Workspace of a 3- PRRS type parallel manipulator

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Abstract-In this paper, methods of workspace analysis of a 3-PRRS type parallel manipulator are described. The equations of spheres and circles on these spheres, along which the center of the moving platform can move, are derived, and it is shown that the total reachable area of these spheres is the workspace of the considered parallel manipulator. Numerical examples of defining the workspace of the 3-PRRS type parallel manipulator are presented.

Keywords - parallel manipulator, moving and fixed platform, workspace.

I. INTRODUCTION

Most parallel manipulators with six degrees of freedom (DOF) have six legs [1-3]. Parallel manipulators with six DOF and three legs or tripods, in comparison with parallel manipulators with six legs or hexapods, have a larger workspace and less singular configurations. The following types of tripods are known: 3-URS [4], 3-ESR [5], 3-PRPS [6], 3-RES [7-9], 3-PPSR [10], 3-PRPS [11], 3-CRS [12], 3-CCC [13].

We have developed a novel parallel manipulator - tripod of a 3-PRRS type with six DOF (Fig. 1) which in comparison with the existing tripods has a large workspace. In [14, 15], the geometry of this parallel manipulator was studied and the inverse and direct kinematics were solved.

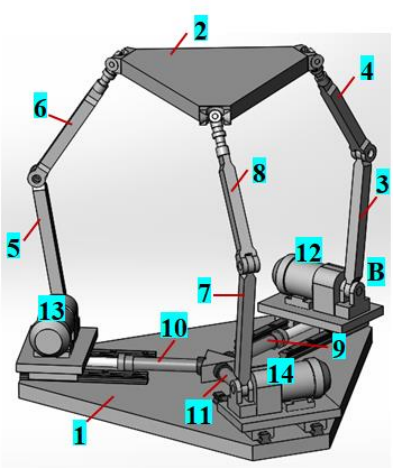

Fig.1. 3-PRRS type parallel manipulator

This paper is devoted to a workspace analysis of a 3-PRRS type parallel manipulator.

II. WORKSPACE ANALYSIS

To analysis the geometry, kinematics and workspace of a 3-PRRS type parallel manipulator, two coordinate systems *UVW* and *XYZ* are fixed to each element of kinematic pairs, the *W* and *Z* axes of which are directed along the axis of rotational and translational motions of the kinematic pairs, and the *U* and *X* axes are directed along the direction of the perpendicular drawn from the *W* axis to the *Z* axis. Figure 2 shows one of the legs 3-4 of the parallel manipulator with the chosen coordinate systems.

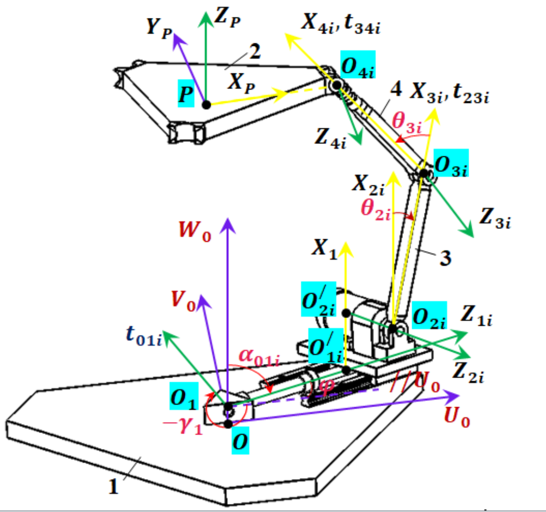

Fig.2. Leg 3-4 of the parallel manipulator

In [14], the transformation matrices between the chosen coordinate systems, having six parameters, were derived and the following expressions for determining the coordinates of the spherical joints of the moving platform 2 in the absolute coordinate system $OU_0V_0W_0$, were obtained

$$\left.\begin{aligned}
&-b_i c\gamma_i + s_i s\gamma_i - L_{1i} s\gamma_i s\theta_{2i} - \\
&-L_{2i} s\gamma_i s(\theta_{2i}+\theta_{3i}) = U_{O_{4i}} \\
&-b_i s\gamma_i - s_i c\gamma_i + L_{1i} c\gamma_i s\theta_{2i} + \\
&+L_{2i} c\gamma_i s(\theta_{2i}+\theta_{3i}) = V_{O_{4i}} \\
&c_i + a_i + L_{1i} c\theta_{2i} + \\
&+L_{2i} c(\theta_{2i}+\theta_{3i}) = W_{O_{4i}}
\end{aligned}\right\}, i = 1,2,3, \qquad (1)$$

where a_i, b_i, c_i, γ_i are parameters, characterizing the geometry of links, L_{1i} and L_{2i} are the lengths of the legs, c and s denote cos and sin.

Multiplying the first and the second equations of the system (1) on $\mathrm{c}\gamma_i$ and $\mathrm{s}\gamma_i$, and add them, we obtain the following three equations of planes along which three RRS type dyads $O_{2i}O_{3i}O_{4i}$ move

$$\mathrm{c}\gamma_i \cdot U_{O_{4i}} + \mathrm{s}\gamma_i \cdot V_{O_{4i}} + b_i = 0, \; (i = 1, 2, 3). \qquad (2)$$

Adding the squares of the first and the third equations of the system (1), we obtain

$$\begin{aligned}&(W_{O_{4i}} - c_i - L_{1i} \cdot \mathrm{c}\theta_{2i})^2 + \\ &+ (s_i - \frac{U_{O_{4i}} + b_i \cdot c\gamma_i}{s\gamma_i} - L_{1i} \cdot \mathrm{s}\theta_{2i})^2 - L_{2i}^2 = 0.\end{aligned} \qquad (3)$$

Determining b_i from system (2), and substituting into the equations (3), we obtain

$$(X_{2i} - L_{1i} \cdot \mathrm{c}\theta_{2i})^2 + (Y_{2i} - L_{1i} \cdot \mathrm{s}\theta_{2i})^2 - L_{2i}^2 = 0, \qquad (4)$$

where X_{2i}, Y_{2i} are the positions of the absolute coordinate system $OU_0V_0W_0$ in the local coordinate system $O_{2i}X_{2i}Y_{2i}Z_{2i}$ defined by the equations

$$\left.\begin{aligned}X_{2i} &= W_{O_{4i}} - c_i - a_i, \\ Y_{2i} &= s_i - U_{O_{4i}} \cdot \mathrm{s}\gamma_i + V_{O_{4i}} \cdot \mathrm{c}\gamma_i\end{aligned}\right\}. \qquad (5)$$

Equations (4) are reduced to the form

$$X_{2i} \cdot \mathrm{c}\theta_{2i} + Y_{2i} \cdot \mathrm{s}\theta_{2i} - \frac{X_{2i}^2 + Y_{2i}^2 + L_{1i}^2 - L_{2i}^2}{2 \cdot L_{1i}} = 0, \qquad (6)$$

that have solutions in the case when [16]

$$\left(\frac{X_{2i}^2 + Y_{2i}^2 + L_{1i}^2 - L_{2i}^2}{2 \cdot L_{1i}}\right)^2 - (X_{2i}^2 + Y_{2i}^2) \le 0. \qquad (7)$$

Equating the equations (7) to zero, we obtain two equations of circles

$$\left.\begin{aligned}X_{2i}^2 + Y_{2i}^2 &= (L_{2i} + L_{1i})^2 \\ X_{2i}^2 + Y_{2i}^2 &= (L_{2i} - L_{1i})^2\end{aligned}\right\}, \qquad (8)$$

that are the outer and inner boundaries of the workspace of the dyads $O_{2i}O_{3i}O_{4i}$.

Let write the equations (8) in the absolute coordinate system $OU_0V_0W_0$

$$\left.\begin{aligned}&(U_0 - O_{X_{2i}})^2 + (V_0 - O_{Y_{2i}})^2 + (W_0 - O_{Z_{2i}})^2 = \\ &\qquad = (L_{2i} - L_{1i})^2 \\ &(U_0 - O_{X_{2i}})^2 + (V_0 - O_{Y_{2i}})^2 + (W_0 - O_{Z_{2i}})^2 = \\ &\qquad = (L_{2i} + L_{1i})^2\end{aligned}\right\}, \qquad (9)$$

where $O_{X_{2i}}, O_{Y_{2i}}, O_{Z_{2i}}$ are the coordinates of the centers of the local coordinate system $O_{2i}X_{2i}Y_{2i}Z_{2i}$ relative to the absolute coordinate system $O_0U_0V_0W_0$. Fig. 3 shows the graphs of the circles (9) with the following parameters [15]: $a_i = 15$, $b_i = 8$, $c_i = 5$, $L_{1i} = 60$, $L_{2i} = 70$ of the legs and $h = 43$, $\gamma_1 = 90^0 + \varphi$, $\gamma_2 = 210^0 + \varphi$, $\gamma_3 = \varphi - 30^0$, $d = h\sqrt{3}, \varphi = 10^\circ 72'$ of the moving and fixed platforms.

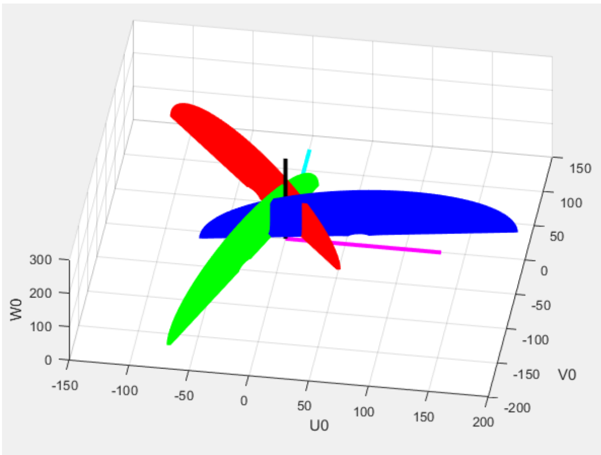

Fig.3. Graphs of the dyads $O_{2i}O_{3i}O_{4i}$ circles

Determine the coordinates of the center (point P) of the moving platform 2 by the following equations

$$\left.\begin{aligned}&X_P = U_{O_{4i}} - U_{O_{4Pi}} \cdot \mathrm{s}\gamma_i \cdot \mathrm{s}(\theta_{2i} + \theta_{3i}) - \\ &- V_{O_{4Pi}} \cdot \mathrm{s}\gamma_i \cdot \mathrm{c}(\theta_{2i} + \theta_{3i}) - W_{O_{4Pi}} \cdot \mathrm{c}\gamma_i \\ &Y_P = V_{O_{4i}} + U_{O_{4Pi}} \cdot \mathrm{c}\gamma_i \cdot \mathrm{s}(\theta_{2i} + \theta_{3i}) + \\ &+ V_{O_{4Pi}} \cdot \mathrm{c}\gamma_i \cdot \mathrm{c}(\theta_{2i} + \theta_{3i}) - W_{O_{4Pi}} \cdot \mathrm{s}\gamma_i \\ &Z_P = W_{O_{4i}} + U_{O_{4Pi}} \cdot \mathrm{c}(\theta_{2i} + \theta_{3i}) - \\ &- V_{O_{4Pi}} \cdot \mathrm{s}(\theta_{2i} + \theta_{3i})\end{aligned}\right\}, \qquad (10)$$

where $U_{O_{4Pi}}, V_{O_{4Pi}}, W_{O_{4Pi}}$ are the coordinates of the center P of the moving platform in the local coordinate systems $O_{4i}X_{4i}Y_{4i}Z_{4i}$.

Multiplying the first equations of the system (10) on $\mathrm{s}\gamma_i$, and the second equations on $-\mathrm{c}\gamma_i$, and adding the first and second equations, we obtain

$$\left.\begin{array}{l}(X_P-U_{O_{4i}})\cdot s\gamma_i-(Y_P-V_{O_{4i}})\cdot c\gamma_i=\\=-U_{O_{4Pi}}\cdot s\gamma_i\cdot s(\theta_{2i}+\theta_{3i})-\\-V_{O_{4Pi}}\cdot s\gamma_i\cdot c(\theta_{2i}+\theta_{3i})\\Z_P-W_{O_{4i}}=U_{O_{4Pi}}\cdot c(\theta_{2i}+\theta_{3i})-\\-V_{O_{4Pi}}\cdot s(\theta_{2i}+\theta_{3i})\end{array}\right\}. \quad (11)$$

Multiplying the first equations of the system (11) on $s(\theta_{2i}+\theta_{3i})$, and the second equations on $-c(\theta_{2i}+\theta_{3i})$, and add for them, we obtain

$$\begin{array}{l}U_{O_{4Pi}}=\left[(Y_P-V_{O_{4i}})\cdot c\gamma_i-(X_P-U_{O_{4i}})\cdot s\gamma_i\right]\cdot\\\cdot s(\theta_{2i}+\theta_{3i})+(Z_P-W_{O_{4i}})\cdot c(\theta_{2i}+\theta_{3i}).\end{array} \quad (12)$$

To determine $V_{O_{4Pi}}$, we also add the two equations of the system (11), previously multiplying the first equations on $c(\theta_{2i}+\theta_{3i})$, and the second equations on $s(\theta_{2i}+\theta_{3i})$, and obtain

$$\begin{array}{l}V_{O_{4Pi}}=\left[(Y_P-V_{O_{4i}})\cdot c\gamma_i-(X_P-U_{O_{4i}})\cdot s\gamma_i\right]\cdot\\\cdot c(\theta_{2i}+\theta_{3i})-(Z_P-W_{O_{4i}})\cdot s(\theta_{2i}+\theta_{3i}).\end{array} \quad (13)$$

To determine $W_{O_{4Pi}}$, we add the first and the second equations of the system (10), previously multiplying the first equation on $c\gamma_i$, and the second equation on $s\gamma_i$, and obtain

$$W_{O_{4Pi}}=-(X_P-U_{O_{4i}})\cdot c\gamma_i-(Y_P-V_{O_{4i}})\cdot s\gamma_i. \quad (14)$$

Since the distances from the center P of the moving platform to the centers of the spherical joints are equal to h in the local coordinate systems $O_{4i}X_{O_{4i}}Y_{O_{4i}}Z_{O_{4i}}$ (Fig. 4), the following equality is rightly

$$U_{O_{4Pi}}{}^2+V_{O_{4Pi}}{}^2+V_{O_{4Pi}}{}^2=h^2. \quad (15)$$

Substituting the equations (12), (13), (14) into the equation (15), we obtain three equations of the sphere in the absolute coordinate system $OU_0V_0W_0$ along which the point P moves

$$\begin{array}{c}(U_{O_{4Pi}}-X_P)^2+(V_{O_{4Pi}}-Y_P)^2+\\+(V_{O_{4Pi}}-Z_P)^2=h^2.\end{array} \quad (16)$$

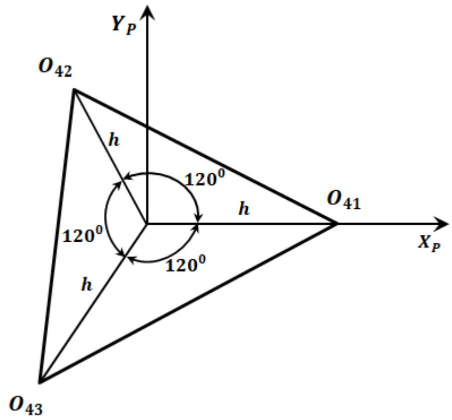

Fig.4. Moving platform

Thus, the legs of the parallel manipulator move in circles (9) relative to the absolute coordinate system $OU_0V_0W_0$, and the center of the moving platform moves in three spheres relative to points O_{4i} according to the equations (16). The total reachable area of three spheres is the workspace of the parallel manipulator.

Let consider the algorithm for determining the workspace of the considered parallel manipulator by solving the inverse kinematics problem. In [14], the transformation matrix $\mathbf{T}_{OP}$ between the local coordinate system $PX_PY_PZ_P$ of the moving platform and the absolute coordinate system $OU_0V_0W_0$ was derived

$$\mathbf{T}_{OP}=\left[\begin{array}{c|c|c|c}1&0&0&0\\\hline\begin{array}{l}a_{OP}\cdot c\gamma_{OP}+\\+b_{OP}\cdot s\gamma_{OP}\cdot s\alpha_{OP}\end{array}&\begin{array}{l}c\gamma_{OP}\cdot c\beta_{OP}-\\-s\gamma_{OP}\cdot c\alpha_{OP}\cdot s\beta_{OP}\end{array}&\begin{array}{l}-c\gamma_{OP}\cdot s\beta_{OP}-\\-s\gamma_{OP}\cdot c\alpha_{OP}\cdot c\beta_{OP}\end{array}&s\gamma_{OP}\cdot s\alpha_{OP}\\\hline\begin{array}{l}a_{OP}\cdot s\gamma_{OP}-\\-b_{OP}\cdot c\gamma_{OP}\cdot s\alpha_{OP}\end{array}&\begin{array}{l}s\gamma_{OP}\cdot c\beta_{OP}+\\+c\gamma_{OP}\cdot c\alpha_{OP}\cdot s\beta_{OP}\end{array}&\begin{array}{l}c\gamma_{OP}\cdot c\alpha_{OP}\cdot c\beta_{OP}-\\-s\gamma_{OP}\cdot s\beta_{OP}\end{array}&-c\gamma_{OP}\cdot s\alpha_{OP}\\\hline c_{OP}+b_{OP}\cdot c\alpha_{OP}&s\alpha_{OP}\cdot s\beta_{OP}&s\alpha_{OP}\cdot c\beta_{OP}&c\alpha_{OP}\end{array}\right] \quad (17)$$

or

$$\mathbf{T}_{OP}=\left[\begin{array}{c|c|c|c}1&0&0&0\\\hline X_P&t_{11}&t_{12}&t_{13}\\\hline Y_P&t_{21}&t_{22}&t_{23}\\\hline Z_P&t_{31}&t_{32}&t_{33}\end{array}\right]. \quad (18)$$

In the inverse kinematics problem, the matrix (17) or (18) is given. Let define the coordinates of the centers of the spherical joints in the absolute coordinate system $OU_0V_0W_0$ through the matrix $\mathbf{T}_{OP}$

$$\begin{bmatrix} 1 \\ U_{O_{41}} \\ V_{O_{41}} \\ W_{O_{41}} \end{bmatrix} = \mathbf{T}_{OP} \cdot \begin{bmatrix} 1 \\ h \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ U_{O_{42}} \\ V_{O_{42}} \\ W_{O_{42}} \end{bmatrix} = \mathbf{T}_{OP} \cdot \begin{bmatrix} 1 \\ -h/2 \\ h\cdot\sqrt{3}/2 \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} 1 \\ U_{O_{43}} \\ V_{O_{43}} \\ W_{O_{43}} \end{bmatrix} = \mathbf{T}_{OP} \cdot \begin{bmatrix} 1 \\ -h/2 \\ -h\cdot\sqrt{3}/2 \\ 0 \end{bmatrix}. \qquad (19)$$

Substituting the coordinates of the centers of the spherical joints from the equation (19) into the equations (2), we obtain a system of equations connecting the components $t_{11}, t_{12}, t_{21}, t_{22}$ of the direction cosines of the moving coordinate system $PX_PY_PZ_P$

$$\left.\begin{aligned} & t_{11}\cdot h\cdot\frac{c\gamma_1}{s\gamma_1} + t_{21}\cdot h + \frac{c\gamma_1}{s\gamma_1}\cdot X_P + Y_P + \frac{b_i}{s\gamma_1} = 0 \\ & -\frac{h}{2}\cdot\frac{c\gamma_2}{s\gamma_2}\cdot t_{11} + \frac{h\cdot\sqrt{3}}{2}\cdot\frac{c\gamma_2}{s\gamma_2}\cdot t_{12} - \frac{h}{2}\cdot t_{21} + \\ & + \frac{h\cdot\sqrt{3}}{2}\cdot t_{22} + \frac{c\gamma_2}{s\gamma_2}\cdot X_P + Y_P + \frac{b_i}{s\gamma_2} = 0 \\ & \frac{h}{2}\cdot\frac{c\gamma_3}{s\gamma_3}\cdot t_{11} - \frac{h\cdot\sqrt{3}}{2}\cdot\frac{c\gamma_3}{s\gamma_3}\cdot t_{12} - \frac{h}{2}\cdot t_{21} - \\ & - \frac{h\cdot\sqrt{3}}{2}\cdot t_{22} + \frac{c\gamma_3}{s\gamma_3}\cdot X_P + Y_P + \frac{b_i}{s\gamma_3} = 0 \end{aligned}\right\}. \quad (20)$$

From the sum of the three equations of system (20) we obtain

$$t_{12} = \frac{-m_1\cdot t_{11} - m_3\cdot X_P - 3\cdot Y_P - m_4}{m_2}, \qquad (21)$$

where

$$m_1 = h\cdot(\frac{c\gamma_1}{s\gamma_1} - \frac{c\gamma_2}{2\cdot s\gamma_2} + \frac{c\gamma_3}{2\cdot s\gamma_3}),\ m_2 = \frac{h\cdot\sqrt{3}}{2}(\frac{c\gamma_2}{s\gamma_2} - \frac{c\gamma_3}{s\gamma_3}),$$

$$m_3 = \frac{c\gamma_1}{s\gamma_1} + \frac{c\gamma_2}{s\gamma_2} + \frac{c\gamma_3}{s\gamma_3}.$$

From the second equation of system (20), we subtract the third equation and obtain

$$t_{22} = \frac{-n_1\cdot X_P - n_2\cdot Y_P - n_3\cdot t_{11} - n_5}{n_4}, \qquad (22)$$

where

$n_1 = (c\gamma_1\cdot c\gamma_2\cdot s\gamma_3 + c\gamma_1\cdot c\gamma_3\cdot s\gamma_2 + 4\cdot c\gamma_2\cdot c\gamma_3\cdot s\gamma_1)\cdot n_6$,

$n_2 = 3\cdot(c\gamma_2\cdot s\gamma_1\cdot s\gamma_3 + c\gamma_3\cdot s\gamma_1\cdot s\gamma_2)\cdot n_6$,

$n_3 = h\cdot(c\gamma_1\cdot c\gamma_2\cdot s\gamma_3 + c\gamma_1\cdot c\gamma_3\cdot s\gamma_2 - 2\cdot c\gamma_2\cdot c\gamma_3\cdot s\gamma_1)\cdot n_6$,

$n_4 = h\cdot\sqrt{3}\cdot(c\gamma_3\cdot s\gamma_1\cdot s\gamma_2 - c\gamma_2\cdot s\gamma_1\cdot s\gamma_3)\cdot n_6$,

$n_5 = b_{12}\cdot(2\cdot c\gamma_2\cdot s\gamma_1 + 2\cdot c\gamma_3\cdot s\gamma_1 + c\gamma_2\cdot s\gamma_3 + c\gamma_3\cdot s\gamma_2)\cdot n_6$,

$n_6 = 1/(s\gamma_1\cdot c\gamma_3\cdot s\gamma_2 - s\gamma_1\cdot c\gamma_2\cdot s\gamma_3)$.

From the first equation of system (20) we determine t_{12}

$$t_{21} = \frac{1}{h}\cdot\left(-t_{11}\cdot h\cdot\frac{c\gamma_1}{s\gamma_1} - \frac{c\gamma_1}{s\gamma_1}\cdot X_P - Y_P - \frac{b_i}{s\gamma_1}\right). \quad (23)$$

Thus, we set t_{11}, and from the equations (21), (22), (23) determine t_{12}, t_{21}, t_{22}. The remaining components of the 3x3 rotation matrix are determined from the following condition

$$\left.\begin{aligned} t_{11}^2 + t_{21}^2 + t_{31}^2 = 1 \\ t_{12}^2 + t_{22}^2 + t_{32}^2 = 1 \end{aligned}\right\}. \qquad (24)$$

From the equation (24) and the orthogonal conditions we obtain

$$\left.\begin{aligned} & t_{31} = \pm\sqrt{1 - t_{11}^2 - t_{21}^2} \\ & t_{32} = \pm\sqrt{1 - t_{12}^2 - t_{22}^2} \\ & t_{13} = t_{21}\cdot t_{32} - t_{22}\cdot t_{31} \\ & t_{23} = -t_{11}\cdot t_{32} + t_{12}\cdot t_{31} \\ & t_{33} = t_{11}\cdot t_{22} - t_{12}\cdot t_{21} \end{aligned}\right\}. \qquad (25)$$

Therefore, all components of the guiding cosines of the moving coordinate system $PX_PY_PZ_P$ are expressed in terms of t_{11}. Next, we set the values of the coordinates X_P, Y_P, Z_P of the point *P*, and change the values of t_{11} from -1 to 1 with a certain step, and find the points with the coordinates $U_{O_{4i}}, V_{O_{4i}}, W_{O_{4i}}$ for which the following conditions are satisfied

$$\left.\begin{aligned} & (U_{O_{41}} - U_{O_{42}})^2 + (V_{O_{41}} - V_{O_{42}})^2 + \\ & + (W_{O_{41}} - W_{O_{42}})^2 = d^2 \\ & (U_{O_{41}} - U_{O_{43}})^2 + (V_{O_{41}} - V_{O_{43}})^2 + \\ & + (W_{O_{41}} - W_{O_{43}})^2 = d^2 \\ & (U_{O_{42}} - U_{O_{43}})^2 + (V_{O_{42}} - V_{O_{43}})^2 + \\ & + (W_{O_{42}} - W_{O_{43}})^2 = d^2 \end{aligned}\right\}, \qquad (26)$$

where d is the distance between the centers of the spherical joints.

If the conditions (26) are satisfied, then we solve the inverse kinematics problem, and in the case when $\theta_{2i} < 0$ и $\theta_{3i} > 0$, the computer program puts an point in a space. Fig. 5 shows the workspace of the considered parallel manipulator.

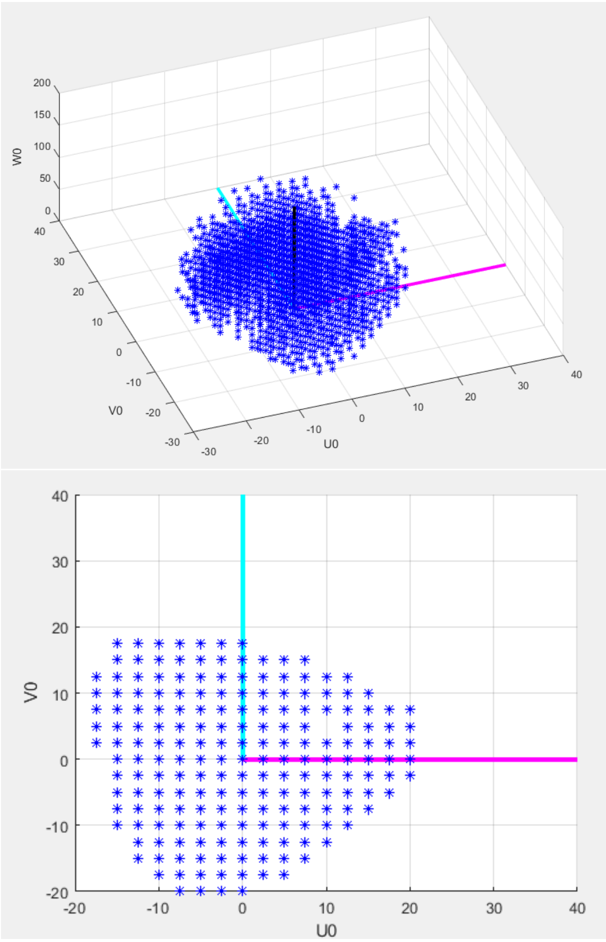

Fig.5. Workspace by invers kinematics

The workspace of this parallel manipulator can also be drawn according to the direct kinematics problem. To do this, we change the values of the angles θ_{2i} in the interval $\left[-\frac{\pi}{2}, 0\right]$ with a step $\frac{\pi}{18}$ in three cycles and draw the workspace (Fig. 6).

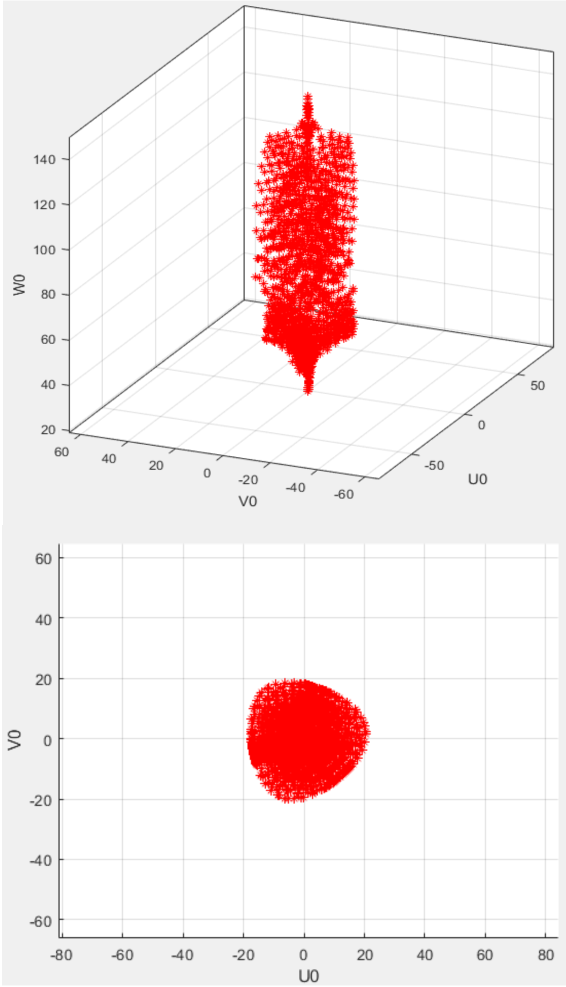

Fig.6. Workspace by direct kinematics

III. CONCLUSIONS

The workspace of a 3-PRRS type parallel manipulator is defined, in this paper. It is shown, that three legs of the parallel manipulator move in circles, and the center of the moving platform moves in three spheres relative to the centers of the spherical kinematic pairs. The total reachable area of the three spheres is the workspace of the considered parallel manipulator. The numerical results of the 3-PRRS type parallel manipulator's workspace analysis are obtained.

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