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Difference Time domain Modeling (FDTD) by  
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# Simulation of a wave propagation using Finite Difference Time domain Modeling (FDTD) by Matlab

Amro Emad Awad Ali

**Abstract**— the purposes from this paper are driving and simulating the propagation of a wave by using Finite Difference Time Domain Modeling Analysis (FDTD), by drive the corresponding FDTD codes from Maxwell's equation and simulate these codes in MATLAB.

**Index Terms**—Matlab, Model, Simulation, Maxwell, FDTD.

## I. INTRODUCTION

Finite difference time domain modeling method is a numerical analysis technique used for modeling computational electrodynamics, introduced by Yee in 1966. At that time it was the first direct time domain solutions of Maxwell's differential (curl) equations on special grids or lattices [1].

When Maxwell's differential equations are applied, it can be seen clearly that the electric field (E) and the electromagnetic field (H) are interleaved in both space and time, also the electromagnetic field values are assumed to be located between the electric field values. In addition to the new value of electric field are calculated from its previous value and the most recent value of electromagnetic field [1].

The main objective of this paper is to derive corresponding FDTD code from Maxwell's equations and beside consider the states of the media and simulated boundary. After that simulate the propagation of the given source wave in these FDTD code in MATLAB for one-dimension. Then plot the propagation for this wave in different time; to show the behavior for this wave in this given media.

The rest part of the Report is organized as follows: Section II briefly describes in details the derivation of FDTD code from Maxwell's equations, Matlab code for the problem and the output of the described code. Section III Conclusion of these problems. .

This paper is a simulation of how a wave propagates in both space and time by using a clear explanation of maxwell's equations.

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## II. SOLVING PROBLEMS (CODE AND RESULTS)

For explanation of this issues Matlab (R2015b) has been used. The main idea for solving this problem is developing code from Maxwell's differential equations to simulate the propagation of a sinusoidal wave as a source signal hitting on a lossy dielectric medium.

### One dimension FDTD:

I will derive the corresponding 1-D FDTD code from the Maxwell's equations and simulate a sinusoidal wave hitting on a lossy dielectric material , starting from the middle of the problem space, with a dielectric constant of 3.8 and a conductivity of 0.02 S/m. The source is a 1GHz sinusoidal wave originates at cell 10.i am going to take the problem space as 400 cells and also plotting Ez with four values of ' T' (four graphs) to show the propagation of the wave throughout the problem space.

I will start by driving 1-D FDTD code from Maxwell's equations as follow:

$$\varepsilon \frac{\partial E}{\partial t} = \nabla \times H - J \quad (1)$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} \nabla \times E \quad (2)$$

$$J = \sigma \cdot E \quad (3)$$

Where  $J$  is the current density,  $\sigma$  is the conductivity, and  $\varepsilon$  is the dielectric constant.

Now substitute (3) in (1) and dividing the whole equation with  $\varepsilon$  which is equal to  $\varepsilon_0 \varepsilon_r$  .

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_r} \nabla \times H - \frac{\sigma}{\varepsilon_0 \varepsilon_r} E \quad (4)$$

Assume  $E$  is along x axis,  $H$  is along y axis, and the propagation is along z axis.

$$\frac{\partial E_x(t)}{\partial t} = -\frac{1}{\varepsilon_0 \varepsilon_r} \cdot \frac{\partial H_y(t)}{\partial z} - \frac{\sigma}{\varepsilon_0 \varepsilon_r} E_x(t) \quad (5)$$

$$\frac{\partial H_y(t)}{\partial t} = -\frac{1}{\varepsilon_0 \mu_0} \cdot \frac{\partial E_x(t)}{\partial z} \quad (6)$$

Now take the normalization of  $E_x(t)$  in equations (5) and (6).

$$\frac{\partial \tilde{E}_x(t)}{\partial t} = -\frac{1}{\epsilon_r \sqrt{\epsilon_0 \mu_0}} \cdot \frac{\partial H_y(t)}{\partial z} - \frac{\sigma}{\epsilon_0 \epsilon_r} \tilde{E}_x(t) \quad (7)$$

$$\frac{\partial H_y(t)}{\partial t} = -\frac{1}{\sqrt{\epsilon_0 \mu_0}} \cdot \frac{\partial \tilde{E}_x(t)}{\partial z} \quad (8)$$

Now take the finite difference approximations for both the temporal and spatial derivatives (FDTD code).

$$\begin{aligned} \frac{\tilde{E}_x^{n+1/2}(k) - \tilde{E}_x^{n-1/2}(k)}{\Delta t} &= -\frac{1}{\epsilon_r \sqrt{\epsilon_0 \mu_0}} \cdot \frac{H_y^n(k+1/2) - H_y^n(k-1/2)}{\Delta x} \\ &\quad - \frac{\sigma}{\epsilon_0 \epsilon_r} \frac{\tilde{E}_x^{n+1/2}(k) + \tilde{E}_x^{n-1/2}(k)}{2} \end{aligned} \quad (9)$$

Where  $\Delta x$  is the cell size which is  $\Delta t = \Delta x / 2c_0$  then:

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}} \frac{\Delta t}{\Delta x} = c_0 \cdot \frac{\Delta x / 2 \cdot c_0}{\Delta x} = \frac{1}{2}$$

Now equation (9) becomes:

$$\begin{aligned} \tilde{E}_x^{n+1/2}(k) \left[ 1 + \frac{\Delta t \cdot \sigma}{2\epsilon_0 \epsilon_r} \right] &= \tilde{E}_x^{n-1/2}(k) \left[ 1 - \frac{\Delta t \cdot \sigma}{2\epsilon_0 \epsilon_r} \right] \\ &\quad - \frac{1/2}{\epsilon_r} \left[ H_y^n(k+1/2) - H_y^n(k-1/2) \right] \end{aligned} \quad (10)$$

Or

$$\begin{aligned} \tilde{E}_x^{n+1/2}(k) &= \frac{\left( 1 - \frac{\Delta t \cdot \sigma}{2\epsilon_0 \epsilon_r} \right)}{\left( 1 + \frac{\Delta t \cdot \sigma}{2\epsilon_0 \epsilon_r} \right)} \tilde{E}_x^{n-1/2}(k) \\ &\quad - \frac{1/2}{\epsilon_r \left( 1 + \frac{\Delta t \cdot \sigma}{2\epsilon_0 \epsilon_r} \right)} \left[ H_y^n(k + \frac{1}{2}) \right. \\ &\quad \left. - H_y^n(k - 1/2) \right] \end{aligned} \quad (11)$$

Matlab code has been written for four "T" (100,200,300 and 400) to show the propagation of the sinusoidal wave across the medium.

```
clear;
clc;
%epsr is the dielectric constant
epsr=3.8;
%epsz0 is the permittivity of space
epsz0=8.854223e-12;
%sigma is the conductivity
sigma=0.02;
%KE is the number of cells to be used
KE=400;
dx=zeros(KE,1);
ex=zeros(KE,1);
hy=zeros(KE,1);
ix=zeros(KE,1);
gax=ones(KE,1);
```

```
gbx=zeros(KE,1);
%kc is the number of cell in which asinusoidal
wave originates
kc=10;
%f0 the corresponding frequency of the wave
f0=1e9;
%c0 the speed of light in avacuum
c0=3e8;
lambdaM=c0/(sqrt(epsr)*f0);
ddx=lambdaM/10;
dt=ddx./(2*c0);
ex_low_m2=0;
ex_low_m1=0;
T=0;
NSTEPS=400;c=0;
%main FDTD loop
for k=200:KE;
    gax(k,1)=1/(epsr+(sigma*dt/epsz0));
    gbx(k,1)=sigma*dt/epsz0;
end
for m=100:100:400;
    c=c+1;
    for n=1:m;
        T=T+1;
        for k=2:KE;
            dx(k)=dx(k)+.5*(hy(k-1)-hy(k));
        end
        source=sin(2*pi*f0*n*dt);
        dx(kc)=dx(kc)+source;
        %calculate the e field of the FDTD
        for k=2:KE;
            ex(k)=gax(k,1)*(dx(k)-ix(k));
            ix(k)=ix(k)+gbx(k,1)*ex(k);
        end
        %absorbing boundary condition added
        ex(1)=ex_low_m2;
        ex_low_m2=ex_low_m1;
        ex_low_m1=ex(2);
        %calculate the h field of the FDTD
        for k=1:(KE-1);
            hy(k)=hy(k)+.5*(ex(k)-ex(k+1));
        end
        figure(c); plot(ex,'b'); hold on
        xa=[200 200];ya=[-1.5 1.5];
        plot(xa,ya,'r','LineWidth',2,'LineStyle','--')
        title(['T = ',num2str(n)]);
        ylabel('Propagation of E')
        xlabel('FDTD ')
        axis([1 KE -2 2])
        text(250,-1.15,'Conductivity(sigma)=0.02')
        text(250,-1.3,'Epsilon=3.8')
        text(50,-1.3,'Epsilon=1','color','k')
        hold off
    end
end
```

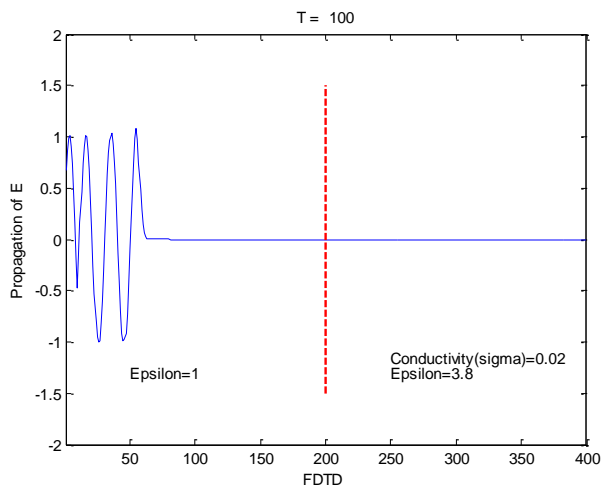


Fig. 1. 1-D FDTD sinusoidal wave start propagating from cell 10 in a lossy dielectric material, output at T=100.

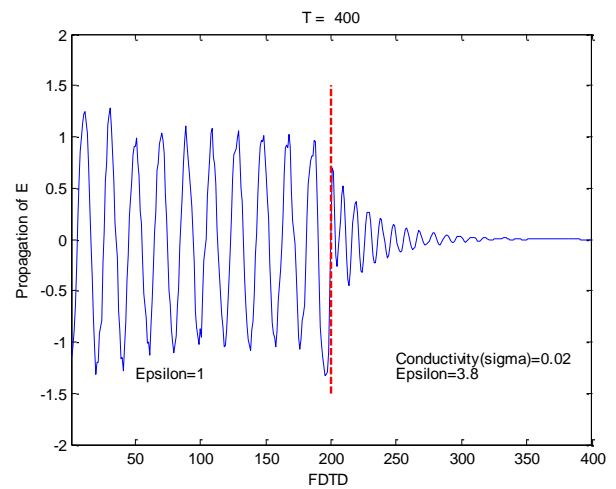


Fig. 4. 1-D FDTD sinusoidal wave start propagating from cell 10 in a lossy dielectric material, output at T=400.

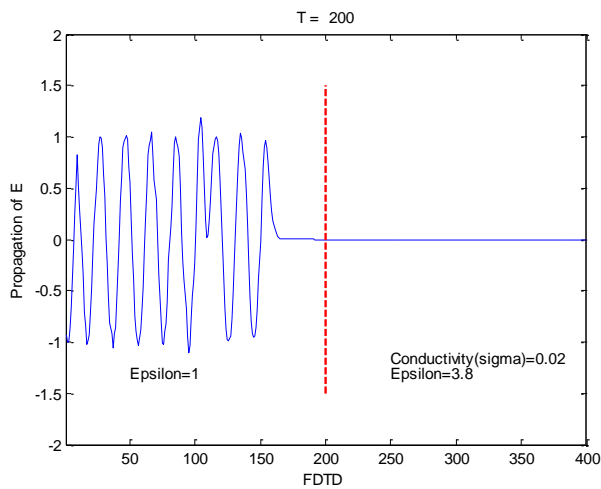


Fig. 2. 1-D FDTD sinusoidal wave start propagating from cell 10 in a lossy dielectric material, output at T=200.

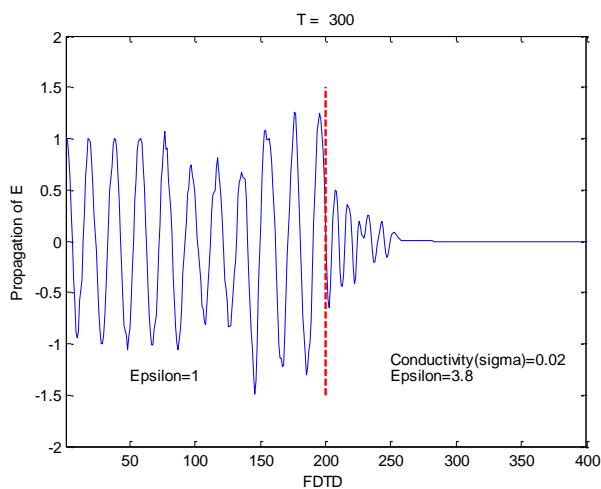


Fig. 3. 1-D FDTD sinusoidal wave start propagating from cell 10 in a lossy dielectric material, output at T=300.

### III- Conclusion

One dimension FDTD simulation has been done for a sinusoidal wave hitting a lossy dielectric material.

From all of the above it can be clearly seen that FDTD is accurate, robust and easy to do by applying a systematic approach.

### REFERENCES

- [1] Dennis M.Sullivan"Electromagnetic Simulation Using The FDTD Method", Institute of Electrical and Electronics Engineers Inc, New York, 2000.



**Amro Emad Awad Ali** received the B.S.c degree in electrical engineering (control) from Sudan University of science and Technology, Khartoum, Sudan, in 2014. He is currently performing a postgraduate studies (a master degree) in electrical engineering (computer and control systems) in King Saud University.