The Critical Role of Homotopy Continuation in Robotic-Assisted Surgery - Future Perspective

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***Abstract*—This article provides an introduction and overview of the mathematical concept of homotopy continuation and its applications - especially for path tracing - in robotic-assisted surgery. It explains the importance of homotopy continuation in solving path-planning and restriction problems and image reconstruction. The opinion article starts with a short introduction to robotic-assisted surgery and also provides several examples of how homotopy continuation has been used in other areas of robotics. Further advancements in homotopy continuation techniques are anticipated and with the combination of machine and deep learning it will likely enable researchers to plan out safer surgeries with the assistance of homotopy-guided robots.**

***Index Terms*—Homotopy continuation, robotic assisted surgery, topological space, mathematical models.**

I. INTRODUCTION

A. OVERVIEW OF ROBOTICALLY ASSISTED SURGICAL SYSTEMS

Robotically Assisted Surgical Systems (RASS) aim to relieve and support clinicians in their current tasks and to also enable procedures that are currently difficult to undertake. Ergonomic and user-friendly system designs can contribute to physical relief and simplified instrument handling, which enhances surgical precision and reduces cognitive burden [1]–[3]. Minimally invasive surgery (MIS) has become the standard of care for many procedures due to improved recovery and decreased postoperative pain, but conventional laparoscopy may not be adequate for the management of complex anatomy. RASS allows surgeons to address difficult cases while preserving the benefits of MIS. From a clinical point of view, RASS can increase procedural safety, standardize surgical quality, decrease hospital length of stay and cost, and improve patient outcomes.

Although creation and marketing of RASS is complicated and costly, there are over 40 systems on the market that can be used for different kinds of surgery, and about the same number are currently in development. Effective systems that can make semi-autonomous and, in the future, autonomous movements and decisions need to be guided by machine learning and foresight path planning (see Figure 1).

This paper introduces an engineering algorithm called homotopy continuation which has the potential to improve the quality of RASS through path-planning and obtained surgical

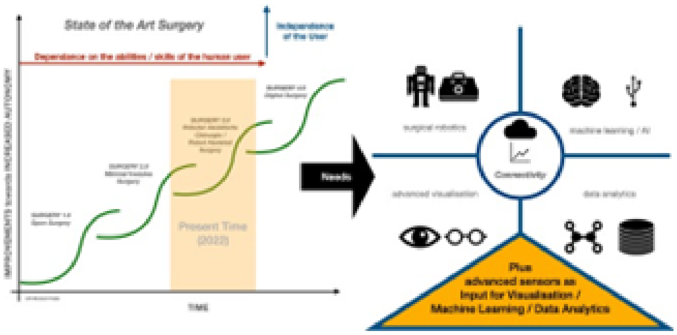

Fig. 1. Robotic Surgery at its Finest. Moving to the next level - DIGITAL SURGERY - necessitates the integration of various exponential technologies, which will eventually result in more independence of the individual user's capabilities (adapted from [4])

precision. The tools also need to avoid collisions and adapt to unexpected movements in real time. By mapping out alternative paths from one location to another and calculating the odds of hitting an object on a particular path, homotopy continuation allows for collision warnings between robotic arms or between a robotic arm and a surgical assistant, as well as can provide alternative paths for the surgeon in control of the robotic arms.

II. METHODS

A. *OVERVIEW OF HOMOTOPY CONTINUATION ALGORITHM*

Homotopy continuation (HC) enables real-time feedback and control during surgical procedures. By continuously tracking paths through the 3-d surgical space needed that a surgeon must traverse during the surgery, it provides surgeons with immediate information about the hazards along the surgical route, helping them make informed decisions during complex surgical tasks. This real-time feedback can enhance surgical accuracy, reduce the risk of complications, and improve patient outcomes [5].

HC was originally developed as a tool to allow researchers to slowly transform one equation solution into another equation solution while tracking the progress of the transformation [5]. Practically, this means that an equation with an unknown solution can be solved by transforming a similar equation with a known solution into the desired solution.

In a trivial example, HC could be used to transform the equation x2=y to x3=y (shown in Figure 2a and 2b) by slowly deforming the existing parabola solution into a cubic solution (Figure 3). While this tool is mainly used in engineering and machine learning, its origin lies in a field of mathematics called topology, which is concerned with global properties. Homotopy is a path-tracing tool that allows mathematicians to classify objects based on path congruences, where objects like holes act as obstacles to these paths [6].

HC has solved important problems in robotic navigation and for collision avoidance with humans and other devices/tools. One of the current robotic surgical systems provides error warnings when the instruments are out of view, at the maximum rotational limits, or when the surgeon needs to relax on the hand grip. The system also provides “targeting,” where the surgeon indicates the target pathology and the arms are automatically configured to provide optimal access to that location through HC.

Within the context of gynecological surgery, robotic surgery typically involves multiple robotic arms, an endoscopic camera, and a human assistant located at the bedside who utilizes a laparoscopic port for retraction, suction, and insertion of suture. Collision of the robotic arms with the human assistant occurs frequently because the surgeon at the console cannot see what is happening at the bedside. Minimizing collision with the human assistant is necessary for a successful surgical outcome.

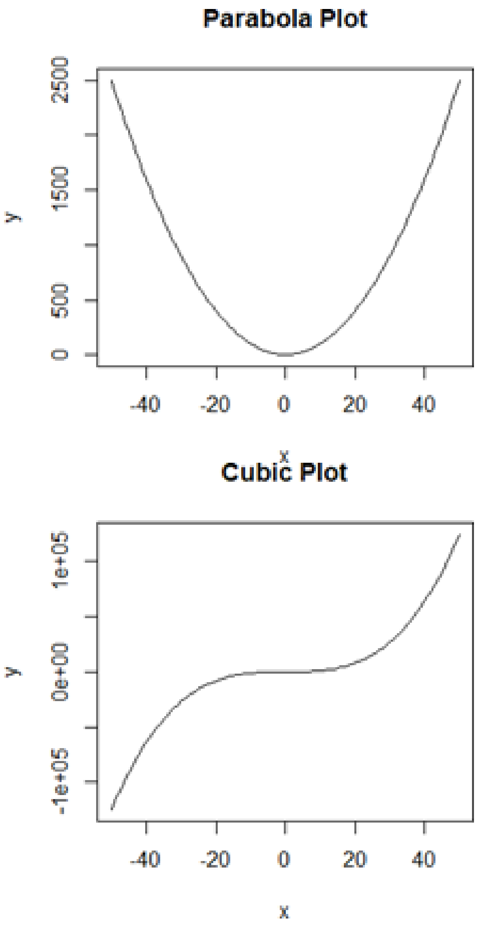

Fig. 2. A plot of a square function (parabola shown in Fig. 2.a upper) and a cubic function (shown in Fig. 2.b below)

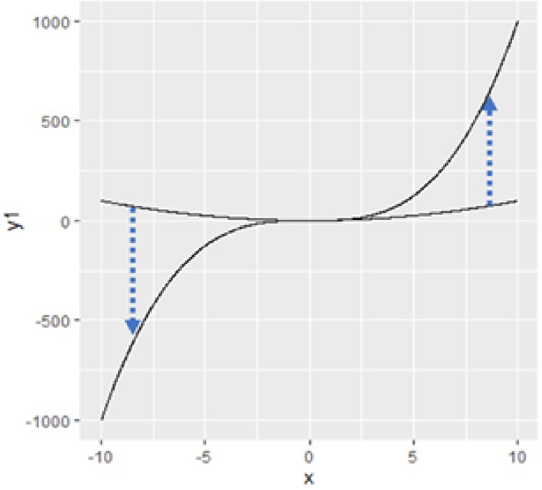

Fig. 3. Arrows showing the deformation directions to change Figure 2a into Figure 2b.

Within the pelvis, the target pathology may be located in close proximity to critical structures and unexpected collisions between the assistant and the robotic arms can result in damage to surrounding organs. For example, Figure 4 demonstrates the proximity of the right fallopian tube and ovary to the right external iliac vein and artery in a patient with normal anatomy.

In Figure 5, the surgeon has opened the retroperitoneal space along the left pelvic sidewall in order to remove lymph nodes along the left external iliac vessels. This dissection must be presicse as vascular injury in this area can result in catastrophic bleeding.

In addition, addressing areas of the body (for instance, when endometriosis exists in other areas of the abdomen than the area initially targeted for surgery) is often required in complex surgeries. The robotic arms need to pivot and avoid collisions with other arms, with the assistant, and with bodily structures as they are moved within the patient. As shown in Figure 6, the two left-sided robotic arms and a laparosocpic suction tip are attempting to operate in the same area, resulting in limited work space. The surgeon at the console has to intentionally move the instruments out of the way in order to allow the assistant to suction the target area. Furthermore, patients with a short torso or thin body habitus may have limited space along the abdomen for adequate spacing between arm placements. With alternative path planning shown on the monitor to guide movements and pivot when a possible collision path is detected, HC can allow surgeons to pivot plans quickly and along predetermined best paths rather than adapting on the fly.

Homotopy constraints (in which constraints are added to the homotopy continuation process) are useful in object avoidance. HC in general has been used to plan optimal routes for robotic movement, along with backup plans in case an optimal path is blocked by an object suddenly [6]–[12].

When using most commercially available robotic systems, a warning flag is seen on the surgeon's monitor when a collision is likely given current motion trajectories. For instance, if an assistant's grasper moves too close to robotic arms, an alert is sent to avoid the collision. This information can be critical when working in areas of vascular anatomy or where collisions between the assistant and the robotic arms could induce tissue damage.

We will focus on this application of HC in the following sections, including collision avoidance (with other robotic arms or with the surgical assistant), alternative path planning (to allow for multiple routing options), and image in-filling (in cases where some of the image is blurred or obstacles to avoid are obstructed).

III. RESULTS AND FUTURE PROSPECTIVE

In robotic surgeries, the HC algorithm is used to iteratively compute optimal robot trajectories by continuously changing robot motion based on real-time feedback from the surgical environment. It uses algebraic equations to represent the robot's motion and adaptively adjust the trajectory to avoid obstacles, optimize efficiency, and enhance surgical precision. This method aids in the optimization of the robot's path planning and control, ensuring precise and safe surgical techniques [6], [7].

HC allows the robot to adapt to changes in the operating environment and enhance surgical results by providing the robot with efficient and reliable trajectory planning during surgery. It has been used in a wide range of robotic surgical applications, including urology, gynecology, gastrointestinal surgery, and more, to enhance surgical precision and patient safety.

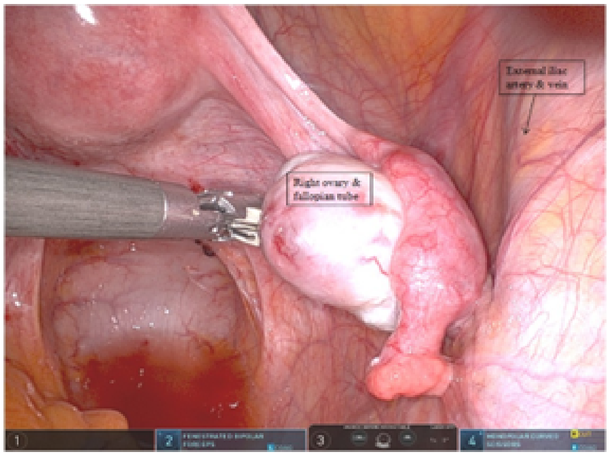

Fig. 4. Normal female pelvic anatomy demonstrating proximity of the right ovary and fallopian tube to the right external iliac artery and vein, the major blood supply of the lower extremity.

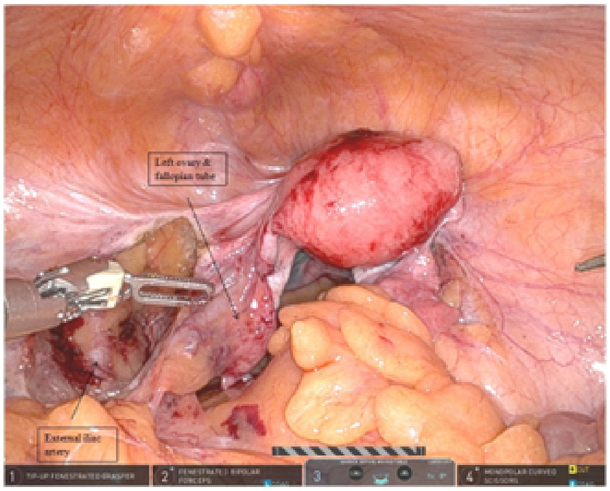

Fig. 5. Female pelvic anatomy after dissection of the left retroperitoneal space during lymphadenectomy for endometrial cancer.

A. *Collison and Object Avoidance*

Object avoidance is important in the surgical context, as surgical assistants may move their tools unexpectedly, creating the possibility of a collision with robotic arms; alerts on the monitor can alert the surgeon controlling the robotic arms and allow for a path correction with one of the homotopic paths outlined by the algorithm.

As we have previously discussed, homotopy-based constraints on arm movement in particular hold promise for improvements in robotic surgery, where collision avoidance is critical for human assistants and robotic arms. For instance, a robotic arm assisting in surgery could be routed around sensitive areas like arteries to avoid introducing risk to a patient

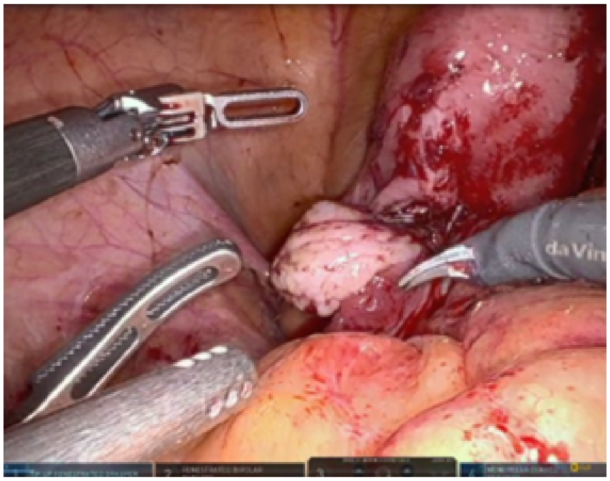

Fig. 6. Example of collision from suction tip (laparoscopic assistant) and left-sided robotic arms during a complicated endometriosis surgery.

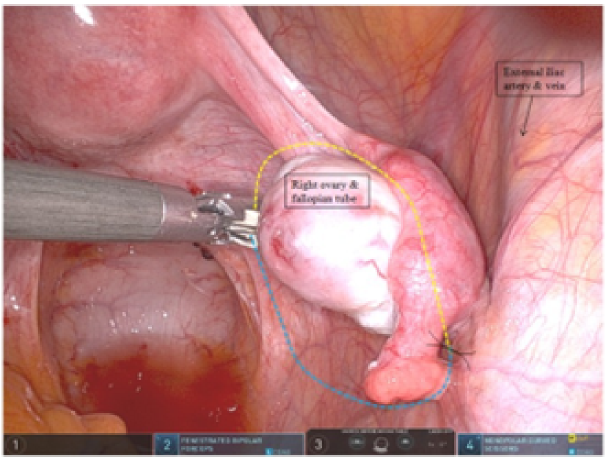

Fig. 7. In this figure yellow and blue lines show different possible path options.

undergoing brain resection or an emergency hysterectomy. In the future, homotopy constraints and continuation methods will continue to play an important role in robotic path planning and robotic arm movement coordination [6], [7], [12], and we envision many possible uses building off extant tools based on homotopy continuation. For instance, we may be able to have multiple surgeon-guided arms rather than surgical assistants with scalpels; this would limit the potential collisions by coordinating arms and removing the human unpredictability factor (slips of tools, arm cramps…). In addition, as HC improves in quality, the robotic program may become another agent in the surgery, with the surgeon choosing pre-planned actions and serving as an emergency backup to the robot. Autonomous vehicles function in this manner, with the algorithm driving and making most decisions but equipped with a manual override should the driver see a need to correct an action.

B. *Alternative Trajectories*

Alternative paths are beneficial, as they allow the robot or a human guiding robotic parts to choose the best path among many possible paths from one point to another. In the context of surgery, this allows a surgeon to choose the best path from insertion of the arm to the surgery locations; this is typically shown on the monitor as a series of paths from which to choose and their paths through the 3-dimensional surgical space.

C. *Image In-Filling and Spatial Interpolation*

One recent advance in HC, spatial interpolation (where features in 3-dimensional space that may not have been captured in datasets are inferred through HC), holds promise for use in robotic surgery [13]. Imaging may miss abnormalities encountered during surgery, and HC offers a way to infer what might exist when imaging does not match real-time data during the surgery. This would allow surgeons to avoid potential complications due to abnormalities or structures that were not captured well in pre-surgery imaging (or not imaged based on patient symptoms prior to surgery, such as additional disc/vertebral pathology present during a laminectomy) [14], [15].

Collaboration among homotopy researchers, robotics experts, and surgeons is likely to yield new advances in robotic surgery, including more accurate movements of arms and avoidance of sensitive structures that could cause major complications. In addition, collaboration between medical imaging researchers and homotopy experts could yield tools that are able to infer missing pieces of medical imaging, including nearby areas that were not directly imaged prior to surgery but require surgical intervention during the surgery. Ultimately, these tools can make surgeries easier for surgeons and safer for patients.

D. *Limitations*

An important limitation to HC includes the ability to visualize structures using RASS and relay this information to the HC algorithm. As surgeons operate, intrabdominal contents can obscure the camera(s) and decrease image quality. Planning for instances where HC may not be able to relay confident object avoidance is important. Further, the use of alternative paths through HC may not all yield the most optimal surgical outcomes. For example, naturally occurring tension between organs and surrounding tissues make specific surgical incisions more appropriate than others.

IV. CONCLUSION

HC is a new avenue of research in robotic-assisted surgeries. Given its success in other areas of robotics, including homotopy-based constraints to avoid collisions and homotopy continuation for navigational path planning, it is likely that this technology will improve surgical logistics and patient outcomes. In addition, HC offers a potential way to supplement pre-operative imaging that is obscured by artifacts or incomplete. Recent advances in the fields of robotics and imaging suggest that these opportunities will work well when applied to the robotic-assisted surgery.

Additional research opportunities and fields using HC and RASS include:

1) Augmented reality guidance: HC can be integrated with augmented reality (AR) systems, providing real-time guidance to surgeons during surgery. AR overlays can display the solutions of the system of equations in the surgical field, assisting surgeons in making precise decisions and improving surgical accuracy.
2) Enhanced surgical planning: HC can be used for preoperative surgical planning, allowing surgeons to simulate and optimize surgical procedures in a virtual environment.
3) Adaptive robotic control to dynamically adjust robotic control during surgery for increased safety and surgical efficiency.
4) Personalized and precision medicine can be supported by developing patient-specific surgical plans considering the individual patient anatomy, tissue properties, and surgical goals.
5) Collaborative surgery, where HC can be integrated with collaborative robotic systems, allowing multiple surgeons to work together remotely in real-time. This can enable surgical expertise to be shared across different locations, improving access to specialized surgical care and reducing the need for patient travel.

Many other applications exist for HC based methods, e.g., in robot-assisted rehabilitation [16], robotic transportation of medical waste [17], and robot-assisted fitting of prosthetic devices [18], some of them are already implemented in clinical settings.

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