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Abstract In the auction market, allocation and pricing will affect participants' behavior, honesty, and the success of the auction. A proper mechanism will help to achieve higher utility. In view of the problem of allocation and pricing of resource such as cloud resource allocation and spectrum auction, this paper designs the TCD4GB mechanism based on the scene of combined double auction, which determines winners, allocates goods and calculates payments. The concept of unit difference is introduced into this paper in order to solve winner determination problem in the group-buying mechanism. The matched sellers who have the minimum cost is directly chosen to be the winning sellers in the process of selecting the winning buyers. The mechanism also calculates payment by using the unit difference of overlapping buyers and apportion it to the matching sellers through the idea of second-price in VCG mechanism. This mechanism avoids the higher utility of buyers when they report falsely. Through theoretical and simulation experiments, it is proved that the TCD4GB mechanism satisfies the economic attributes of individual rationality and budget balance.

Key words: combinatorial double auction; allocation and pricing; winner determination problem

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1 Introduction

Auction is a kind of method to transfer foods in market in which participants' profit should be considered. Pricing is the process of determining the cost charged from users who receive the goods.Setting up an appropriate pricing model will help to get higher profits. The fixed pricing mechanism does not have economic efficiency and cannot reflect the equilibrium price of market supply and demand. That is why a dynamic pricing mechanism based on combined double auction can make more reasonable pricing. In current market, especially in cloud computing services, most market-based allocation systems are biased towards suppliers in an unregulated trading environment. Paper[2]proposed a market based optimal resource allocation, which comprehensively considers the utilities of users and suppliers, and proved that the combined double auction model is more suitable for the problem of cloud computing resource allocation than the fixed pricing mechanism. Paper[3]put forward a combinatorial double auction market, which improved the utilities of suppliers and users in the market and reduces the waste of resources. In the actual market, there are many problem models such as the allocation of cloud resources. Many sellers offer goods for sale and many buyers have demand for the goods. These participants submit reports to the auctioneers and the auctioneer determines the final winners according to the reports of these participants. It can be seen that the combination double auction mechanism is more suitable for such auction problem.

In this paper, factor of sellers' volume discount is added to the combinatorial double auction, that is, when sellers set dynamic price, the issue of commodity allocation and pricing is realized. The goal is to realize the effective allocation of goods and to calculate the payment of the corresponding participants under the auction situation that allows multiple sellers to sell multiple discounted goods and multiple buyers combined bidding, and to ensure that the design mechanism satisfies the individual rationality, budget balance and incentive compatible economic attributes.

The rest of this paper is organized as follows. The second section introduces the relevant work of this study; the third section describes the model of the combined double auction mechanism and gives the relevant definitions; the fourth section gives the algorithm and explanation of the mechanism proposed in this paper and proves the properties by theoretical proof; the fifth section designs experiments to validate the mechanism; the last section summarizes this paper.

2 Problem description

The basic process of auction is shown in Figure 1. Sellers and buyers submit their own reports to the auctioneer (that is, the mechanism operator). The auctioneer executes the mechanism. The mechanism collects information from participants and determines winners and then assigns goods and calculates payments. The mechanism returns the result to participants and the auctioneer collects the payment of transaction goods from buyers and then pay to the sellers. The following is a detailed description of the specific contents and implementation of the mechanism.

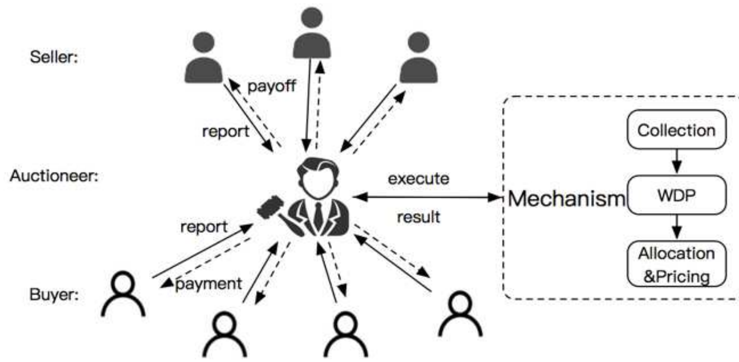

Fig. 1: Basic framework map of double auction

The seller set is $S = \{s_1, s_2, \ldots, s_m\}$. Each seller applies to the auctioneer to sell goods and report the relative information including the type of the goods, the quantity that sold, and price. There are K kinds of goods sold in auctions, which can be represented as $Kind = \{1, \ldots, K\}$. The report given by the seller $s_j(s_j \in S)$ is a triple $rs_j =< q_j^k, c_j^k, D_j >$. q_j^k indicates the number of commodity k sold by the seller s_j , and c_j^k indicates the unit price that the seller is willing to sell. The seller in this chapter acquiescence is honest and reliable. A single seller's commodities can only be assigned to a buyer. If the seller matches the buyer successfully, the seller will no longer sell the goods to other buyers. We define that the discount function of seller s_j is $D_j : qs_j^k \to d$, and qs_j^k means the number of commodity k sold by the seller s_j,and $D_j \in [0,1]$. On the report submitted by the seller s_j, here can be seen as the price function of the seller's report, the maximum number of sales is q_j^k, the initial price is c_j^k.When a certain quantity is met, the price is the price after the discount.

There are n buyers, and the buyers set is: $B = \{b_1, b_2, \ldots, b_n\}$. Buyers submit reports to the mechanism including the types of goods they want to buy, the quantity they purchase, and the price they are willing to pay. The report of buyer $b_i(b_i \in B)$ is $rb_i =< Q_i, B_i >$, where Q_i is a vector, indicating that buyer b_i needs to buy a portfolio of goods, $Q_i = (q_i^1, \ldots, q_i^k, \ldots, q_i^K)$. q_i^k indicates the number of product k that need to be purchased in the portfolio. B_i means the price the buyer is willing to pay, $B_i = (b_i^1, \ldots, b_i^k, \ldots, b_i^K)$, b_i^k is a bid for commodity k.

When the number of goods purchased by buyer b_i is not 0, $V_i = (v_i^1, \ldots, v_i^k, \ldots, v_i^K)$, v_i^k represents this buyer's valuation of product k; when the number of goods purchased by buyer is 0, the value is 0. The buyer's valuation is the buyer's private information, only the buyer himself knows. If the buyer's price is the same as his true valuation, that is $V_i = B_i$, then the buyer b_i is honest. If all participants are honest, the mechanism is a truthful mechanism.

After all reports are submitted, the mechanism collects the information to determine the set of winners: W, then allocate goods and calculates the payment P. O^* is a matrix of $n*m$, which indicates a matching relationship between buyers and sellers. If a buyer is a winner and assigned a sellers' goods to him, the buyer and the seller are matched. In the matrix, arbitrary $O_i j^* \in \{0,1\}$,$(i \in n, j \in m)$. $O_i j^*$ is the matching relationship between buyer b_i and seller s_j. $O_i j^* = 0$ indicates that there is no matching relationship between buyer b_i and seller s_j. $O_i j^* = 1$ means the seller s_j is matched with the buyer b_i. The buyer who matched successfully will get the reported number of goods.

3 Volume discount allocation and pricing mechanism based on combinatorial double auction

This paper proposes an incentive compatible combination double volume discount mechanism based on auction mechanism, which is called TCD4GB (Truthful Combinatorial Double Auction Mechanism for Group-Buying) mechanism. The TCD4GB mechanism includes two parts: allocation and pricing. The allocation determines the WDP (Winner Determination Problem) and the product allocation of winners, called as the TCD4GB-WD mechanism. The proof for some attributes in this mechanism is demonstrated bellow.

3.1 TCD4GB-WD Winner Determination Mechanism

Algorithm 1 embodies the execution process of TCD4GB-WD mechanism. Input the reports of sellers and buyers in the algorithm, and match the buyers and sellers according to the contents of the reports, and get winners. There are many buyers and sellers in the auction model. The winner determination problem in the combinatorial double auction is a NP-hard problem[**?**] , so the mechanism uses greedy algorithm to solve the winner determination problem. The detailed description of the algorithm is as follows.

The maximum unit difference is selected in each round selection, and the unit difference shows the difference between the total bids of the buyer and the total cost of the matched sellers in each unit commodity. The higher the unit difference buyer creates the greater social welfare. The mechanism determines winners in this way, and the selection of each round is greedy to choose the participants who can maximize the social surplus so that the mechanism can effectively allocate commodities.

Algorithm 1: TCD4GB-WD Mechanism

Input: Reports submitted by Buyers: $RB = \{rb_1, \dots, rb_i, \dots, rb_n\}, rb_i =< Q_i, B_i >$ and Reports submitted by Seller: $RS = \{rs_1, \dots, rs_j, \dots, rs_m\}, rs_j =< q_j^k, c_j^k, D_j >$

Output: Set of Winners: WS, WB

```
initiate WS ← ∅, WB ← ∅, O_nm = 0;
initiate TWS ← S, TWB ← B;
while TWS ≠ ∅ and TWB ≠ ∅ do
    for i ∈ TWB do
        for k ∈ K do
            if q_i^k ≠ 0 and q_j^k ≤ 0(∀ j ∈ TWS) then
                TWB ← TWB − {i};
            end
        end
    end
    for i ∈ TWB do
        TB_i = ∑_{k∈K} b_i^k ∗ q_i^k;
        TC_i = ∑_{k∈K}(q_i^k ∗ min_{j∈TWS}(D_j(q_i^k) ∗ c_j^k));
        TS_i = arg min_{j∈TWS}(D_j(q_i^k) ∗ c_j^k);
        ud_i = (TB_i − TC_i)/∑_{k∈K} q_i^k;
    end
    if max_{i∈TWB}(ud_i) ≥ 0 then
        WB ← WB + {arg max_{i∈TWB}(ud_i)};
        WS ← WS + {TS_i};
        TWS ← TWS − {TS_i};
        TWB ← TWB − {i};
    else
        TWB ← ∅
    end
end
return WB,WS
```

3.2 Attribute proof

Theorem 1. *TCD4GB mechanism is incentive compatibility.*

Proof. According to Definition 6, that is to prove that the buyer will not get a higher utility after the buyer has lied about the bid. The payment of the buyer b_i is $p_i = ud_{i'} * |Q_i| + TC_i$. The parameters in payment have nothing to do with their bidding, so the real utility of the buyer will not change, no matter how the buyer can still be the buyer of the winner. Then assuming that the higher bid of buyer b_i makes the buyer's unit difference a lot larger and can give priority to the allocation. Then the unit difference of the overlapping buyer is likely to increase when calculating the buyer's payment. The buyer's payment will increase, and the real utility will be reduced. Assuming that the lower bid of buyer b_i makes the buyer's unit difference a lot smaller and it is very likely that the buyer cannot be a winner. All participants in the mechanism are sealed reports, and the information is not shared among buyers. Therefore, the buyer b_i cannot control how bids reduced to make it still be the winner. Even if the buyer becomes winner, it will not change the payment and real utility has not changed.

To sum up, buyers falsely reporting may loss or unable to increase their individual utility. Therefore, buyers do not have to report their information falsely. Theorem 1 is proved.

Theorem 2. *The social welfare is the largest under the allocation model of TCD4GB mechanism.*

Proof. In paper[1] , Myerson indicated that there is no mechanism to meet the three economic attributes of budget balance, incentive compatibility and maximization of social welfare. So, we discuss the social welfare SW under the allocation mode of the maximum unit difference in this mechanism. According to Definition 4, assuming the existence of $SW < SW' = \sum_{i \in WB'}(TB'_i - TC'_i)$ under the mechanism. According to Algorithm 1, TB_i is the sum of bids of the buyer whose unit difference is the biggest, and TC_i is the minimum total price of goods that buyer b_i needs. That is $TB'_i > TB_i$ or $TC'_i < TC_i$. This is in contradiction with the mechanism algorithm, so $SW < SW'$ is not set up. Theorem 2 is proved.

4 Experiment

This section uses simulation experiments to verify the feasibility of the proposed TCD4GB mechanism. The parameters of this simulation experiment are in Table 1. The result of a complete TCD4GB mechanism execution is shown in Table 2. In Table 2, the Rotation indicates the cycle times of determining winner, ud indicates the winner's unit difference, TB is the total bids, TC is the corresponding minimum selling cost, TS is the matching sellers sequence, ud' is the winner who is the overlapping buyer of the original winner and the unit difference less than that of the original winner, and the value of ud' is used to calculate the winner's payment P.

The results of Table 2 show that the payments of the winning buyers are less than their respective total bidding price, and the amounts of payments is higher than the total cost of the matched sellers, which proves that the TCD4GB mechanism is individual rationality. There is little difference between ud' and ud. It can be seen that if the buyer falsely bids, it is not likely to be the winning buyer. Even if the information control is in the range of success, the payment is not related to its own bidding price, and the collection of information also needs cost, so the buyer does not have motivation to lie.

Table 1: simulation experimental parameters

parameter	description	range
$Kind$	Kind of goods	6
m	Number of sellers	[20,200]
q_j^k	Quality of goods seller sold	[1,30]
c_j^k	Unit price by selle	[10,20]
D_j	Volume discount	((15,0.9),(20,0.8))]
n	Number of buyers	[20,100]
q_i^k	Demand of buyer	[0,25]
b_i^k	Bids of buyer	[8,22]

Table 2: Mechanism execution results

Rotation	Winner	ud	TB	TC	TS	ud'	p
1	No.10	11.23	778	385.9	02,03,05	10.33	746.55
2	No.8	9.40	520	285.2	01,13	8.67	501.68
3	No.9	9.33	640	360.8	12,15	8.67	620.00
4	No.6	8.67	488	280.6	22,04	8.33	479.92
5	No.4	8.04	480	287.0	14,25,06	6.59	445.30
6	No.5	7.33	630	410.5	32,23	6.72	611.69
7	No.7	6.25	927	631.0	11,43,24	4.96	864.17
8	No.2	5.26	439	318.4	35,16	2.33	371.59
9	No.1	4.62	502	382.8	52,33	2	434.00
10	No.3	1.67	500	448.8	43,45	\	450.00

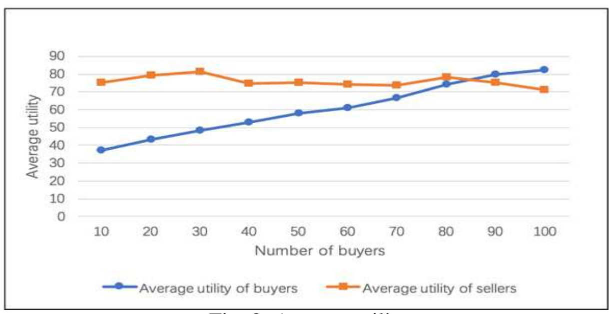

Fig. 2: Average utility

In Figure 3, 200 sellers are set up, and 10 to 100 increased by 10 people. In the experiment, the average utility of all winning participants under different numbers of buyers after operating allocation and pricing mechanism is calculated. From the picture, the average utility of winning sellers is basically stable. We default that sellers are honesty, the payoffs of sellers are buyers' payments, so the sellers' profit are not undulating. And the average utility of buyers is increasing slowly with the number of buyers, which is because the more buyers participate, the greater the gap between the unit difference, the greater the gap between the buyers' payment and the sum of the bids, the more utility buyers achieve. Therefore, the average utility of buyers will increase slowly with the increase of the number of buyers.

5 Related Work and Conclusion

Anand[4] assumed some atomic buyers, the participation of these buyers has no effect on the price in the model. Considering the limited supply of goods and the private information of buyers, Chen[6] analyzed the bidding strategy of buyers in the volume discount auction. Based on this analysis, paper[5] further compared the volume discount auction mechanism with the fixed bidding mechanism. Paper[7] first introduced the concept of volume discount in cloud computing.

There are many models with allocation and pricing algorithms based on double auction. Paper[8] proposed an auction based bidirectional resource allocation algorithm according to the different commodity types and performance characteristics. The algorithm allocates goods according to the performance characteristics of goods and buyers' demand. Paper[9] proposed a dynamic commodity allocation mechanism based on the auction to meet the needs of the users and charge the corresponding cost. This mechanism enables suppliers to allocate resources effectively and gain higher benefits. Paper[10] proposed an auction framework under an entirely competitive situation, and designed a double auction mechanism based on Bayesian game designed for resource allocation and pricing. A resource allocation model and pricing algorithm based on combined double auction was proposed in paper[11] , which is designed to maximize transaction surplus. Paper[12] calculated the transportation cost into suppliers' selling cost, and proposed an effective mechanism to improve suppliers' utility by controlling buyers' payment.

In this paper, TCD4GB mechanism is proposed to allocate and price volume discount commodities. It determines the winning participant, assigns the commodity and calculates payment. In view of the winner determination problem, the concept of unit difference is introduced to determine winning buyers and to determine winning sellers with lowest selling price. The mechanism also uses the second price payment idea from VCG mechanism to calculate the buyers' payment. It is proved that the TCD4GB mechanism is an incentive compatible mechanism and the mechanism satisfies the economic attributes of individual rationality and budget balance.

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References

1. Myerson R B, Satterthwaite M A. Efficient Mechanism for Bilateral Trading[J]. Journal of Economic Theory, 1983, 29(2):265-281.
2. Samimi P, Teimouri Y, Mukhtar M. A combinatorial double auction resource allocation model in cloud computing[M]. Elsevier Science Inc. 2016.
3. Tafsiri S A, Yousefi S. Combinatorial double auction-based resource allocation mechanism in cloud computing market[J]. Journal of Systems and Software, 2018, 137:322-334.
4. Anand K S, Aron R. Group Buying on the Web: A Comparison of Price-Discovery Mechanisms[J]. Management Science, 2003, 49(11):1546-1562.
5. Chen J, Chen X, Song X. Comparison of the group-buying auction and the fixed pricing mechanism[J]. Decision Support Systems, 2007, 43(2):445-459.
6. Chen J, Chen X, Song X. Bidder's strategy under group-buying auction on the Internet[J]. IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, 2002, 32(6):680-690.
7. Lee C, Wang P, Niyato D. A Real-Time Group Auction System for Efficient Allocation of Cloud Internet Applications[J]. IEEE Transactions on Services Computing, 2015, 8(2):251-268.
8. Kang Z, Wang H. A Novel Approach to Allocate Cloud Resource with Different Performance Traits[C]// IEEE International Conference on Services Computing. IEEE Computer Society, 2013:128-135.
9. Nejad M M, Mashayekhy L, Grosu D. Truthful Greedy Mechanisms for Dynamic Virtual Machine Provisioning and Allocation in Clouds[J]. IEEE Transactions on Parallel and Distributed Systems, 2015, 26(2):594-603.
10. Nejad M M, Mashayekhy L, Grosu D. Truthful Greedy Mechanisms for Dynamic Virtual Machine Provisioning and Allocation in Clouds[J]. IEEE Transactions on Parallel and Distributed Systems, 2015, 26(2):594-603.
11. Li L I, Liu Y A, Liu K M, et al. Pricing in combinatorial double auction-based grid allocation model[J]. The Journal of China Universities of Posts and Telecommunications, 2009, 16(3):59-65.
12. Chen R R, Roundy R O, Zhang R Q, et al. Efficient Auction Mechanisms for Supply Chain Procurement[J]. Management Science, 2005, 51(3):467-482.