Improving the Accuracy of Newmark-Based Time Integration Methods

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Abstract

The generalized-α method [3] is a Newmark-type integrator that has become a standard approach for the time integration of the equations of motion of multibody systems. An extension of generalized-α for multibody systems with a configuration space with Lie group structure has been proposed in Ref. [1], which we refer to as Lie group generalized-α in the following. A primary motivation for the present study arises from recent investigations Ref. [4], showing that the conventional generalized-α method often achieves higher accuracy than Lie group generalized-α when simulating rigid multibody systems. In this work, we present improvements for the time integration of constrained multibody systems, using Lie group methods applied to Newmark-based time integration methods.

It was shown in Ref. [5], that the Lie group Newmark method, which is a special case of Lie group generalized-α [1], involves simplifications that can reduce Lie group Newmark's accuracy. Lie group Newmark can be considered a simplified scheme, because the kinematic equations relating velocities $\mathbf{v}$, accelerations $\dot{\mathbf{v}}$, and the time derivatives of the incremental motion $\mathbf{x}$

$$\mathbf{v} = \mathbf{T}(\mathbf{x})\dot{\mathbf{x}}\,, \tag{1}$$

$$\dot{\mathbf{v}} = \mathbf{T}(\mathbf{x})\ddot{\mathbf{x}} + \dot{\mathbf{T}}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}\,, \tag{2}$$

are incorporated in the (simplified) form

$$\mathbf{v} = \dot{\mathbf{x}}\,, \tag{3}$$

$$\dot{\mathbf{v}} = \ddot{\mathbf{x}}\,, \tag{4}$$

in the integration formulae, which is also the case for Lie group generalized-α, cf. Ref. [1]. By construction, Lie group Newmark and Lie group generalized-α satisfy only the relations Eqs. (3-4) in each integration step instead of the nonlinear relations Eqs. (1-2), which may cause lower accuracy. Nevertheless, using the Eqs. (3-4) when deriving Lie group Newmark or Lie group generalized-α yields simple and efficient time integration methods with second-order convergence, cf. Refs. [1, 3]. As a key motivation for the present work, it was shown in Ref. [5] for unconstrained mechanical systems, that the incremental form of Lie group Newmark can be modified using the tangent operator $\mathbf{T}$ and its time derivative $\dot{\mathbf{T}}$ such that Eqs. (1-2) are satisfied in each integration step, which yields improved accuracy.

In this paper, we extend the approach proposed in Ref. [5] from unconstrained to constrained multibody systems. Furthermore, we attempt to enhance the accuracy of Lie group generalized-α by incorporating the approach presented in Ref. [5]. We would like to note, that incorporating Eqs. (1-2) into Lie group Newmark-based integrators is not new at all, cf. Ref [2]. However, the accuracy and computational efficiency of the resulting methods as compared to the conventional Newmark and generalized-α method have not been investigated in detail yet. For the evaluation of the proposed approaches, we use several typical examples of rigid multibody systems and evaluate the accuracy and computational efficiency of the proposed approaches compared to conventional generalized-α, Lie group generalized-α, conventional Newmark, and Lie group Newmark. In our investigations, we consider the Lie group

$\mathbb{R}^3 \times SO(3)$ as the configuration space of a single free rigid body and use the Cartesian rotation vector (RV) for parametrizing the rotation group $SO(3)$, cf. Ref. [4] for details.

Preliminary Results

In Figure 1, the convergence of the error of the benchmark heavy top's center of mass position is illustrated for different time integration methods and rotation parameters. The error is computed as shown in Eq. (99) in Ref. [1]. For generalized-α (Gα), a conventional Euler parameter (EP) formulation has been used. The heavy top is modeled with kinematic constraints as in Ref. [1].

As depicted in Figure 1, the proposed modified Lie group Newmark method (mod. LGNM) exhibits an accuracy nearly one magnitude higher than that of Lie group Newmark (LGNM) and Lie group generalized-α (LGGα). Moreover, the modified Lie group Newmark's accuracy is even higher than the accuracy of the conventional generalized-α method. Figure 1 also reveals that the proposed approach requires fewer Newton iterations compared to the conventional Newmark and generalized-α method, aligning with the performance of Lie group generalized-α and Lie group Newmark.

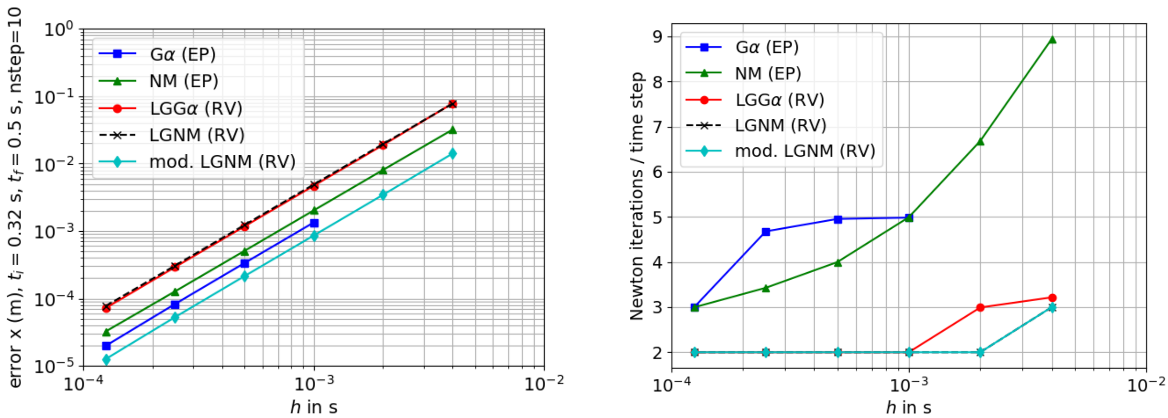

Figure 1: *Left:* Convergence. *Right:* Average number of Newton iterations required for simulation.

Conclusion and Outlook

In conclusion, this study presents improved Newmark-based Lie group time integration methods for constrained multibody systems. It turns out, that the proposed Lie group Newmark approach exhibits higher accuracy than the conventional Newmark and generalized-α method, as well as their Lie group extensions. Besides Lie group Newmark and Lie group generalized-α, the methodology presented in this work could be applied to other implicit Lie group time integration methods, potentially increasing their accuracy. Concerning time integration, the proposed approaches could also improve the accuracy of other Lie group formulations such as the $SE(3)$ Lie group formulation for example.

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