Fuzzy causal Logic: Fuzzy Conditional Inference and Approximate Reasoning

Venkata Subba Reddy Poli

June 1, 2020

Fuzzy causal Logic: Fuzzy Conditional Inference and Approximate Reasoning

Poli Venkata Subba Reddy

Abstract

We consider fuzzy conditional inference of the form "if x is P then y is Q ", "if x is P then y is Q else y is R"and "if x is P_1 and/or P_2 and/or $\cdots$ and/or $P_n$then y is R". in this paper. We propose four method of inference applying logical constructs developed by Mizumoto. We show how these methods satisfy our intuitions under several criteria.

Keywords: Fuzzy sets, Fuzzy logic, logical constructs, Fuzzy conditional inference, Approximate reasoning

,

1. Introduction

Mathematical logics are dealing with variables and are unable reason with words which the propositions may contain uncertain, vague, or imprecise propositions. Fuzzy logic is reasoning such propositions or statements. Zadeh [81 and Mamdani [2] proposed methods for fuzzy reasoning for fuzzy conditional proposition contain "if ... then ..."propositions. The consequences inferred by Zadeh [8] and Mamdani [2]. In their methods do not fit our intuitions. Mizumoto [2] developed logical constructs for fuzzy implications and the Godel definition and Standard sequence methods. Some of the logical constructs satisfy and some of them do not satisfy. We developed a method and apply on local constructs of the propositions containing "if ... then ..."and "if ... then ... else...". The proposed method satisfy all the fuzzy intuitions.

Considered three criteria.

Criteria-1

If x is P then y is Q
x is P_1

---

y is ?

If *Apple* is *red* then *Apple* is *ripe*
apple is very *ripe*

---

y is ?

Criteria-2
If x is P then y is Q else y is R
x is P_1

---

y is ?

If *Apple* is *Ripe* then *Apple* is *Taste* else *Apple* is *Sour*
apple is very *ripe*

---

y is ?

Criteria-3
If x is P and x is Q or x is R then y is S
x is P_1 and x is Q_1 or x is R_1

---

y is ?

If x is Red or x is ripe and x is big then x is taste
x is red or x is ripe and x is very big

---

y is ?

2. Fuzzy plausibility

Consider the causal logical inference [10]
Modus Ponens
$p \rightarrow q$

P

——

q

Modus Tollens

$p \rightarrow q$

q'

——

P'

Generalization

$p \vee q = p$

$p \vee q = q$

Specialization

$p \wedge q = p$

$p \wedge q = q$

causal Logic	**Proposition**	**Inference**
Modus Ponens	x is P	y is Q
Modus Ponens	x is not P	y is not Q
Modus Tollens	y is Q	x is P
Modus Tollens	y is not Q	x is not P

Table 1: Causal logic

Plausibility theory will perform inconsistent information into consistent.

Generalization

$p\vee$ q, μ =p, μ

$p\vee$ q, μ = q, μ

Specialization

$p\wedge$ q , μ=p, μ

$p\wedge$ q = q, μ

2.1. Fuzzy Conditional Inference

A fuzzy set P is define by its characteristic function $\int \mu_P(x)/x$, $x \epsilon X$, where x is individual and X is universe of discourse.

P=$\int \mu_P(x)/x$

P'=1-$\int \mu_P(x)/x$
$P \vee Q$=max $\{(\int \mu_P(x), \int \mu_Q(y))(x,y)\}$
$P \wedge Q$=min $\{(\int \mu_P(x), \int \mu_Q(y))(x,y)\}$
$P \oplus Q$=min { 1, $(\int \mu_P(x) + \int \mu_Q(y))(x,y)\}$
The fuzzy conditional propositions of the form "if (precedent part) then (consequent part)".
Consider the proposition of type "if x is P then y is Q "
Zadeh [8] definition for fuzzy conditional inference is given by

$P \rightarrow Q$= $P' \oplus Q$= { 1 $\wedge$ 1 - $(\int \mu_P(x) + \int \mu_Q(y))\}$
Consider the proposition of type "if x is P then y is Q else x is R".
It may be defined as "if x is P then y is Q $\vee$ if x is P' then x is R"
It is given by
"if x is P then y is Q"
"if x is P' then x is R"

$P \rightarrow Q$= $P' \oplus Q$= min { 1, 1 - $(\int \mu_P(x) + \int \mu_Q(y))\}$
$P' \rightarrow R$= $P \oplus R$= min { 1, $(\int \mu_P(x) + \int \mu_R(y))\}$
Mamdani [1] definition for fuzzy conditional inference is given by

$P \rightarrow Q$= $P' \oplus Q$= { $(\int \mu_P(x) X \int \mu_Q(y))\}$
Logical system of Standard sequence S is given by

$$v(P \rightarrow Q) = \begin{cases} 1 & v(P) \leq v(Q) \\ 0 & v(P) > v(Q) \end{cases}$$

Logical system of Godelian sequence G is given by

$$v(P \rightarrow Q) = \begin{cases} 1 & v(P) \leq v(Q) \\ v(Q) & v(P) > v(Q) \end{cases}$$

2.2. Improved method
The consequent part is derived from precedent part for fuzzy conditional inference [5].
$A \rightarrow B$= A, i.e., $\int \mu_B(y)$= $\int \mu_A(x)$, i.e., $B \subseteq A$ and $A \subseteq B$ (2.1)

Consider fuzzy quantifiers very, more or less etc., A^α and B^α
$A^\alpha \subseteq B$, i.e., $A^\alpha \leq B$

$B^\alpha \subseteq A$, i.e., $B^\alpha \leq A$

The fuzzy conditional inference is given by using Mamdani fuzzy conditional inference

if x is A then y is $B = \{A \times B\}$

The fuzzy conditional inference is give by using (2.1)

$A \to B = \int \mu_B(y) \times \int \mu_A(x)$

$A \to B = \int \mu_B(y) \wedge \int \mu_A(x)$

Fuzzy conditional inference is given by

$$(A \to B) = \{\ \int \mu_A(x) \quad \int \mu_A(x) \Longleftrightarrow \int \mu_B(x) \tag{2.2}$$

3. Fuzzy Conditional Inference

3.1. Verification of Criteria-1

The fuzzy conditional inference may be given for Criteria-1 by

Intuition	Proposition	Inference
I-1	x is P	y is Q
I-2	y is Q	x is P
II-1	x is very P	y is very Q
II-2	y is very Q	x is very P
III-1	x is more or less P	y is more or less Q
III-2	y is More or less Q	is more or less P
IV-1	x is not P	y is not Q
IV-2	y is not Q	x is not P

Table 2: Criteria-1

Consider the fuzzy conditional inference

$\int \mu_P(x)/x \to \int \mu_Q(y)/y = \{\int \mu_P(x)/x\}$

and

$\int \mu_P(x) = \int \mu_Q(y)$

3.2. Verification of fuzzy Intuitions for Criteria-1

2.1.1 In the case of Intuition I-1, II-1 and III-1

P^α o (P$\rightarrow$ Q)
$= \int \mu_{P^\alpha}(x)/x$ o $(\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$
$= \int \mu_{P^\alpha}(x)/x$ o $(\int \mu_P(x)/x)$
$= \int \mu_{Q^\alpha}(y)/y \wedge (\int \mu_Q(y)/y$

$\int \mu_{Q^\alpha}(y)/y$
$=y$ is $Q^\alpha(y)/y$
Intuition I-1, II-1 and III-1 are satisfied.

2.1.2 In the case of Intuition I-2,II-2 and III-2

$(P \rightarrow$ Q) o Q^α
$= (\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$ o $\int \mu_{Q^\alpha}(y)/y$
$= \int \mu_P(x))$ o $\int \mu_{Q^\alpha}(y)$
$= \int \mu_P(x)) \wedge \int \mu_{P^\alpha}(x)$

$= \int \mu_{P^\alpha}(x)/x$
x is P^α
Intuition I-2, II-2 and III-2 are satisfied.

2.1.3 In the case of Intuition IV-1

P' o (P$\rightarrow$ Q)
$= \int \mu_{P'}(x)/x$ o $(\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$
$= \int \mu_{P'}(x)/x$ o $(\int \mu_P(x)/x)$
$= \int \mu_Q(y)/y \wedge (\int \mu_Q(y)/y$

$= \int \mu_{Q'}(y)/y$
y is not Q
Intuition IV-1 satisfied.

2.1.4 In the case of Intuition IV-2

$(P \rightarrow$ Q) o Q'
$= (\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$ o $\int \mu_{Q'}(y)/y$
$= \int \mu_P(x))$ o $\int \mu_{Q'}(x)$
$= \int \mu_P(x)) \wedge \int \mu_{P.}(x)$

$= \int \mu_{P.}(x)/x$
x is not P
Intuition IV-2 satisfied.
Criteria-1 is satisfies I-1,I-2, II-1, II-2, III-1 and III-2, IV-1, IV-2.

3.3. Verification of Criteria-2

Consider the fuzzy conditional inference

$\int \mu_{P'}(x)/x \to \int \mu_R(y)/y$ =$\{\int \mu_{P'}(x)/x\}$
and
$\int \mu_{P'}(x) = \int \mu_R(y)$

The fuzzy conditional inference may be given for Criteria-2 by

Intuition	**Proposition**	**Inference**
I-1	x is P	y is Q
I-2	y is Q	x is P
II-1	x is very P	y is very Q
II-2	y is very Q	x is very P
III-1	x is more or less P	y is more or less Q
III-2	y is More or less Q	is more or less P
IV-2	y is not R	x is not P
I'-1	x is P'	y is R
I'-2	y is R	x is P'
II'-1	x is very P'	y is very R
II'-2	y is very R	x is very P'
III'-1	x is more or less P'	y is more or less R
III'-2	y is More or less R	is more or less P'
IV'-2	x is R'	y is P

Table 3: Criteria-2

Criteria-1 is verified for I-1,I-2, II-1, II-2, III-1 and III-2, IV-2 in Criteria-1.

2.2.1 In the case of Intuition I'-1, II'-1 and III'-1

P'^{α} o (P' → R)
$= \int \mu_{P'^{\alpha}}(x)/x$ o $(\int \mu_{P'}(x)/x \to \int \mu_R(y)/y)$
$= \int \mu_{P'}(x)/x$ o $(\int \mu_{P'}(x)/x)$
$= \int \mu_{R'^{\alpha}}(y)/y \wedge \int \mu_R(y)/y$

$= \int \mu_{R'^{\alpha}}(y)/y$

y is R'^{α}
Intuition I'-1, II'-1 and III'-1are satisfied.

2.2.2 In the case of Intuition I'-2, II'-2 and III'-2

(P' $\rightarrow$ R)o R^{α}
$= (\int \mu_{P'}(x)/x \rightarrow \int \mu_R(y)/y)$ o $\int \mu_{R^{\alpha}}(y)/y$
$= \int \mu_{P'}(x))$ o $\int \mu_{R^{\alpha}}(y)$
$= \int \mu_{P'}(x))/x \wedge \int \mu_{R^{\alpha}}(x)/x$

$= \int \mu_{P'^{\alpha}}(x)/x$
x is R'^{α}
Intuition I'-2, II'-2 and III'-2 are satisfied.

2.2.7 In the case of Intuition IV'-2

(P' $\rightarrow$ R)o R'^{α}
$= (\int \mu_{P'}(x)/x \rightarrow \int \mu_R(y)/y)$ o $\int \mu_{R'^{\alpha}}(y)/y$
$= \int \mu_{P'}(x))$ o $\int \mu_{R'^{\alpha}}(y)$
$= \int \mu_{P'}(x))/x \wedge \int \mu_{P^{\alpha}}(x)/x$

$= \int \mu_{P^{\alpha}}(x)/x$
x is P^{α}
Intuition IV'-2 satisfied.
Criteria-2 is satisfies I'-1,I'-2, II'-1, II'-2, III'-1, III'-2, IV'-2.

4. Verification of fuzzy Intuitions for Criteria-3

Consider fuzzy conditional inference

If x is P and x is Q or x is R then y is S
x is P_1 and x is Q_1 or x is R_1

---

y is ?

Fuzzy inference is given by using Specialization and Generalization

If x is P then y is S
x is P_1

---

y is ?

If x is x is Q then y is S
x is x is Q_1

---

y is ?

If x is R then y is S
x is R_1

---

y is ?

Fuzzy inference may be verified in the similar lines of Criteria-1

5. Business Application

The Business intelligence needs commonsense. The Business data is defied with fuzziness with linguistic variables.

For example

If x is *Demand* then *Apple* is *Production*
apple is very *Demand*

---

y is ?

If *Apple* is *Sales* then *Price* is *Taste* else *Apple* is *Stock*
apple is very *Sales*

---

y is ?

If x is *Demand* or x is *Sales* and x is *Price* then y is *Production*
x is *more Demand* or x is *very Sales* and x is *Price*

---

y is ?

These Criteria shall be studied with Criteria-1, Criteria-2 and Criteria-3.

6. Conclusion

In this paper, we consider the fizzy condition inference

If x is P then y is Q
x is P_1

---

y is ?

If x is P then y is Q else y is R
x is P_1

---

y is ?

If x is P and x is Q or x is R then y is S
x is P_1 and x is Q_1 or x is R_1

---

y is ?

We try to prove three criteria with our method using fuzzy plausibility and it is approximate reasoning.

References

[1] E.H.Mamdani,Application of Fuzzy Logic to Approximate Reasoning Using Linguistic Synthesis. IEEE Trans. Computers. vol.26, no.12, pp.1182-1191, 1977.

[2] S. Fukami, M. Muzumoto,K. Tanaka, Some Considerationas on Fuzzy Conditional Inference, Fuzzy Seta and Systems, vol.4, pp.243-273, 1980.

[3] C. Howson, Successful Business Intelligence, McGraw-Hill, 2014.

[4] N. Rescher, Many-valued Logic, McGrow-Hill, New York, 1969.

[5] Poli Venkata Subba Reddy, M. Syam Babu, Some methods of reasoning for conditional propositions, Fuzzy Sets and Systems, vol.52, no.3, pp.229-250, 1992.

[6] Poli Venkata Subba Reddy, Fuzzy conditional inference for medical diagnosis, Proceedings, Second International Conference on Fuzzy Theory and Technology, Durham, FT&T'93, vol.3, pp.193-195, 1993.

[7] Poli Venkata subba reddy, Fuzzy logic based on Belief and Disbelief membership functions, Fuzzy Information and Engineering, Vol.9, no.4, pp.405-422, 2017.

[8] L. A Zadeh, "Calculus of fuzzy Restrictions", In Fuzzy set s and their Applications to Cognitive and Decision Processes, L. A. Zadeh, King-Sun FU, Kokichi Tanaka and Masamich Shimura (Eds.), Academic Press, New York, pp.1-40, 1975.

[9] L.A. Zadeh, Fuzzy sets. Information and Control, vol.8, pp.338-353., 1965.

[10] A. Bochman, A logic for causal reasoning,Proceedings IJCAI2003,Morgan Kaufmann,2003.