \rightarrow \tilde{R})$$

$$= \int \mu_{\text{very}\tilde{P}'}(x) \circ (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y))$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$= \int \mu_{\text{very}\tilde{R}'}(y) \circ (\int \mu_{\tilde{R}}(y))$$

$$= \int \mu_{\text{very}\tilde{R}'}(y) \wedge (\int \mu_{\tilde{R}}(y))$$

Using specialization

$$= \int \mu_{\text{very}\tilde{R}}(y)$$

$$= y \text{ is very } \tilde{R}$$

intuition II'-1 satisfied.

#### 4.2.4 In the case of intuition II'-2 $(\tilde{P}' \rightarrow \tilde{R}) \circ \text{very } \tilde{R}$

$$= (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\text{very}\tilde{R}}(y)$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$= \int \mu_{\tilde{P}'}(x) \circ \int \mu_{\text{very}\tilde{P}'}(x)$$

$$= \int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\text{very}\tilde{P}'}(x)$$

Using specialization

$$= \int \mu_{\text{very}\tilde{P}'}(x)$$

$$= x \text{ is very } \tilde{P}'$$

intuition II'-2 satisfied.

#### 4.2.5 In the case of intuition III'-1

$$\text{more or less } \tilde{P}' \circ (\tilde{P}' \rightarrow \tilde{R})$$

$$= \int \mu_{\text{moreorless}\tilde{P}'}(x) \circ (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y))$$



Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$\begin{aligned} &= \int \mu_{\text{moreorless}\tilde{R}}(y) \circ (\int \mu_{\tilde{R}}(y)) \\ &= \int \mu_{\text{moreorless}\tilde{R}}(y) \wedge (\int \mu_{\tilde{R}}(y)) \end{aligned}$$

Using specialization

$$\begin{aligned} &= \int \mu_{\text{moreorless}\tilde{R}}(y) \\ &= y \text{ is more or less } \tilde{R} \end{aligned}$$

intuition III'-1 satisfied.

#### 4.2.6 In the case of intuition III'-2

$$\begin{aligned} &(\tilde{P}' \rightarrow \tilde{R}) \circ \text{more or less } \tilde{R} \\ &= (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\text{moreorless}\tilde{R}}(y) \end{aligned}$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$\begin{aligned} &= \int \mu_{\tilde{P}'}(x) \circ \int \mu_{\text{moreorless}\tilde{P}'}(x) \\ &= \int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\text{moreorless}\tilde{P}'}(x) \end{aligned}$$

Using specialization

$$\begin{aligned} &= \int \mu_{\text{moreorless}\tilde{P}'}(x) \\ &= x \text{ is more or less } \tilde{P}' \end{aligned}$$

intuition II'-2 satisfied.

#### 4.2.7 In the case of intuition IV'-1

$$\begin{aligned} &\text{not } \tilde{P}' \circ (\tilde{P}' \rightarrow \tilde{R}) \\ &= \int \mu_{\text{not}\tilde{P}'}(x) \circ (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y)) \end{aligned}$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$

$$\begin{aligned} &= \int \mu_{\text{not}\tilde{R}}(y) \circ (\int \mu_{\tilde{R}}(y)) \\ &= \int \mu_{\text{not}\tilde{R}}(y) \wedge (\int \mu_{\tilde{R}}(y)) \end{aligned}$$

Using specialization

$$\begin{aligned} &= \int \mu_{\text{not}\tilde{R}}(y) \\ &= y \text{ is not } \tilde{R} \end{aligned}$$

intuition IV'-1 satisfied.

#### 4.2.8 In the case of intuition IV'-2

$$\begin{aligned} &(\tilde{P}' \rightarrow \tilde{R}) \circ \text{not } \tilde{R} \\ &= (\int \mu_{\tilde{P}'}(x) \rightarrow \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\text{not}\tilde{R}}(y) \end{aligned}$$

Considering  $\tilde{P}' \rightarrow \tilde{R} = \tilde{P}'$

Considering  $\tilde{R} = \tilde{P}'$   
 $= \int \mu_{\tilde{P}'}(x) \circ \int \mu_{\text{not}\tilde{P}'}(x)$   
 $= \int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\text{not}\tilde{P}'}(x)$   
Using specialization  
 $= \int \mu_{\text{not}\tilde{P}'}(x)$   
 $= x \text{ is not } \tilde{P}'$   
intuition IV'-2 satisfied.

Criteria-1 is suitable for I'-1, I'-2, II'-1, II'-2, III'-1, III'-2, IV'-1 and IV'-2.

## 5. Fuzzy Granular Propositions with Truth Variables

Zadeh [16] defined quantification of true variables as composition of fuzzy set and true variables.

The evidence is granular if it consists of collection of propositions,

$$E = \{g_1, g_2, \dots, g_n\}$$

$$g_1 = x_1 \text{ is } \tilde{A}_1 \text{ is } \lambda_1$$

$$g_2 = x_2 \text{ is } \tilde{A}_2 \text{ is } \lambda_2$$

...

$$g_n = x_n \text{ is } \tilde{A}_n \text{ is } \lambda_n$$

Suppose we have granular propositions

$$g_1 = \text{Rama is very young is true}$$

$$g_2 = \text{Rama is young is very true}$$

$$g_3 = \text{Sita is beautiful is very true}$$

What is the fuzziness of the granular fuzzy propositions?

The granular fuzzy proposition is "x is  $\tilde{A}$  is  $\lambda$ ".

Where  $\lambda$  is true, false, very true, more or less false, very true etc.

The fuzzy granular propositions may contain "if ... then ... else ..."  
and "and/or".

$$\text{If } x \text{ is } \tilde{P} \text{ then } y \text{ is } \tilde{Q} \text{ else } y \text{ is } \tilde{R} \text{ is } \lambda$$

$$\text{If } x \text{ is } \tilde{P} \text{ and } x \text{ is } \tilde{Q} \text{ or } x \text{ is } \tilde{R} \text{ then } y \text{ is } \tilde{S} \text{ is } \lambda$$

**Definition 5.1** *The quantification of fuzzy true variables for fuzzy set of fuzzy proposition of the type "x is A is  $\lambda$ " is defined as  $\mu_A^{-1}(x) \circ \lambda$ , where  $\mu_A(x)^{-1}$  is inverse of comparability function of A, "o" is composition and  $\lambda$  is fuzzy true variable like true, false, very true etc.*

**Definition 5.2** The composition of fuzzy true variables for “a two fold fuzzy set” of fuzzy proposition of the type “ $x$  is  $\tilde{A}$  is  $\lambda$ ” may be defined as

$$\tilde{A}(x)_\lambda = \mu_{\tilde{A}}(x)_\lambda = \{\mu_A^{true}(x), \mu_A^{false}(x)\} \circ \lambda$$

where quantification of true variable applied on respective true functions. i.e.,

$$\tilde{A}(x)_{\lambda_1} = \{\mu_A(x)_{\lambda_1}, \mu_A^{false}(x)\}, \text{ where } \lambda_1 = \text{not true, very true, more or less true e.tc.}$$

For instance,  $\lambda_1 = \text{very true}$

$$\tilde{A}(x)_{\lambda_1} = \{\mu_A(x)^2, \mu_A^{false}(x)\}$$

$$\tilde{A}(x)_{\lambda_2} = \{\mu_A(x)^{true}, \mu_A^{\mu_2}(x)\}, \text{ where } \lambda_2 = \text{not false, very false, more or less false e.tc.}$$

For instance,  $\lambda_2 = \text{more or less false}$

$$\tilde{A}(x)_{\lambda_1} = \{\mu_A(x)^{true}, \mu_A^{0.2}(x)\}$$

The true functional modification of fuzzy proposition “ $x$  is  $\tilde{A}$  is very true” is given

$$\{\mu_A^{true}(x), \mu_A^{false}(x)\} \circ \text{very true} = \{\mu_{very A}^{true}(x), \mu_A^{false}(x)\}$$

The true functional modification of fuzzy proposition “ $x$  is  $\tilde{A}$  is very false” is given

$$\{\mu_A^{true}(x), \mu_A^{false}(x)\} \circ \text{very false} = \{\mu_A^{true}(x), \mu_{very A}^{false}(x)\},$$

The true functional modification of fuzzy proposition “ $x$  is tall is very true” is given as

$$\begin{aligned} \tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\ &\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}, \\ \text{very } \tilde{\text{tall}} &= \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, \\ &\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}. \end{aligned}$$

The true functional modification of fuzzy proposition “ $x$  is tall is very false” is given as

$$\begin{aligned} \tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\ &\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}, \\ \text{very } \tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\ &\quad 0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}. \end{aligned}$$

The nested fuzzy propositions of the form

$$x \text{ is } \tilde{A} \text{ is } (\lambda_1 \text{ is } (\lambda_2 \dots \text{ is } \lambda_n)) = x \text{ is } \tilde{A} \circ \lambda_1 \circ \lambda_2 \circ \dots \circ \lambda_n.$$

Consider quantification of true variables for fuzzy inference Type-1

If  $x$  is  $\tilde{P}$  and  $x$  is  $\tilde{Q}$  or  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$  is  $\lambda$   
 $x$  is  $\tilde{P}_1$  and  $x$  is  $\tilde{Q}_1$  or  $x$  is  $\tilde{R}_1$

---

$y$  is ?

The fuzzy inference is given for Type-1 using generalization and specialization

If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{S}$  is  $\lambda$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

If  $x$  is  $\tilde{Q}$  then  $y$  is  $\tilde{S}$  is  $\lambda$   
 $x$  is  $\tilde{Q}_1$

---

$y$  is ?

If  $x$  is  $\tilde{R}$  then  $y$  is  $\tilde{S}$  is  $\lambda$   
 $x$  is  $\tilde{R}_1$

---

$y$  is ?

Confider fuzzy inference Type-2

If  $x$  is  $\tilde{P}$  then  $y$  is  $\tilde{Q}$  else  $y$  is  $\tilde{R}$  is  $\lambda$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

The fuzzy inference is given for Type-2 using generalization and specialization

indent (if  $x$  is  $\tilde{P}$  then  $x$  is  $\tilde{Q}$ ) is  $\lambda$   
 $x$  is  $\tilde{P}_1$

---

$y$  is ?

$$\begin{array}{l}
(\text{if } x \text{ is } P' \text{ then } x \text{ is } \tilde{R}) \text{ is } \lambda \\
x \text{ is } \tilde{P}_1 \\
\hline
y \text{ is ?}
\end{array}$$

From fuzzy conditional inference Type-1 and Type-2, two criteria may be given as

$$\begin{array}{l}
\textbf{Criteria-}I_\lambda - 1 \\
\text{If } x \text{ is } \tilde{P} \text{ then } y \text{ is } \tilde{S} \text{ is } \lambda \\
x \text{ is } \tilde{P}_1 \\
\hline
y \text{ is ?}
\end{array}$$

$$\begin{array}{l}
\textbf{Criteria-}I_\lambda - 2 \\
(\text{if } x \text{ is } P' \text{ then } x \text{ is } \tilde{R}) \text{ is } \lambda \\
x \text{ is } \tilde{P}'_1 \\
\hline
y \text{ is ?}
\end{array}$$

The fuzzy inference is drawing a conclusion from fuzzy propositions.  
The fuzzy intuitions for Criteria- $I_\lambda - 1$ .

Fuzzy Conditional Inference is straight forward based on verification of fuzzy intuitions for Criteria-1

Table 3: Fuzzy inference for Criteria- $I_\lambda - 1$ .

Intuition	Proposition	Inference
$I_\lambda - 1$	$x$ is $\tilde{P} \circ \lambda$	$y$ is $\tilde{S}_\lambda$
$I_\lambda - 2$	$y$ is $\tilde{S} \circ \lambda$	$x$ is $\tilde{P}_\lambda$
$II_\lambda - 1$	$x$ is very $\tilde{P} \circ \lambda$	$y$ is very $\tilde{S}_\lambda$
$II_\lambda - 2$	$y$ is very $\tilde{S} \circ \lambda$	$x$ is very $\tilde{P}_\lambda$
$III_\lambda - 1$	$x$ is more or less $\tilde{P} \circ \lambda$	$y$ is more or less $\tilde{S}_\lambda$
$III_\lambda - 2$	$y$ is more or less $\tilde{S} \circ \lambda$	$x$ is more or less $\tilde{P}_\lambda$
$IV_\lambda - 1$	$x$ is not $\tilde{P} \circ \lambda$	$y$ is not $\tilde{S}_\lambda$
$IV_\lambda - 2$	$y$ is not $\tilde{S} \circ \lambda$	$x$ is not $\tilde{P}_\lambda$

Criteria-1 is suitable for  $I_\lambda - 1, I_\lambda - 2, II_\lambda - 1, II_\lambda - 2, III_\lambda - 1, III_\lambda - 2, IV_\lambda - 1$  and  $IV_\lambda - 2$ .

The fuzzy intuitions for Criteria- $I_\lambda - 2$ .

$I'_\lambda - 1$   
 if  $x$  is  $\tilde{P}'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $x$  is  $\tilde{P}'$

---

$y$  is  $\tilde{R}_\lambda$

$I'_\lambda - 2$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $y$  is  $\tilde{R}$

---

$x$  is  $\tilde{P}'_\lambda$

$II'_\lambda - 1$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $x$  is very  $\tilde{P}'$

---

$y$  is very  $\tilde{R}_\lambda$

$II'_\lambda - 2$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $y$  is very  $\tilde{R}$

---

$x$  is very  $\tilde{P}'_\lambda$

$III'_\lambda - 1$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $x$  is more or less  $\tilde{P}'$

---

$y$  is more or less  $\tilde{R}_\lambda$

$III;_\lambda - 2$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 is more or less  $\tilde{R}$

---

$y$  is more or less  $\tilde{P}'_\lambda$

$IV'_\lambda - 1$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $x$  is not  $\tilde{P}'$

---

$y$  is not  $\tilde{R}_\lambda$

$IV'_\lambda - 2$   
 if  $x$  is  $P'$  then  $y$  is  $\tilde{R}$  is  $\lambda$   
 $y$  is not  $\tilde{R}$

---

$x$  is not  $\tilde{P}'_\lambda$

The inference is given for Criteria-2 according to intuitions.

Table 4: Fuzzy inference for Criteria- $I_\lambda - 2$ .

Intuition	Proposition	Inference
$I'_\lambda - 1$	$x$ is $\tilde{P}'$	$y$ is $\tilde{R}_\lambda$
$I'_\lambda - 2$	$y$ is $\tilde{R}$	$x$ is $\tilde{P}'_\lambda$
$II_\lambda - 1$	$x$ is very $\tilde{P}'$	$y$ is very $\tilde{R}_\lambda$
$II'_\lambda - 2$	$y$ is very $\tilde{R}$ o	$x$ is very $\tilde{P}'$
$III'_\lambda - 1$	$x$ is more or less $\tilde{P}'$	$y$ is more or less $\tilde{R}_\lambda$
$III'_\lambda - 2$	$y$ is More or less $\tilde{R}$	is more or less $\tilde{P}'_\lambda$
$IIV'_\lambda - 1$	$x$ is not $\tilde{P}'$	$y$ is not $\tilde{R}_\lambda$
$IV'_\lambda - 1$	$y$ is not $\tilde{R}$ o is	$x$ is not $\tilde{P}'_\lambda$

Fuzzy Conditional Inference is straight forward based on verification of fuzzy intuitions for Criteria-2

Criteria-1 is suitable for  $I'_\lambda - 1, I'_\lambda - 2, II'_\lambda - 11, II'_\lambda - 2, III'_\lambda - 1, III'_\lambda - 2, IV'_\lambda - 1$  and  $IV'_\lambda - 2$ .

## 6. Fuzzy Certainty Factor

The fuzzy certainty factor (FCF) shall made as single fuzzy membership functions with two fuzzy membership functions to eliminate the conflict of evidence between “true ” and “false”.

**Definition 4.1** *The FCF of  $\mu_{\tilde{A}}$  for propositions “ $x$  is  $\tilde{A}$ ” is characterized by its membership function  $\mu_{\tilde{A}}^{FCF}(x) \rightarrow [0, 1]$ , where  $\mu_{\tilde{A}}^{FCF}(x) = \{\mu_{\tilde{A}}^{true}(x) - \mu_{\tilde{A}}^{false}(x)\}/x$ ,*

*$\mu_{\tilde{A}}^F CF(x) < 0, \mu_{\tilde{A}}^F CF(x) = 0$  and  $\mu_{\tilde{A}}^F CF(x) > 0$  are the redundant, insufficient and sufficient respectively.*

The FCF will compute the conflict of evidence of the incomplete information.

For Example

$$\mu_{young}^{true}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\}$$



$$\mu_{young}^{false}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\}$$

The fuzzy intuitions may be studied with fuzzy certainty factor as studied in the above.

## 7. Application for Fuzzy Intuitions

The Business intelligence needs reasoning. The Business data is defied with fuzziness with linguistic variables.

If  $x$  is *Demand* then  $y$  is *Profit*  
 $x$  is very Demand

---

$y$  is very Profit

Consider the fuzzy data sets.

Table 5: Fuzzy data set Demand

Item No.	Demand
Item1	(0.4,0.1)
Item2	(0.6,0.1)
Item3	(0.9,0.2)
Item4	(1.0,0.2)
Item5	(1.0,0.0)

if  $x$  is Demand then  $x$  is Profit

The fuzzy conditional inference using (3.1) given by

Table 6: Fuzzy data set Profit

Item No.	Profit
Item1	(0.4,0.1)
Item2	(0.6,0.1)
Item3	(0.9,0.2)
Item4	(1.0,0.2)
Item5	(1.0,0.0)

Table 7: Fuzzy data set very Profit

Item No.	very Profit
Item1	(0.16,0.01)
Item2	(0.36,0.01)
Item3	(0.81,0.04)
Item4	(1.0,0.04)
Item5	(1.0,0.0)

If  $x$  is *not Demand* then  $y$  is *Loss*  
 $x$  is not more or less Demand

---

$y$  is more or less Loss

Consider the fuzzy data sets for production.

Table 8: Fuzzy data set not Demand

Item No.	not Demand
Item1	(0.6,0.9)
Item2	(0.4,0.9)
Item3	(0.1,0.8)
Item4	(0.0,0.8)
Item5	(0.0,1.0)

The fuzzy conditional inference using (3.1) given by

Table 9: Fuzzy data set Loss

Item No.	loss
Item1	(0.6,0.9)
Item2	(0.4,0.9)
Item3	(0.1,0.8)
Item4	(0.0,0.8)
Item5	(0.0,1.0)

If  $x$  is *Demand* then  $y$  is *Profit* is very true  
 $x$  is *very Demand*

---

$y$  is *Profit* is very true

Table 10: Fuzzy data set Profit is very true

Item No.	very Profit
Item1	(0.09,0.1)
Item2	(0.25,0.1)
Item3	(0.49,0.2)
Item4	(1,0,0,2)
Item5	(1.0,0.0)

If  $x$  is *not Demand* then  $y$  is *Loss* is more or less is true  
 $x$  is *not Demand*

---

$y$  is *Loss* is more less is true

Table 11: Fuzzy data set Loss is more or less true

Item No.	more or less Loss
Item1	(0.09,0.1)
Item2	(0.25,0.1)
Item3	(0.49,0.2)
Item4	(1,0,0,2)
Item5	(1.0,0.0)

## 7. Conclusion

We considered fuzzy intuitions and fuzzy granular intuitions of the form “if  $\dots$  then  $\dots$  else  $\dots$ ”, “if  $\dots$  then  $\dots$ ” and “and/or” and fuzzy inference

is studied using twofold fuzzy sets. we found the all fuzzy intuitions are satisfied with our method.

### **Acknowledgments**

The Author would like to thank the Sri Venkateswara University, Tirupati, India authorities for providing facility to carry out this work.

### **References**

- [1] E.H.Mamdani, Application of Fuzzy Logic to Approximate Reasoning Using Linguistic Synthesis. IEEE Trans. Computers. vol.26, no.12, pp.1182-1191, 1977.
- [2] S. Fukami, M. Muzumoto, K. Tanaka, Some Considerations on Fuzzy Conditional Inference, Fuzzy Sets and Systems, vol.4, pp.243-273, 1980.
- [3] C. Howson, Successful Business Intelligence, McGraw-Hill, 2014.
- [4] N. Rescher, Many-valued Logic, McGraw-Hill, New York, 1969.
- [5] Poli Venkata Subba Reddy, M. Syam Babu, Some methods of reasoning for conditional propositions, Fuzzy Sets and Systems, vol.52, no.3, pp.229-250, 1992.
- [6] Poli Venkata Subba Reddy, Fuzzy conditional inference for medical diagnosis, Proceedings, Second International Conference on Fuzzy Theory and Technology, Durham, FT&T'93, vol.3, pp.193-195, 1993.
- [7] Poli Venkata subba reddy, Fuzzy logic based on Belief and Disbelief membership functions, Fuzzy Information and Engineering, Vol.9, no.4, pp.405-422, 2017.
- [8] L. A Zadeh, "Calculus of fuzzy Restrictions", In Fuzzy set s and their Applications to Cognitive and Decision Processes, L. A. Zadeh, King-Sun FU, Kokichi Tanaka and Masamich Shimura (Eds.), Academic Press, New York, pp.1-40, 1975.
- [9] L.A. Zadeh, Fuzzy sets. Information and Control, vol.8, pp.338-353., 1965.
- [10] A. Bochman, A logic for causal reasoning, Proceedings IJCAI2003, Morgan Kaufmann, 2003.