A Novel Bi-Penalty Method for Stable Contact-Impact Analysis

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Abstract

The contact-impact analysis focuses on the short period contact that results in a change in the direction of a body's velocity. In the finite element method, an adequate expression of the contact stress for discretized fields should be formulated to obtain accurate results. In general, the Lagrange multiplier and penalty method, which originate from the optimization fields, have popularity in fulfilling the Karush-Kuhn-Tucker conditions, which means the impenetrability condition. The latter method regularizes the impulsive response of the contact force and generates the contact stress using nonphysical springs on the contact surfaces. Thus an arbitrary stiffness parameter should be defined, and the simple representation of the contact stress can reduce the implementation difficulty and computational cost. The penalty method with a large stiffness penalty accurately constrains the contact conditions and reduces temporal potential energy loss by decreasing the penetration. However, in the explicit finite element method, the stability is pushed to its limit at the same time, and it requires a smaller time step to satisfy the stability condition [1], which is defined as

$$dt \leq 2\sqrt{\lambda}_{\max}, \tag{1}$$

where λ_{max} is the maximum eigenvalue of systems. In addition, the exact stiffness penalty is impossible to find for the general case, and the selection of the proper stiffness penalty is a crucial factor especially in the dynamic case. Furthermore, the simple expression of the contact stress causes spurious oscillation that can not be avoided. To overcome the deficiencies of the penalty method, the bi-penalty method can be considered, which is formed by adding an mass penalty to the penalty method [2] as

$$W_c = \frac{1}{2}\int \varepsilon_k g_N^2 d\Gamma_c + \frac{1}{2}\int \varepsilon_m \dot{g}_N^2 d\Gamma_c, \tag{2}$$

where ε_k and ε_m are the stiffness and mass penalty parameters. g_N and $\dot{g}_N$ are the normal gap and gap velocity, respectively. Considering the discrete surface and shape function, the matrix form can be expressed as

$$[\mathbf{M}+\mathbf{M_p}]\,\ddot{\mathrm{u}} + [\mathbf{K}+\mathbf{K_p}]\,\mathrm{u} = \mathbf{f}, \tag{3}$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrix. The subscript $\mathbf{p}$ means the penalty matrix.
The advantage of the bi-penalty method can be summarized next. First, the mass penalty restricts the stiffness penalty and can be defined using a ratio of the stiffness penalty and eigenvalues. Then the eigenvalue of the bipenalized system is limited under the maximum eigenvalue of the unpenalized system, which can be described as

$$\lambda_{\max} \geq [\lambda_{\mathbf{p}}]_{\max}, \tag{4}$$

and using the definition of the penalty method and, in terms of eigenvalue, we can rewrite Equation 4 as

$$\lambda_{\max} \geq \varepsilon_k/\varepsilon_m. \tag{5}$$

Then the limitation of the magnitude of the stiffness penalty can be removed, the stability limit can not violated by the nonphysical springs. Second, the temporal kinematic energy loss can be minimized when

a contact is activated. The mass penalty term makes a temporal kinematic energy loss caused by the velocity gap when the mass penalty parameter has a dominant value than body mass. The proposed method defines the mass penalty using the maximum eigenvalue as the denominator, which can minimize the loss. Third, the zig-zag effect [3] is prevented. A large stiffness penalty reduces a gap and temporal potential energy loss. However, at the same time, high-frequency oscillation occurs around the gap, which can trigger the zig-zag effect. The bi-penalty method makes more stable oscillations and reduces the defining difficulty of the penalty parameters than the penalty method by using an approximated stiffness penalty that has a similar magnitude to the stiffness of the contact elements or application of the stabilization method [3]. Then, the effect can be avoided.

The numerical examples are performed on 1D contact-impact problems, as shown in Figure 1, with the central difference method to show the validity of the method, and the results are compared with exact solution. The initial velocity is 10mm/s. The length **L** is 100mm and divided into 200 elements. The young's modulus and density are 100MPa and $1\text{ton}/\text{mm}^3$, respectively. Figure 2 describe the solution gap and energy result. In the bi-penalty gap result, the spurious oscillation is suppressed, which is opposite to the penalty method's result. On top of that, the reduced oscillation allows us to use the larger stiffness penalty, and the more accurate energy result can be obtained by reduced temporal potential energy loss. The bi-penalty energy shows that the smooth total energy and stable internal energy. In addition, the penalty energies are maintained at sufficiently small values, indicating that the penalty parameters are appropriately defined. Compared to the initial state, the penalty terms are nonzero during the contact, and this affects the total energy. This is an unavoidable phenomenon to expressing contact stress with the penalty method. For more accurate results, using the finer elements and the stabilization method [3] can be employed.

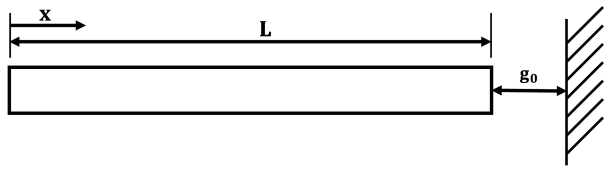

Figure 1: Illustration of rigid wall contact.

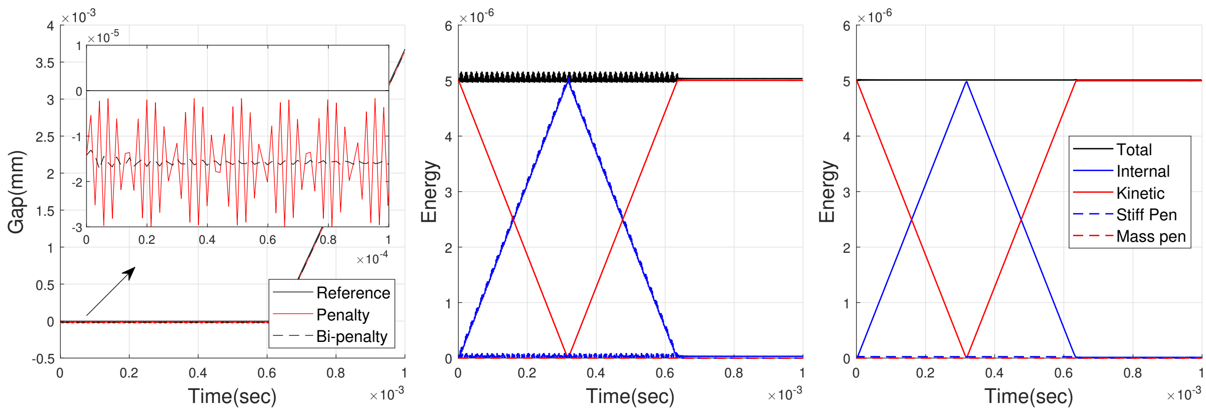

Figure 2: Solution gap(left), and energy of the penalty method(middle) and the bi-penalty method(right).

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