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Deep Bhattacharjee, Soumendra Nath Thakur, Priyanka Samal  
and Onwuka Frederick

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# Analytical Representation of the Degrees of Freedom: Referencing Quantum Cosmology

DEEP BHATTACHARJEE\*, Integrated Nanosciences Research, India

SOUMENDRA NATH THAKUR\*, Tagore's Electronic Lab., India

PRIYANKA SAMAL, Integrated Nanosciences Research, India

ONWUKA FREDERICK, Ekiti State University, Nigeria

\*Corresponding authors: Deep Bhattacharjee, Email: [itsdeep@live.com](mailto:itsdeep@live.com) and Soumendra Nath Thakur, Email: [postmasterenator@gmail.com](mailto:postmasterenator@gmail.com). †All authors hereby declare no conflict of interests related to this paper. This paper is a modified and updated version of the paper; the PDF of which can be downloaded from: <https://vixra.org/pdf/2108.0087v1.pdf>

Concepts: Starting from the fundamental concepts, this paper delves further into the logic and representing of complex geometry and the degrees of freedom associated with it referencing the Calabi-Yau geometry considering the superstring formalisms along.

Additional Key Words and Phrases: Hypersurface, Higher Dimensions, Glome, Torus, Dimensional Analysis, Quantum Gravity, Fundamental Constants, Calabi-Yau Manifolds, M Theory, Duality, Multiverse, Mobius Strip, Klein Bottle,  $D(p)$ -Branes.

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**Abstract:** This exploration delves into the world of dimensional analysis, a fundamental tool in mathematics and physics taking into effect the topological criterion associated with it notably; supersymmetric string theory and other compactified dimensions with further insights into the geometry of Klein bottle, mobius strip and curled up compactified extra dimensions like 6-D Calabi-Yau manifold. Dimensions, such as length, width, height, and time, are the basic attributes that define our physical reality. The powers of these dimensions play a pivotal role in understanding how various physical quantities are interrelated. This study introduces the concept of dimensional transitions, both from lower to higher dimensions and vice versa, shedding light on how new dimensions and powers are introduced, presenting mathematical challenges, and deepening our comprehension of the physical world. The study showcases mathematical equations central to quantum mechanics and quantum gravity, elucidating the intricate relationship between energy, length scales, and fundamental constants. Of particular interest is the interplay of dimensions and powers in equations that encompass spatial and temporal dimensions, emphasizing the influence of the fourth dimension, often associated with time. This analysis demonstrates that by following this method, dimensional analysis becomes a powerful tool for exploring the relationships between dimensions in the physical world and their role in mathematical and physical equations. In summary, this work unravels the complexities of dimensional analysis and its applications, offering insights into how it contributes to our understanding of the fundamental laws of physics.

\* Part of this research is being carried out at Tagore's Electronic Lab., West Bengal, India/v1 on dimensional analysis.

# I. OVERVIEW

The realms of mathematics and physics are inherently governed by the fundamental attributes of our physical world, known as dimensions. These dimensions encompass the very essence of our existence, including length, width, height, and time. The interplay of these dimensions and their powers forms the basis of dimensional analysis, a powerful tool that unravels the relationships between these attributes and how they influence the world around us. At its core, dimensional analysis provides a systematic approach to examining how physical quantities are connected, offering a deeper understanding of the intricate tapestry of the universe. One of the most intriguing aspects of this analysis is the exploration of transitions between dimensions, whether from lower to higher dimensions or the reverse journey. These transitions introduce novel dimensions and powers, a mathematical puzzle that deepens our appreciation of the physical world. This exploration embarks on a journey into the heart of dimensional analysis, presenting a mathematical voyage into the core concepts that underpin quantum mechanics and quantum gravity. These equations vividly illustrate the complex interrelationships between energy, length scales, and fundamental constants. Among these equations, the significance of the fourth dimension, often intertwined with the concept of time, shines brightly, emphasizing the profound influence of temporal dimensions on the physical universe. By following this analytical method, we embark on an odyssey through the realms of dimensions, revealing the secrets they hold within mathematical and physical equations. In the following discussion, we unravel the complexities of dimensional analysis and its applications, shedding light on how it contributes to our understanding of the fundamental laws of physics.

Higher dimensions are impossible to visualize as the size of dimension varies inversely proportional to its level. The more the dimension ranges, the least its size. We are a set of points living in a particular point of space and a particular frame of time. i.e, we live in space-time. The space has more dimensions that meets the human eye. We are living in a world of hyperspace. Our world being a smaller dimension is floating in higher dimensions. The quest for the visually of higher dimensions has been a fantasy to mankind but this aspect of nature is completely locked. We can transform dimensions i.e., from higher to lower dimensions, or from lower to higher dimensions, but only through mathematics. The relative notion of mathematics helps us to do the thing, which is perhaps impossible in the experimental part of physical reality. Humans, being an element of 3 Dimensions – length, breath, height can only perceive one higher dimension, that is space-time. but beyond that the notion of dimension itself changes. The dimensions got curled up in every intersection of the coordinates of space in such a way that the higher dimensions remain stable to us. But in reality, it is highly unstable. In the higher dimensions, above 4, the space is tearing apart and joining again spontaneously, but the tearing portion itself covered by 2 dimensional Branes which acts as a stabilizer for the

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<sup>†</sup>Author address:

Deep Bhattacharjee: itsdeep@live.com,

Soumendra Nath Thakur: postmasterenator@gmail.com,

Priyanka Samal: priyankasamal9437@gmail.com,

Onwuka Frederick: frederickonwuka14@gmail.com

<sup>\*</sup>This work is carried out on the basis of the topological aspects of detailed research on dimensions and its associated properties on relativistic formulations and string theory.



## **1. Flatland vs. Higher Dimensions:**

The minimum dimensions possible theoretically for an object to exist is 2D. So, a piece of paper if be a universe of Flatland then all the square, circles, triangles are existent in that dimension. They can move freely sideways but cannot move upwards and downwards because in doing so the Flatlander's will require to jump from 2 Dimensions to 3 Dimensions world and that is not possible in viewpoint of Physics. Similarly, we live in a 4Dimensional world but in actual our co-dimension is  $4-1=3$ . Why? Because the temporal dimension of Space-Time is beyond our reach. It exists as if a continuous flow encompassing the lower dimensions.

## **2. Why do we exist in a 3-Dimensional world?**

We are being an object of 3 Dimensions are capable of moving up, down, top, bottom, sideways – But we cannot behold the higher dimensions. If we behold our higher dimensions, then our way of direction will emerge in a more variety of way and we can see things from different perspectives that are not possible within our dimensions. Even if any higher dimensional object comes in our dimensions, then we can only see its 3D cross-section, but we cannot perceive its actual structure. We will see them as Blobs. In the same way a flatlander will perceive the 3Dobject like Sphere, Cube, Cone, Pyramid into a 2D cross-sections like for Sphere they will see a circle, for cube they will see a square, for cone they will see a triangle and for pyramid they will only see 3 separate triangles. We are being an object of 3D cannot perceive anything beyond our own. There may be some hidden directions, hidden perspectives which are completely blind to us in our 3D world. So, if the aliens visited us and they are of higher dimensional object then we will only see their 3D cross-sections. A square when extended its edges to 90 Degree becomes a Cube and a Cube when extends its edges to 90 Degree becomes a Tesseract or hyper-cube. We can see only their projections, but we can't see their real structure.

## **3. What are the advantages of Higher Dimensions?**

There are many and some are we can pass through a barrier easily; we don't have to face difficulty is untying knots and we can mage surgery without any penetration.

## **4. What is Hyperspace?**

According to String Theory our Space has up to 10Dimensions. Although sounds weird but still its a mathematical fantasy. Our Space can be imagined as a grid of numerous lines and within the intersection two lines there exists numerous higher dimensions compactified in a K-Manifold or Calabi-Yau manifold. Our space has more dimensions that can meet our eye. We are still unaware of many of the hidden mysteries of dimensions of space and these dimensions are called as hyperspace. Every known element of this nature like Na, Ca, K, Mg, etc. has a link to these higher dimensions. They are made of quarks which are again made up of strings which vibrates at Planks Length that is  $1.6 \cdot 10^{-36}$  meter and they are linked with the higher dimensional universe. The strings vibrate and each mode of vibration corresponds to a specific element of this known universe. Our observable universe has more than 4Dimensions but in essence only 3-Dimensions are tangible. And our duplicate universe in a multiverse model has 6-Dimensions but in essence there can be more than 10Dimensions but “No Ghost Theorem” forbids them to enter into physics because of their negative norms. And there exists any universe of a much more dimensions but they can be beyond our known intuition & a variety of complicated

creatures might live inside it. To us the higher dimensions are complicated just like the 2D flatlanders with the 3 Dimensions [1-8].

### **III. PARAMETERS**

#### **1. Introduction to Dimensions and Dimensional Analysis:**

Begin by introducing the fundamental concept of dimensions and the role they play in the physical world. Explain that dimensional analysis serves as a bridge between these dimensions and their powers, aiding in understanding their relationships.

#### **2. Defining the Problem: A Transition between Different Dimensions:**

Emphasize the central theme of examining transitions between dimensions, whether from lower to higher dimensions or vice versa. Highlight the novelty these transitions bring by introducing new dimensions and powers, thereby deepening our understanding of the physical world.

#### **3. Planck Equation and Energy-Frequency Relationship:**

Dive into the Planck equation and its significance in quantum mechanics. Explain how this equation relates energy to the frequency of particles or quanta. Break down its components, including the energy  $E$ , Planck constant ( $h$ ), and frequency ( $f$ ), to showcase the fundamental relationship.

#### **4. Planck Length Conversion:**

Explore the Planck Length Conversion equation, which links the Planck length ( $\ell_P$ ) to essential constants. Discuss how this equation establishes a fundamental length scale within the context of quantum gravity, shedding light on the interconnectedness of length scales.

#### **5. Conceptual Framework Equation:**

Analyze the Conceptual Framework Equation, illustrating how it connects energy  $E$ , Planck length ( $\ell_P$ ), the speed of light  $c$ , and fundamental constants. Elaborate on its role in quantum gravity and emphasize the relationships it unveils within the realm of energy, length scales, and fundamental constants.

#### **6. Boundaries of Perception and the Fourth Dimension Equation:**

Introduce the concept of the fourth dimension, often associated with time, and discuss its significance. Explain how the Fourth Dimension Equation captures the transition from three-dimensional space to the fourth-dimensional time and back to three-dimensional space, emphasizing the interconnectedness of spatial and temporal dimensions.

#### **7. Analyzing Powers and Dimensional Changes:**

Explore the intertwined relationship between dimensions and their powers when transitioning between different dimensions. Provide examples, such as the transition from one-dimensional length ( $L$ ) to two-dimensional space ( $L^2$ ) and the transition back, which

involves powers of dimensions. Highlight the mathematical representations that capture these dimensional changes.

### **8. Application to Quantum Mechanics and Quantum Gravity:**

Delve into the application of dimensional analysis in quantum mechanics and quantum gravity. Discuss the variation in the power of the speed of light  $c$  between equations related to spatial and temporal dimensions. Emphasize how these variations are tied to the number of spatial dimensions considered and the introduction of the temporal dimension.

### **9. Dimensional Transitions in Quantum Gravity:**

Explore scenarios in quantum gravity where the power of the speed of light  $c$  differs between spatial dimensions and the time dimension. Discuss the transition from a four-dimensional time frame to a three-dimensional space, representing a change in dimensionality. Mathematically represent this transition to highlight the dimensional relationship.

### **10. String Theoretical Approach:**

The dimensions of D-Branes regarding IIA is ODD that is, 1,3,5,7,9 while the dimensions of D-Branes regarding IIB is 0,2,6,4,8 which is EVEN. The closed strings can move freely while the open strings are attached to the D-Branes with their two open ends. Apart from the Bosonic theory there were 5 string theories which can't be regarded as the candidate for the TOE. This theory accounts for 10 Space-Time dimensions where the rest 6 dimensions are compactified to a very small curled up single dimension with the structure of a Calabi-Yau manifold. These 5 String theories can be called as a Supersymmetric string theory because of the notation of supersymmetry which again can be unified by a single unified theory called the 'M'-Theory considering the Supergravity along with 11 spatial Dimensions. Two theories when are completely different, then they can be linked with the concept of T & S duality thereby making the different theories into a single unified framework by the option of one theory being dual to the other by the incorporation of the dualistic Principles. In String theories there are 10 Space-Time dimensions which means there are 9-Dimensions of Space along with 1-dimensions of time. Now if 1-dimensions can be assumed as the circumference of the Circle then one can return from the point where they started. A string when travelling around the circle then there exists different states of momentum and energy and in addition the strings wrapped around thereby forming a tension force of stretching and these wrapping modules are called the winding number. Now, the momentum of the strings in a curled-up dimensions of radius  $R$  is similar to the winding number of the strings in a circle of radius  $1/R$  and vice versa, which makes the strings inter-related to each other by the notion of this Duality Principle. The large distance of momentum varies with the small distance of winding number and the small distance of winding number varies with the large distance of momentum. In any way the 5 different string theories can be interrelated by a maximum of 2 to 3 theories. T-duality relates Type IIA string Theory with Type IIB String Theory and compactify their dimensions curled up in a circle and if a worldsheet is considered then they get compactified into a cylinder where the large radius is inherently linked with small radius and the momentum is related with the winding number. The notion of large and the notion of small are became one and same, their properties inseparable from one another. They got wind up in higher dimensions. A string can be either left moving or right moving with the center of the string momenta is considered as the sum of the right and

left fields whereas the stretch in the middle is considered as the difference. Now, there comes a second concept of S-Duality which considers a particular type of aspect called coupling constant which means the ability of the string to emit or absorb other strings within itself. Strings carry mass and electric charge, and they may do the process of emitting and absorbing other strings or attached and decay to other strings by a method called perturbation theory. String theories have a coupling constant which follows the phenomena of perturbation theory which depends upon the mode of the oscillation of the strings having the coupling constant small or large. If two strings are linked by S-Duality, then a string with weak coupling constant is linked with strong coupling constant and by this process the two string theories can be linked with each other. By means of string theory Type I is related to Heterotic SO (32) and Type IIB theory with itself. Further type IIA is of strong coupling constant and behaves as a 11-Dimensional theory with 10 Space-Time dimensions and 1 Time dimension which incorporates the 11-Dimensional M-theory. In case of T-Duality the momenta take on discrete values that means if the energy of one string is 1-Joule and the other string is 3-Joule then there can't be no values in between them. The values are always discrete in nature. The existence of these dualities are very important in superstring theories which helps to unify the theories into a single structural framework of M-Theory. In case of the duality, the geometry of the Space-Time breaks into Planck's scale Physics of  $1.6 \times 10^{-35}$  meter and T-Duality is closely related to another concept of symmetry called as mirror symmetry which is a part of the enumerative algebraic geometry.

The concept of duality makes the two different theories to be dual in a nontrivial way such that the properties of one are dual to the other. In Particle physics, Particles are of 0-Dimensional points which can be further extended to 1-Dimensional strings. The mathematics of the string theory can be studied in various higher dimensions apart from the Three Spatial dimensions of Length, Breadth, Height that is Up/Down, left/Right, Backward/Forward... indeed it takes 1 more Dimensions of Now/Then called the Time-dimensions along with the other 6-Dimensions compactified & curled up in circles. The idea of this curled up circles comes from the Kaluza-Klein theory of 5<sup>th</sup> Dimensions. If one watches a wire from its near, then the thin wire appears to be of a cylindrical object. But, if he moves away further and further from that wire then the wire will become like a 1-Dimensional String or rod rather than 3-Dimensional wire. So, an ant walking over the wire will see the wire as a straight line from its point of view but from your point of view the wire is of 3-dimensional. But the ant can't recognize it. This is because of the fact that the radius of the wire is getting smaller as it is moving farther from you and ultimately got curled up into a 1-Dimensional object but from the viewpoint of the Ant, the wire is just like flat. So, the higher dimensions are so curled up that its impossible for us to notice stat just like the ant if we stayed on that wire. Winding number is a number by which a curve circles a plane from a given point counterclockwise. If each string is closed without any end points, then the winding number is the number of turns or curves the string makes. If the counterclockwise rotation is 8 times and clockwise rotation is 4 times, then the winding number is equal to  $8-4$  that is 4 turns. The counterclockwise turns behave as negative while the clockwise turns behave as positive. Their difference will result in each winding number. The momenta of the strings are quantized, and the theories can be simultaneously replaced by the changing of momentum and winding number. The duality of the strings is the equivalence principle with the total energy remains unchanged.

If string theory considers 10-Dimensions, then the 4 are the spatial dimensions whereas the other 6-Dimensions are compactified into a single curled up dimension forming a structure called the Calabi-Yau manifold. The strings propagate over these manifolds and



vibrates in a chaotic order with ends either open or closed. It was eventually found that a single Calabi-Yau manifold doesn't determine the property of the string analogy rather two distinct Calabi-Yau manifolds represent the same analogy by a process called as the Mirror Symmetry. This is an important duality in string theory as because they are mirror to one another and helps to solve many complicated problems in string analogy. If the Calabi-Yau manifold is dissected into two pieces, then also the Mirror Symmetry can be observed in accordance with T-Duality. A torus can be called as the simplest manifestation of the Calabi-Yau manifold which can be treated as the product of two circles. The circles are organized within the torus space and the torus itself acts as a space between the circles. In this case the mirror symmetry can be viewed as the T-Duality acting itself on the longitudinal circles of the torus. T-Duality is called topological duality whereas S-Duality is called as strong weak duality.

In String theory, the 0-dimensional particles are replaced by 1-dimensional vibrating strings having mass and charge and they propagate throughout Space-Time freely without any restrictions, in fact they are very chaotic in nature, this strings can be open as a curved line or closed as a loop but when the strings are viewed from the macro perspective they can be represented by particles which are several in nature depends upon the distinct vibrational pattern. Two small strings combine to form a large string and one large string can get splits up into two small strings. String theory needs extra dimensions of about 10 for mathematical simplicity and the extra dimensions are compacted to form a comparatively curled up lower dimensions.

String models have been trying to develop in which strings represent the high energy Physics and so for this the extra dimensions must be compactified to lower dimensions for the simplicity. Or in other ways, the extra dimensions can be restricted. When Calabi-Yau manifolds have become a notion for compactifying the extra dimensions then two versions of the string theory can be compactified into two distinct features of the Calabi-Yau manifolds of Type IIA and Type IIB thereby producing a Mirror Symmetry. Mirror Symmetry states that two different models called A-Model and B-Model are equivalent in the same way such that there is a duality in between them. Actually, these two Calabi-Yau manifolds give rise to the same physics by incorporating The Mirror Symmetry. The application of mirror symmetry belongs to the branch of mathematics called Enumerative Geometry which raises the questions of counting the number of solutions to a geometric question. Calculations of the mirror Symmetry of the B-Models is much easier than the A-Models. Gauge theory is also related to this type of Symmetry.

When the notion of strings is expressed in higher Dimensions then there comes the notion of P-Branes, P being a variable incorporate many dimensions. The world Brane comes from the "membrane" which represents 2-dimensional Branes. D-Brane is an important part of the Brain that arises when one considers the open strings whose end points are attached to the Branes. Whereas Closed Strings can move freely from one brane to another or one dimension to another. Just as Graviton is an example of Closed Strings, the photons are examples of Open Strings, so, as the end point of photons are attached to the Branes, they can only vibrate within the boundary area of the Brane but not away from it. The letter 'D' in D-Brane refers to Dirichlet Boundary Conditions. Sub manifolds are manifolds which rests within a manifold. A sub manifold is a surface embedded inside the Calabi-Yau manifolds and D-Branes are just like the sub manifolds, but the end point of the strings contains the charges. In A-model D Branes can also be viewed as sub manifolds within the Calabi - Yau manifolds but their length, breadth and height are minimized to half. A torus is made up of infinitely many circles which can decompose the Torus. These

circles parameterize the circles when they are decomposed meaning that there is a correspondence with the circles and the points. A Torus is the union of these two circles and the circles lay down one after the other to form the Torus. This auxiliary space plays a very important role in SYZ conjecture. The Calabi-Yau manifold has 6-Dimensions. They can be divided into 3-Tori which is a topological manifold of 3-Genus or holes and a 3-Sphere. In a normal sphere or a 2 Sphere there is a 2-dimensional boundary over a 3-dimensional sphere but in a 3-Sphere, there is a 3-dimensional boundary of a 4-dimensional hypersphere. If a Torus represents a Space-Time, then the strings can propagate through the Space-Time. According to the T-Duality a string when propagating through Space-Time can have momentum as well as winding number. If the Torus can be split into many circles and the T-Duality can be applied then, a new Torus will be formed which is the mirror symmetry of the Calabi-Yau Manifold [1-6, 9-16].

## **IV. SHAPE OF THE UNIVERSE**

Our universe is not a square or a triangle. The circle being a universal shape dominates our universe in the form of multiple coordinates or higher dimensions. Nobody knows for sure what exactly is the shape of the universe? I did some little research a few months ago and came into a conclusion which is rather imperfect but still holds true in many aspects. Our universe is a glome. A higher dimensional form of sphere where there are 4 Spatial coordinates along with additional time coordinates which is independent and orthogonal to the spatial coordinates. Excluding time, if A 3D Sphere can be projected via stereographic projection into a 2D Screen we will see a mere flattened circle or an ellipse. Similarly, the glome or 3-sphere which is 4D if projected on a 3D background then this will turn to an ellipsoid. An ellipse with volume. This is rather unphysical, how can there be only one ellipsoid when there are numerous ellipsoids stacked themselves in all the Coordinates from every possible direction with parallels, meridians and hyper meridians that stacks up a hypersphere. It is no doubt that our universe is higher dimensional and what we perceive is just a lower dimensional blob. A 3D Cross section of a 4D universe. Our universe is a closed curve. Every lines from the starting point will eventually curved back and end in the own point where it starts. Our universe if is a closed curve with a stack of ellipsoid in it then this can be possible that the homogenous universe, we see is an illusion. There are several gaps between each ellipsoid projected on any one of its 4 axis. The Projection is stereographic or point to point projection and denotes that holographic Principle holds true for such projection. I cannot say on the basis of my thinking that glome is totally an hologram. Rather I would say that universe has no surface just like the sphere has no length. The universe If is surface-less then there must be a huge gap between the two consecutive ellipsoid, and this denotes the dark energy or the expansion of the universe. A sphere which is 3D normally is not visualizable at its Centre. But to drop a dimension and then try to visualize the Centre of a circle is easy. In the same way the universe being a glome or a hypersphere when projected via stereographic point to point projection then it generally shows that the centre of the universe is visible. As I have said that universe is homogeneous and isotopic, the centre of the universe is everywhere. From every location, it seems that you are at the centre of the universe. Although universe is finite but relativistically universe is infinite due to its accelerated expansion. The bridging point between the ellipsoid or where 2 ellipsoid meets in the GLOME can act as a patch of high dark matter disk. When the sun revolves around the black hole at the centre of the galaxy it wiggles up and down due to the gravity of the dark matter which is tremendously heavy in this region. The baryonic matter is less compared to the non-baryonic matter and this leads to the increased or fluctuating gravity which triggers the comets from the Oort cloud which have an impact on the mass extinctions. Scientists showed that the dark

matter disk or the patches of dark matter along the path of solar revolution (Approx 250 million years) can be the potential cause for the wiping out of the dinosaurs due to meteor shower. However, I try to link this theory with the help of the hyperspherical coordinates of the glome. The glome has a volume and one possible fact is that everybody is trapped outside the glome. The hypersphere has both the properties of a topologically curved spaces as well as the 4<sup>th</sup> Dimension. The 5<sup>th</sup> dimension might be time. The extra – 6,7,8,9,10 are the spatial curled up hidden dimensions. If we consider our universe as a Dp(5)Brane then our universe is floating in the bulk of 10D. Colliding universes may exist and this can alter the shape of the coordinates. Generally, a hypersphere having no surface area has a positive curvature. But with the special alignment of the other 4 axis – the hypersphere can be squeezed down into a flat disk and we humans are the 1D objects than relatively compared with the flat 2D disk of the hypersphere, but this never happens. Our universe can also have a chaotic shape like an ameba. Or two amoebas joined together with a tiny tissue. In that case we cannot rule out the effect of hypersphere. We have to think of this universe as a truncation point between the D-7 Branes where all the matter curve exists and their junction may give rise to a U(1) Abelian Gauge Bosons. But that D-7 Brane or the 7D is a part of a higher dimensional Calabi-Yau 4-Fold which is too infinitesimal to notice. We can say for sure that our universe is not a hypercube or a tesseract. But we can say that our universe might be a hypersphere where the extra dimensions are hidden from us and what we see is their projection on the 3D-Dp-Branes. Our universe is floating like a bubble in a multiverse. What if the coordinate of the universe is also changing with the floating speed? That means our universe squeezes from its normal value but that doesn't happen. The curvature can only be either positive or zero, it can't get negative. Negatively curved universe is not our universe as because the coordinates will never align in such a way that the curvature goes below zero. This is mathematically impossible. Therefore, as it's hard to find the proper shape of the universe, it is also hard to determine what probabilistic shape the universe holds that's unknown to us. Holographic Principle is a very nice idea. Moreover, the Conformal mapping of the stereographic projection is angle preserved and the CFT/AdS correspondence holds true for the geometry of the universe.

Well, our universe is a bubble floating among multiple universe or multiverse. But the concept of multiverse is still an oblivion to the mathematician. There can't even be an infinitesimal limit that this conjecture can be proved. So, better left aside the concept of multiverse and proceed towards the topological geometric shape of our universe or perhaps many such universe if an identical universe just as ours is existent somewhere in the multiverse. Perelman proved the Poincare Conjecture there by showing that how a spherical universe can be splitted into two different universes by the formation of singularity in between them. Let's consider our universe as a sphere, then it will elongate from both right or left, thereby expanding it longitudinally like a cylinder. Now after a certain amount of expansion, the sphere will take the shape of a barbell, with the two opposite sides inflating and in between contracting thereby creating a singularity. This singularity will then diminish its size and then thereby ultimately disappears by creating two different universes. But in reality, our universe is not at all so smooth like a sphere. It doesn't have a spherical symmetry either. Then what exactly is the probabilistic shape of our universe? Well, mathematics has the answer. Nobody knows exactly the shape of the universe but instead we have known the size of the so-called observable universe from the CMBR or Cosmic Microwave Background radiation. Our universe is not at all a sphere because it has certain properties which the spherical symmetry doesn't always support. But from the CMBR and the red shifting of the distant supernovas it has been calculated that at any three points from a certain region of the universe will provide a Triangle with

slightly more than 180 degrees. This states that our universe is somewhat flat with a little positive curvature. But that's not at all accurate with a 100% accuracy, maybe it is hyperbolic or maybe it is elliptical or maybe it is flat. Now, the geometry of the universe is somewhat complicated, it is not the geometry of the universe rather it is the geometry of the observable universe. And there are some definite features,

- [1] If one goes from one end of the universe, then he will come to its initial position upside down.
- [2] If one travels from the outside of the universe, then he will eventually get to the inside of the universe and again comes to its surface thereby making the notion that outside meets inside. But that is physically impossible.
- [3] If we see one star in a sky, then we can see its duplicate or another star in the sky, that means for every star there is its duplicate star. Why? Maybe its because the light from one star goes around the edges of the universe and travels back in a final position carrying the reflection of this original star. Maybe its possible if our universe has a positive curvature like that of a cylinder or sphere.

Now if our Universe is a hypercube or simply a cube, although its not, its of a higher dimension, but still if our universe is a cube for the sake of simplicity then if we draw two parallel lines then they will never intersect with each other as because there are no connecting points. Its all open. The lines will go forever and thereby providing a notion that our universe may be infinite. But on the other hand if our universe is some sort of a sphere or any geometrical object of positive curvature then if we draw two straight lines or specifically two geodesics then they will meet with each other after a certain amount of time. This may provide us a notion that our universe is finite with a connecting point. Now, this concludes that we are living in an infinite portion of a finite universe. That means from micro point of view the universe is finite but from macro point of view our universe is infinite. Whatever is infinite is finite and whatever is finite is actually a finite.

Actually, whatever above is happening is true in respect to our observable universe but not from unobservable universe. This is because there are still some light waves that haven't come yet to the earth. Actually, whenever we are looking at the sky, we see only the past. For example, the sunlight is 8 minutes older, the farthest stars are million light years older and so on, so on.

Take a longitudinal piece of paper. Paint one side green and the other side red. Now twist the paper and glue to two ends. You will get a strip called Mobius Strip. It is non-orientable topology and has only one edge and if an ant travels from the outside he will come to its initial position as its mirror image without even crossing any edge. That means mobius strip represents a mirror image of an object if he travels from one point and return to the other. One thing you will notice that, in the Mobius strip which you had made, the green side joins the red side. And the number of twists is 1. So, the next twist will not be a Mobius Strip anymore. Because the green side will join the green side and the red side will join the red side. But in the next turn again a Mobius Strip is created. So, to create a Mobius strip, just before gluing both open ends of the paper twist it in Odd turns like, 1, 3, 5, 7, 9 etc.... And in even turn this notion is completely absent. Any object travelling through the Mobius strip will seem to be on the same side although he has travelled through its both sides. And a mirror image is produced. That means the right-handed man will seem left-handed and vice versa. But before flipping you will be upside down. And after one flip then your exact mirror image will be produced. Mobius Strip is a surface with only one side any

one boundary. Its boundary is a simple closed curve that is homeomorphic to a circle. One of its important properties of depicting it as a single boundary is that, if one draws a line on one side, it will join to the line on the other side without even crossing its edge. If you cut the strip into a scissor, then one will get a long strip with two full twists in it rather than two separate strips. This happens as the strip has only one edge that is twice as long as the strip. Well, what is homeomorphic. It depends on the genus but not on the structure. A coffee mug is homeomorphic to a torus or doughnut as because the torus got one genus that is a puncture in its middle just like the same way a coffee mug got one genus in its handle attaching the mug body. So, although the Mobius Strip is homeomorphic to a circle its boundary is not at all a true circle. But it is possible to embed the Mobius Strip in 3-Dimensions so that the boundary acts like a closed space or a perfect circle under certain circumstances. And, by this way we can get a Klein's bottle, when two Mobius Strips are joined together. A Klein Bottle is an object of 4th Dimension, and it has no genus but it's the surface where the outside meets the inside and one travelling through its surface will experiences no edge or boundary and will ultimately return to its starting point upside down. So, it's time to explore a Klein's bottle now.

A Klein Bottle is a mathematical extended part of the Mobius Strip in 3-Dimensions (For simplicity 3D model is taken although it is of 4D) non-orientable 2-Manifold against which the notion of normal vector can't be continuously determined. It is a Null-Sided surface without any boundary through which a traveler returns to its starting point upside down. If you watch the Klein Bottle properly then you will see that the bottle itself intersects the bottle thereby making a 2-Dimensional Circular cross section. But that is not real. For the visual clarity this model has been adopted. But, for the 4th Dimension there is no notion of intersection into its own body, rather the tube will enter in its own body without any circular 2D Cross-section. Klein Bottle is the only bottle in topology that is self-intersecting. Regarding the portion of the cylindrical self-intersection, there is an abstract 4D analogy – Let's consider that the number 8 is self-intersecting at the middle. But if we higher up one line in the intersecting position then the two lines will never intersect. The same thought experiment can be applied in the Klein Bottle analogy of self-intersection portions. The cylinder or the bottle is growing but is not piercing its existence as it is of 4th dimension. Well, its quite complicated stuff to understand. Klein Bottle is closed with connecting points, and it has a positive curvature. It is homeomorphic to the sum of two projective planes and homeomorphic to a sphere. If you watch the rail lines from the distant then you will notice that the two parallel lines of the single tract intersects at infinity. This is just an optical illusion and called as a projective plane. The traditional mirror Klein bottle image is achiral that means it is not supposed to form mirror symmetry or it is indistinguishable from its original image when projected to a Mirror Plane. If the traditional Klein Bottle is to be cut, then it deconstructs into 2 oppositely Chiral Mobius strips.

Now, what will happen if the Klein Bottle don't self-intersect into itself rather it becomes a continuous closed boundary-less loop, then it will take the shape of a Torus. And this is the actual shape of our observable universe according to CMB.

A Torus can be got by revolving a Circle in 3-Dimensional Space about an axis coplanar with the Circle. When the axis of revolution does not touch the surface then one can get a ring-shaped structure or a toroidal revolution. In Geometry the set of points in space are coplanar if there exists a plane that contains all the points into its surface. A ring torus is

homeomorphic to the Cartesian product of two circles. These circles when rotated generates a 3d Euclidean Space. A torus consists of an infinite alignment of 2 Types of circles, the longitudinal circle or the 'big circle', the latitudinal circle or the 'small circle'. It can also be called as major and minor circle. The Family of 3-Tori makes up a compactified 6D objects in String Theory called the Calabi-Yau Manifold. The Torus being a product of two interlocking Circles of is the key element to be taken into consideration while unveiling the shape of the Universe. The circle in a torus is closed path. A simple manifestation of the Torus is by joining the two ends of the cylinder together. Our universe can be called as a Hypertorus or N-Dimensional Torus. Now, there is an amazing property of Torus. If you take a flat paper of 2D and draw two lines longitudinal & latitudinal then makes up a cylinder and glues up the open ends of the cylinder to make a torus you will find that although the two lines are of same length but when the torus is formed the circles created by the two lines are varying in size. One becomes larger than the other.

Now, if the exact shape of our universe is a Horn torus. In Horn torus the Major Radius is equal to the Minor radius. And in this sub-group of Torus, it has no hole in the middle. The middle point of the Torus acts as the Singularity which creates the big bang. Matter curves space and space tells matter how to move according to GTR. So, the Space itself is just like a 'Bunked Racetrack' where anything can move without accelerating or decelerating or changing its direction. So, as the curvature is must steeper in upper and lower sections so when Big Bang occurs matter erupts from the White Hole with a very giant speed called inflation which occurs about  $10^{-36}$  seconds after the singularity and lasted for  $10^{-32}$  to  $10^{-33}$  seconds. And then as the matter spreads around its curvature so, its speed becomes much less, and the curvature is very less inclined and so the universe takes on an expansion rate rather than an Inflation rate. Now whatever the geometry is, we need to understand what makes up the universe (our universe) and this develops a question why matter dominates anti matter? There are two possible reasons,

- [1] Matter is created more than the Anti-Matter and when the Matter, Anti-Matter Annihilates then the excess matter which is left has become our stars and planets.
- [2] The matter and Anti-Matter has been created at an equal amount but they are still far away to come in contact with each other. So, no question of any annihilation comes [7-24].

## V. DIMENSIONAL ANALYSIS

### 1. Introduction to Dimensions and Dimensional Analysis:

Dimensional analysis is a powerful tool that helps us explore and understand the relationships between dimensions, their powers, and physical quantities. Dimensions, such as length (L), width (W), height (H), and time (T), are the foundational attributes of our physical world. The powers of these dimensions play a pivotal role in revealing how these quantities interact and relate to one another.

A physical quantity (Q) can often be expressed as a function of its fundamental dimensions:

$$Q = f(L, W, H, T)$$

## 2. Defining the Problem: A Transition between Different Dimensions:

One of the most intriguing aspects of dimensional analysis is the examination of transitions between dimensions, whether it's a transition from lower to higher dimensions or vice versa. These transitions introduce new dimensions and powers, presenting mathematical challenges that deepen our understanding of the physical world.

## 3. Planck Equation and Energy-Frequency Relationship:

The Planck equation is a fundamental expression that relates energy ( $E$ ) to the frequency ( $f$ ) of a particle or quantum:

$$E = hf$$

Where:

$E$  is the energy of a quantum.

$h$  is the Planck constant.

$f$  is the frequency of the quantum.

Planck Length Conversion:  $P_\ell = \sqrt{\frac{\hbar G}{c^3}} \Rightarrow$  This equation establishes a vital link between the Planck length ( $\ell_P$ ) and fundamental constants, creating a fundamental length scale within the domain of quantum gravity:

Where:

$P_\ell$  is the Planck length.

$\hbar$  is the reduced Planck constant.

$G$  is the gravitational constant.

$c$  is the speed of light.

Conceptual Framework Equation:  $E = \frac{P_\ell c^3}{\sqrt{\hbar G}} \Rightarrow$  This equation connects energy ( $E$ ) to the Planck length ( $P_\ell$ ), the speed of light ( $c$ ), and fundamental constants. It illustrates the profound relationship between energy, length scales, and fundamental constants within the context of quantum gravity:

Where:

E is the energy associated with quantum gravity.

$P_\ell$  is the Planck length.

c is the speed of light.

$\hbar$  is the reduced Planck constant.

G is the gravitational constant.

#### 4. Boundaries of Perception and the Fourth Dimension Equation:

This equation introduces the concept of the fourth dimension, often linked with time, as a dimension beyond the typical three spatial dimensions. It captures the transition from three-dimensional space to the fourth-dimensional time and back to three-dimensional space:

$$\sqrt{\frac{\hbar G}{c^5}}$$

Where:

The "Fourth Dimension" represents time, distinct from spatial dimensions.

$\hbar$  is the reduced Planck constant.

G is the gravitational constant.

c is the speed of light.

#### 5. Analyzing Powers and Dimensional Changes:

Dimensionality and powers are intertwined when transitioning between different dimensions. For example, consider L as a representation of one-dimensional length and  $L^2$  as a representation of a two-dimensional plane.

*When transitioning from a lower dimension to a higher one while staying within the higher dimension, there's no need to "return" to the lower dimension. The inherent power of the higher dimension itself suffices to encompass the lower dimension. For example, transitioning from one-dimensional length (L) to a two-dimensional plane ( $L^2$ ) within two dimensions involves the power of  $L^2$ , corresponding to the higher space.*

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Defining the Problem: A Transition between Different Dimensions: One of the most intriguing aspects of dimensional analysis is the examination of transitions between dimensions, whether it's a transition from lower to higher dimensions or vice versa. These transitions introduce new dimensions and powers, presenting mathematical challenges that deepen our understanding of the physical world.



*Conversely, when moving from a two-dimensional plane ( $L^2$ ) to a lower dimension (one-dimensional length,  $L$ ) while staying within the lower dimension, this transition involves an increase in dimensionality. Mathematically, this can be represented as  $(L + L^2) = L^3$ , effectively capturing the dimensional relationship between the lower and higher dimensions.*

## **6. Application to Quantum Mechanics and Quantum Gravity:**

Quantum mechanics and quantum gravity theories often explore fundamental constants, such as the speed of light, at extremely small scales or high energies like the Planck scale. The difference in the power of the speed of light ( $c$ ) between two equations (one associated with spatial dimensions and the other with time dimensions) is related to the number of spatial dimensions considered in each context. This discrepancy in the power of  $c$ , exemplified by  $c^3$  in one equation and  $c^5$  in another, corresponds to the number of spatial dimensions contemplated in each context. The fourth dimension, often associated with time, introduces an extra dimension beyond the three spatial dimensions, accounting for the difference in the power of  $c$ .

## **7. Dimensional Transitions in Quantum Gravity:**

Within the realm of quantum mechanics and quantum gravity, there may be situations where the power of the speed of light ( $c$ ) differs between spatial dimensions and the time dimension. An example is the transition from a higher, four-dimensional time frame encompassing length, height, width, and time (denoted as  $L^3+L = L^4$ ) to a lower-dimensional, three-dimensional space ( $L^3$ ) while adhering to the lower dimension (space). In this transition, we witness an augmentation in dimensionality. Since  $(c^3+c)$  equates to  $c^4$  in this instance, it can be mathematically represented as  $(c^4 + c) = c^5$ , effectively capturing the dimensional relationship between the lower and higher dimensions [20-36].

# **VI. DISCUSSIONS**

The mathematical presentation provided delves into the fascinating realm of dimensional analysis, shedding light on the intricate relationships between dimensions and their powers in the context of fundamental physics. This discussion aims to break down the key components and implications of this presentation.

## **1. Dimensions and Dimensional Analysis:**

The introduction of dimensions ( $L$ ,  $W$ ,  $H$ ,  $T$ ) as fundamental attributes of our physical world sets the stage for dimensional analysis. Dimensions are the building blocks of the physical universe, and understanding their role in mathematical equations is fundamental to comprehending the behavior of physical quantities. The concept that a physical quantity ( $Q$ ) can be expressed as a function of these dimensions ( $Q = f(L, W, H, T)$ ) is a cornerstone of dimensional analysis.

## **2. Transition between Different Dimensions:**

One of the central themes of this discussion is the transition between different dimensions, whether from lower to higher dimensions or vice versa. These transitions introduce new dimensions and powers, providing mathematical challenges and deepening our understanding of the physical world. The example involving the transition from one-dimensional length ( $L$ ) to a two-dimensional plane ( $L^2$ ) within two dimensions highlights how the power of the higher dimension is sufficient to encompass the lower dimension. Conversely, when moving from a two-dimensional plane ( $L^2$ ) to a lower dimension (one-dimensional length,  $L$ ), there is an increase in dimensionality. The mathematical representation of  $(L + L^2) = L^3$  effectively captures this dimensional relationship.

## **3. Application to Quantum Mechanics and Quantum Gravity:**

The mathematical presentation explores how dimensional analysis is applied to quantum mechanics and quantum gravity, areas of physics that examine fundamental constants and behaviors at extremely small scales and high energies, such as the Planck scale. A key point of interest is the variation in the power of the speed of light ( $c$ ) between equations associated with spatial and time dimensions. This variation is directly related to the number of spatial dimensions considered in each context. The introduction of the fourth dimension, often linked with time, introduces an extra dimension beyond the three spatial dimensions and plays a critical role in the differences in the power of  $c$ . These equations offer insights into the scales and relationships between energy, length, and fundamental constants across varying scales and dimensional contexts.

## **4. Dimensional Transitions in Quantum Gravity:**

This section of the presentation takes us further into the domain of quantum mechanics and quantum gravity, where the power of the speed of light ( $c$ ) can differ between spatial dimensions and the time dimension. The transition from a higher, four-dimensional time frame ( $L^3 + L = L^4$ ) to a lower-dimensional, three-dimensional space ( $L^3$ ) while adhering to the lower dimension (space) introduces an augmentation in dimensionality. The mathematical representation of  $(c^4 + c) = c^5$  effectively captures this dimensional relationship. This discussion highlights how the dynamics of dimensionality play a critical role in understanding these complex phenomena.

The mathematical presentation underscores the fundamental nature of dimensional analysis as a tool for exploring the relationships between dimensions, their powers, and physical quantities. By understanding how dimensions transition and affect one another, we gain deeper insights into the physical world and the universe's fundamental laws. Dimensional analysis serves as a powerful bridge between the abstract world of mathematics and the tangible world of physics, allowing us to unlock the mysteries of the universe and comprehend the interconnectedness of dimensions in the context of fundamental physics.

## VII. CONCLUSIONS

*This exploration has unveiled the intricacies of dimensional analysis, a cornerstone tool in mathematics and physics, and its profound implications for our understanding of the physical universe. Dimensions, which encompass fundamental attributes such as length, width, height, and time, form the very fabric of our reality. The powers of these dimensions serve as the key to deciphering the interconnections between various physical quantities.*

### 1. Dimensions and Their Powers:

The journey into dimensional analysis commences with the fundamental recognition of dimensions and their pivotal role in shaping the physical world. Length (L), width (W), height (H), and time (T) are the elemental attributes upon which our universe is built. The powers of these dimensions lay the groundwork for comprehending how physical quantities coalesce and correlate within the cosmos.

### 2. Transitions between Dimensions:

One of the core aspects of this study is the examination of transitions between dimensions. Whether traversing from lower to higher dimensions or undertaking the reverse journey, these transitions introduce novel dimensions and powers, bringing forth mathematical complexities that enrich our grasp of the physical realm. The transition from one-dimensional length (L) to a two-dimensional plane ( $L^2$ ) within a two-dimensional framework exemplifies how the power of the higher dimension is sufficient to encapsulate the lower dimension. Conversely, moving from a two-dimensional plane ( $L^2$ ) to a lower dimension (one-dimensional length, L) involves an elevation in dimensionality, captured mathematically as  $(L + L^2) = L^3$ . These transitions illuminate the adaptability and consistency of dimensional analysis in both mathematical and physical domains.

### 3. Applications in Quantum Mechanics and Quantum Gravity:

The exploration further extends to the application of dimensional analysis within the realms of quantum mechanics and quantum gravity. These domains venture into the behavior of fundamental constants, such as the speed of light, at scales as minute as the Planck scale. A particular point of focus is the variance in the power of the speed of light ( $c$ ) across equations tied to spatial and temporal dimensions. This variation directly hinges on the number of spatial dimensions considered in each context. The introduction of the fourth dimension, commonly entwined with time, ushers in an additional dimension beyond the customary three spatial dimensions, influencing the disparities in the power of  $c$ . These equations deliver profound insights into the scales and associations between energy, length, and fundamental constants across diverse scales and dimensional contexts within the realm of fundamental physics.

### 4. Dimensional Transitions in Quantum Gravity:

The journey takes an even deeper plunge into the terrain of quantum mechanics and quantum gravity, where scenarios may arise in which the power of the speed of light ( $c$ ) diverges between spatial dimensions and the time dimension. An exemplification is the transition from a higher, four-dimensional temporal framework, encompassing length, height, width, and time (denoted as  $L^3+L = L^4$ ), to a lower-dimensional, three-dimensional space ( $L^3$ ) while adhering to the lower dimension (space). In this transition, we witness an augmentation in dimensionality. The mathematical representation of ( $c^4 +$

c) =  $c^5$  effectively captures the dimensional relationship in this scenario, revealing the dynamic nature of dimensions in these complex phenomena.

In summary, this comprehensive exploration demystifies the complexities of dimensional analysis and its applications, shedding light on how this fundamental tool contributes to our comprehension of the fundamental laws of physics. By meticulously following this method, we can effectively employ dimensional analysis as a powerful tool for unveiling the secrets of the cosmos, unlocking the enigmatic relationships between dimensions in the physical world, and deciphering their role in both mathematical and physical equations. This journey exemplifies the symbiotic relationship between dimensions and the universe, uniting the abstract realm of mathematics with the tangible world of physics. Ultimately, it reinforces the notion that dimensions, and their powers are the threads that weave the fabric of reality, connecting us with the profound intricacies of the universe [21-36].

## 5. Results:

Summarize the method for effective use of dimensional analysis as a tool to explore and understand the relationships between dimensions in the physical world and their role in mathematical and physical equations. Reiterate the importance of dimensional analysis in unveiling the mysteries of the universe.

## VIII. IMAGE OF CALABI-YAU MANIFOLD

This image shows a local 2D cross-section of the real 6D manifold known in string theory as the Calabi-Yau quintic. This is an Einstein manifold and a popular candidate for the wrapped-up 6 hidden dimensions of 10-dimensional string theory at the scale of the Planck length. The 5 rings that form the outer boundaries shrink to points at infinity, so that a proper global embedding would be seen to have genus 6 (6 handles on a sphere, Euler characteristic -10). The underlying real 6D manifold (3D complex) has Euler characteristic -200, is embedded in  $CP^4$ , and is described by this homogeneous equation in five complex variables:  $z_0^5 + z_1^5 + z_2^5 + z_3^5 + z_4^5 = 0$ . The displayed surface is computed by assuming that some pair of complex inhomogeneous variables, say  $z_3/z_0$  and  $z_4/z_0$ , are constant (thus defining a 2-manifold slice of the 6-manifold), renormalizing the resulting inhomogeneous equations, and plotting the local Euclidean space solutions to the inhomogeneous complex equation  $z_1^5 + z_2^5 = 1$ . This surface can be described as a family of 5x5 phase transformations on a fundamental domain,  $1/25^{\text{th}}$  of the surface, shown (slightly hidden) in blue. Each of the first set of phases mixes in a brighter red color to its patch, and the second set mixes in green. Thus, the color alone shows the geometric parentage of each of the 25 patches. The resulting surface, which is embedded in 4D, is projected to 3D according to one's taste to produce the final rendering. Further details are given in [Courtesy: Andrew J. Hanson, "A construction for computer visualization of certain complex curves," Notices of the Amer. Math. Soc. 41 (9): 1156-1163, (November/December 1994).]

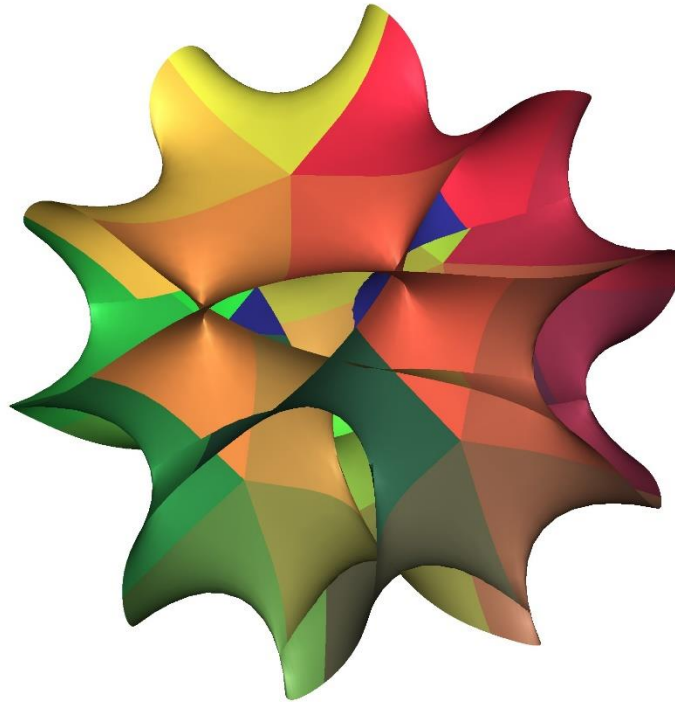


Fig. 1. A 2D slice of a 6D Calabi–Yau quintic manifold [Courtesy: *File:CalabiYau5.jpg* - *Wikimedia Commons*. (2014, January 11). <https://commons.wikimedia.org/w/index.php?curid=30579072>].

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