Approximate Reasoning on Fuzzy Constructs for Data Science

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***Abstract*—We consider fuzzy inference of the form "if $\cdots$ then $\cdots$ ", "if $\cdots$ then $\cdots$ else $\cdots$ "and "if $\cdots$ and/or $\cdots$ then $\cdots$ ". In this paper, we consider fuzzy inference for logical constructs developed by Muzumoto. We propose fuzzy inference applying on logical constructs using our method. We have shown the fuzzy inference method satisfy intuitions under several criteria.**

***Index Terms*—fuzzy logic, fuzzy conditional inference, fuzzy intuitions**

I. Introduction

Zadeh [11], Mamdani [1] and TSK [6] proposed fuzzy conditional inference for fuzzy propositions. Fukami [2] proposed fuzzy intuitions. Fukami [2] adapting the Godel and Standard sequence methods to prove fuzzy intuitions. The fuzzy conditional inference new method is used to prove proposed fuzzy intuitions. We shown the our fuzzy inference method satisfy all the intuitions under several criteria using new method for the fuzzy propositions containing "and/or "and "if $\cdots$ then $\cdots$ else $\cdots$ ".

Type-1
If *x* is *P* then *y* is *Q*
x is P_1

---

y is ?

If *x* is Supply then *x* is increase Profit
x is more Supply

---

y is ?

Type-2
If *x* is *P* and *x* is *Q* or *x* is *R* then *y* is *S*
x is P_1 and *x* is Q_1 or *x* is R_1

---

y is ?

If *x* is Supply or *x* is Demand and *x* is Price then *x* is increase Profit
x is more Supply or *x* is very Demand and *x* is more Price

---

y is ?

Type-3
Consider fuzzy inference

If *x* is *P* then *y* is *Q* else *y* is *R*
x is P_1

---

y is ?

If *x* is Demand then *x* is Profit else *x* is Loss
x is very Demand

---

y is ?

II. Some Methods of Fuzzy Conditional Inference

The fuzzy conditional propositions of the form "if (precedent part) then (consequent part)".

Zadeh [10] fuzzy conditional inference is given as
$A \to B ==$ min$\{1, 1 - A + B\}$.
if x_1 is A_1 and x_2 is A_2 $\cdots$ x_n is A_n, then y is *B* =min$\{1, 1 -$ min$(A_1, A_2, \cdots, A_n) + B\}$

Mamdani [1] fuzzy conditional inference given as $A \to B == \{A \times B\}$.
if x_1 is A_1 and x_2 is A_2 $\cdots$ x_n is A_n, then y is *B* =min$\{$min$(A_1, A_2, \cdots, A_n), B\}$

The consequent part is derived from precedent part for fuzzy conditional inference when the relation between A and B is not known given by [8].

if x is A then y is B= $\{A \times B = A\}$=$\{A \times A\}$=$\{A\}$
if x_1 is A_1 and x_2 is A_2 and $\cdots$ and x_n is A_n then *y* is *B*

$= \min\{A_1, A_2, \cdots, A_n\}$

if x_1 is A_1 and x_2 is A_2 and $\cdots$ and x_n is A_n then *y* is *B*=$\{$min$(A_1, A_2, \cdots, A_n)\}$ (3.1)

if *x* is *A* then *y* is *B*= $\{A\}$ (3.2)

$\int \mu_A(x) \to \int \mu_B(y) = \int \mu_A(x)$

III. Fuzzy Inference on Fuzzy Intuitions

Fuzzy plausibility

Plausibility theory will perform inconsistent propositions into consistent.

Generalization

p $\vee$ q $\vee$ r, μ =p, μ
p $\vee$ q $\vee$ r, μ = q, μ
p $\vee$ q $\vee$ r, μ = r, μ

Specialization

$p\wedge$ q $\wedge$ r , μ=p, μ
$p\wedge$ q $\wedge$ r , μ=q, μ
$p\wedge$ q $\wedge$ r , μ=r, μ

Consider fuzzy inference Type-2

The fuzzy inference is given for Type-1 using generalization and specialization

If *x* is *P* then *y* is *S*
x is P_1

---

y is S_1

If *x* is *Q* then *y* is *S*
x is Q_1

---

y is S_1

If *x* is *R* then *y* is *S*
x is R_1

---

y is S_1

Confider fuzzy inference Type-3

The fuzzy inference is given for Type-2 using generalization and specialization

If *x* is *P* then *y* is *Q* else *y* is *R* =
If *x* is *P* then *y* is *Q* $\vee$ If *x* is *P'* then *y* is *R*

If *x* is *P* then *y* is *Q*
x is P_1

---

y is Q_1

If *x* is *P'* then *y* is *R*
x is P_1

---

y is R_1

From fuzzy conditional inference Type-1, Type-2 and Type-3, the two criterions may be given as

Criteria-1

if *x* is *P* then *y* is *S*
x is P_1

---

y is
textitS_1

Criteria-2

if *x* is *P'* then *y* is *R*
x is P_1'

---

y is R_1

IV. Verification of Fuzzy Intuitions using New Fuzzy Inference

The fuzzy intuitions are give for Criteria-1.

TABLE I
Fuzzy inference for Criteria-1

Intuition	Proposition	Inference
I-1	*x* is *P*	y is *Q*
I-2	*y* is *Q*	x is *P*
II-1	*x* is very *P*	y is very *Q*
II-2	*y* is very *Q*	x is very *P*
III-1	*x* is more or less *P*	y is more or less *Q*
III-2	*y* is More or less *Q*	is more or less *P*
IV-1	*x* is not *P*	y is not *Q*
IV-2	*y* is not *Q*	x is not *P*

Verification of fuzzy intuitions for Criteria-1

4.1.1 In the case of intuition I-1

P o (P$\rightarrow$ Q)
= $\int \mu_P(x)$ o $(\int \mu_P(x) \rightarrow \int \mu_Q(y))$
Considering $P \rightarrow Q = P$
Considering $Q = P$
= $\int \mu_Q(y)$ o $(\int \mu_Q(y))$
= $\int \mu_Q(y) \wedge (\int \mu_Q(y))$
Using specialization
= $\int \mu_Q(y)$
=*y* is *Q*
intuition I-1 satisfied.

4.1.2 In the case of intuition I-2

(*P* $\rightarrow$ Q) o Q
= $(\int \mu_P(x) \rightarrow \int \mu_Q(y))$ o $\int \mu_Q(y)$
Considering $P \rightarrow Q = P$
Considering $Q = P$
= $\int \mu_P(x))$ o $\int \mu_P(x)$
= $\int \mu_P(x)) \wedge \int \mu_P(x)$
Using specialization
= $\int \mu_P(x)$
=*x* is *P*
intuition I-2 satisfied.

4.1.3 In the case of intuition II-1

$very P$ o (*P*$\rightarrow$ Q)
= $\int \mu_{veryP}(x)$ o $(\int \mu_P(x) \rightarrow \int \mu_Q(y))$

Considering $P \to Q = P$
Considering $Q = P$
= $\int \mu_{veryQ}(y)$ o $(\int \mu_Q(x))$
= $\int \mu_{veryQ}(y) \wedge (\int \mu_Q(y))$
Using specialization
= $\int \mu_{veryQ}(y)$
=*y* is *very Q*
intuition II-1 satisfied.

4.1.4 In the case of intuition II-2

(P → Q) o very Q
= $(\int \mu_P(x) \to \int \mu_{veryQ}(y))$ o $\int \mu_Q(y)$
Considering $P \to Q = P$
Considering $Q = P$
= $\int \mu_P(x))$ o $\int \mu_{veryP}(x)$
= $\int \mu_P(x)) \wedge \int \mu_{veryP}(x)$
Using specialization
= $\int \mu_{veryP}(x)$
=*x* is very *P*
intuition II-2 satisfied.

4.1.5 In the case of intuition III-1

more or less *P* o (*P*→ Q)
= $\int \mu_{moreorlessP}(x)$ o $(\int \mu_P(x) \to \int \mu_Q(y))$
Considering $P \to Q = P$
Considering $Q = P$
= $\int \mu_{moreorlessQ}(y)$ o $(\int \mu_Q(x))$
= $\int \mu_{moreorlessQ}(y) \wedge (\int \mu_Q(y))$
Using specialization
= $\int \mu_{moorlessQ}(y)$
=*y* is more or less *Q*
intuition III-1 satisfied.

4.1.6 In the case of intuition III-2

(P → Q) o more or less Q
= $(\int \mu_P(x) \to \int \mu_{moreorlessQ}(y))$ o $\int \mu_Q(y)$
Considering $P \to Q = P$
Considering $Q = P$
= $\int \mu_P(x))$ o $\int \mu_{moreorlessP}(x)$
= $\int \mu_P(x)) \wedge \int \mu_{moreorlessP}(x)$
Using specialization
= $\int \mu_{moreorlessP}(x)$
=*x* is *more or less P*
intuition III-2 satisfied.

4.1.7 In the case of intuition IV-1

not *P* o ($P \to \tilde{Q}$)
= $\int \mu_{notP}(x)$ o $(\int \mu_P(x) \to \int \mu_Q(y))$
Considering $P \to Q = P$
Considering $Q = P$
= $\int \mu_{notQ}(y)$ o $(\int \mu_Q(x))$
= $\int \mu_{notQ}(y) \wedge (\int \mu_Q(y))$
Using specialization
= $\int \mu_{notQ}(y)$
=*y* is *not Q*
intuition IV-1 satisfied.

4.1.8 In the case of intuition IV-2

(P → Q) o not Q
= $(\int \mu_P(x) \to \int \mu_{veryQ}(y))$ o $\int \mu_{notQ}(y)$
Considering $P \to Q = P$
Considering $Q = P$
= $\int \mu_P(x))$ o $\int \mu_{notP}(x)$
= $\int \mu_P(x)) \wedge \int \mu_{notP}(x)$
Using specialization
= $\int \mu_{notP}(x)$
=*x* is *not P*
intuition IV-2 satisfied.

Criteria-1 is satisfies I-1,I-2, II-1, II-2, III-1 and III-2, IV-1, IV-2.

The fuzzy intuitions are give for Criteria-2.

TABLE II
FUZZY INFERENCE FOR CRITERIA-2

Intuition	Proposition	Inference
I'-1	*x* is *P'*	y is *R*
I'-2	*y* is *R'*	x is *P'*
II'-1	*x* is very *P'*	y is very *R*
II'-2	*y* is very *R*	x is very *P'*
III'-1	*x* is more or less *P'*	y is more or less *R*
III'-2	*y* is More or less *R*	is more or less *P'*
IV'-1	*x* is not *P'*	y is not *R*
IV'-2	*y* is not *R*	x is not *P'*

Verification of fuzzy intuitions for Criteria-2

4.2.1 In the case of intuition I'-1 *P'* o (P' → R)

= $\int \mu_{P'}(x)$ o $(\int \mu_{P'}(x) \to \int \mu_R(y))$
Considering *P'* → $R =$ *P'*
Considering $R =$ *P'*
= $\int \mu_R(y)$ o $(\int \mu_R(y))$
= $\int \mu_R(y) \wedge (\int \mu_R(y))$
Using specialization
= $\int \mu_R(y)$
=*y* is *R*
intuition I'-1 satisfied.

4.2.2 In the case of intuition I'-2

(P' → R)o R
= $(\int \mu_{P'}(x) \to \int \mu_R(y))$ o $\int \mu_R(y)$
Considering P' → $R =$ *P'*
Considering $R =$ *P'*
= $\int \mu_{P'}(x))$ o $\int \mu_{P'}(x)$
= $\int \mu_{P'}(x)) \wedge \int \mu_{P'}(x)$
Using specialization
= $\int \mu_{P'}(x)$
=*x* is *P'*
intuition I'-2 satisfied.

4.2.3 In the case of intuition II'-1

very P' o (P' → R)
= $\int \mu_{veryP'}(x)$ o $(\int \mu_{P'}(x) \to \int \mu_R(y))$

Considering $P' \to R = P'$
Considering $R = P'$
= $\int \mu_{veryR}(y)$ o $(\int \mu_R(y))$
= $\int \mu_{veryR}(y) \wedge (\int \mu_R(y))$
Using specialization
= $\int \mu_{veryR}(y)$
=*y* is very *R*
intuition II'-1 satisfied.

4.2.4 In the case of intuition II'-2

(P' $\to$ R)o very R
= $(\int \mu_{P'}(x) \to \int \mu_R(y))$ o $\int \mu_{veryR}(y)$
Considering P' $\to R = P'$
Considering $R = P'$
= $\int \mu_{P'}(x))$ o $\int \mu_{veryP'}(x)$
= $\int \mu_{P'}(x)) \wedge \int \mu_{veryP'}(x)$
Using specialization
= $\int \mu_{veryP'}(x)$
=*x* is *very P'*
intuition II'-2 satisfied.

4.2.5 In the case of intuition III'-1

more or less P' o (P' $\to$ R)
= $\int \mu_{moreorlessP'}(x)$ o $(\int \mu_{P'}(x) \to \int \mu_R(y))$
Considering $P' \to R = P'$
Considering $R = P'$
= $\int \mu_{moreorlessR}(y)$ o $(\int \mu_R(y))$
= $\int \mu_{moreorlessR}(y) \wedge (\int \mu_R(y))$
Using specialization
= $\int \mu_{moreorlessR}(y)$
=*y* is more or less *R*
intuition III'-1 satisfied.

4.2.6 In the case of intuition III'-2

(P' $\to$ R) o more or less R
= $(\int \mu_{P'}(x) \to \int \mu_R(y))$ o $\int \mu_{moreorlessR}(y)$
Considering P' $\to R = P'$
Considering $R = P'$
= $\int \mu_{P'}(x))$ o $\int \mu_{moreorlessP'}(x)$
= $\int \mu_{P'}(x)) \wedge \int \mu_{moreorlessP'}(x)$
Using specialization
= $\int \mu_{moreorlessP'}(x)$
=*x* is *more or less P'*
intuition II'-2 satisfied.

4.2.7 In the case of intuition IV'-1

not P' o ($P' \to$ R)
= $\int \mu_{notP'}(x)$ o $(\int \mu_{P'}(x) \to \int \mu_R(y))$
Considering $P' \to R = P'$
Considering $R = P'$
= $\int \mu_{veryR}(y)$ o $(\int \mu_R(y))$
= $\int \mu_{notR}(y) \wedge (\int \mu_R(y))$
Using specialization
= $\int \mu_{notR}(y)$
=*y* is not *R*
intuition IV'-1 satisfied.

4.2.8 In the case of intuition IV'-2

(P' $\to$ R) o not R
= $(\int \mu_{P'}(x) \to \int \mu_R(y))$ o $\int \mu_{notR}(y)$
Considering P' $\to R = P'$
Considering $R = P'$
= $\int \mu_{P'}(x))$ o $\int \mu_{notP'}(x)$
= $\int \mu_{P'}(x)) \wedge \int \mu_{notP'}(x)$
Using specialization
= $\int \mu_{veryP'}(x)$
=*x* is *not P'*
intuition IV'-2 satisfied.

Criteria-2 is satisfies I'-1,I'-2, II'-1, II'-2, III'-1, III'-2,IV'-1 and IV'-2.

V. Application to Fuzzy Intuitions

The Business intelligence [3] needs reasoning. The Business data is defied with fuzziness with linguistic variables.
If *x* is *Demand* then *y* is *Profit* else *y* is *Loss*
If *x* is *Demand* then *y* is *Profit* $\vee$ If *x* is *not Demand* then *y* is *Loss*
Which is given as
If *x* is *Demand* then *y* is *Profit*
If *x* is *not Demand* then *y* is *Loss*

If *x* is *Demand* then *y* is *Profit*
x is P_1

---

y is R_1

Consider the fuzzy data sets for production The fuzzy conditional inference using is given by
if x is Demand then x is Profit
The fuzzy conditional inference using (3.1) given by The

TABLE III
FUZZY DATA SET DEMAND

Item No.	Denand
Item1	0.3
Item2	0.5
Item3	0.7
Item4	0.8
Item5	1.0

TABLE IV
FUZZY DATA SET PROFIT

Item No.	Profit
Item1	0.3
Item2	0.5
Item3	0.7
Item4	0.8
Item5	1.0

fuzzy conditional inference for Criteria-1 is given by

TABLE V
FUZZY CONDITIONAL INFERENCE

Item No.	I-1	I-2	II-1	II-2	III-1	III-2	IV-1	IV-2
Item1	0.3	0.2	0.09	0.09	.55	0.55	0.7	0.7
Item2	0.5	0.5	0.25	0.25	0.71	0.71	0.5	0.5
Item3	0.7	0.7	0.49	0.49	0.84	0.84	0.3	0.3
Item4	0.8	0.8	0.64	0.64	0.89	0.89	0.2	0.2
Item5	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0

If *x* is *not Demand* then *y* is *Loss*
x is P_1

---

y is R_1

The fuzzy conditional inference using is given by
if x is not Demand then x is Loss
The fuzzy conditional inference using (3.1) given by The

TABLE VI
FUZZY DATA SET LOSS

Item No.	Loss
Item1	0.7
Item2	0.5
Item3	0.3
Item4	0.2
Item5	0.0

fuzzy conditional inference for Criteria-2 is given by

TABLE VII
FUZZY CONDITIONAL INFERENCE

Item No.	I'-1	I'-2	II'-1	II'-2	III'-1	III'-2	IV'-1	IV'-2
Item1	0.7	0.7	0.49	0.49	0.84	0.84	0.7	0.7
Item2	0.5	0.5	0.25	0.25	0.71	0.71	0.5	0.5
Item3	0.3	0.3	0.09	0.09	0.84	0.84	0.3	0.3
Item4	0.2	0.2	0.04	0.04	0.55	0.55	0.3	0.3
Item5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Similarly, the fuzzy conditional inferencing be computed
If *x* is Supply or *x* is Demand and *x* is Price then *x* is increase Profit
x is more Supply or *x* is very Demand and *x* is more Price

---

y is ?

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