Fuzzy Risk Sets for Decision making

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Abstract**—**the information available to the system is incomplete in many applications like Decision Support Systems, Control Systems and Medical Expert Systems. Sometimes decision has to be taken under risk with incomplete information. Fuzzy logic deals with incomplete information with belief rather than likelihood (probability). The fuzzy set is defined with single membership function. The fuzzy set with two membership functions will give more information than single membership function. In this paper, the Fuzzy Certainty Factor (FCF) is studied as difference between fuzzy membership functions "true" and "false" for decision making. The fuzzy certainty factor is studied for fuzzy risk set. The fuzzy inference is studied. Business application is given as an application to fuzzy risk set

Keywords— ***fuzzy set; fuzzy membership functions; fuzzy Certainty Factor;Fuzzy Risk Set; Decision Making***

I. Introduction

The information available to many applications like Business is incomplete. Some times "true" and "false" information to be considered for decision making. The decision making is fuzzy rather than probability. Sometimes the decisions are to be taken under risk. The fuzzy logic is commonsense of the mind. The fuzzy logic is defined with single membership function [9]. The fuzzy set with two membership functions "true" and "false" will give more information than single membership function.

In the following, fuzzy logic with single membership functions is discussed. The fuzzy set is a class of objects with a continuum of grades of membership. The fuzzy set with two fold fuzzy set { $\mu_A^{true}(x)$, $\mu_A^{false}(x)$} is discussed to give more information then the single membership function. The Fuzzy Certainty Factor $\mu_A^{FCF}(x) = \mu_A^{true}(x) - \mu_A^{false}(x)$ is studied to made single fuzzy membership function. The Fuzzy Risk set is studied for FCF for decision making. The Business application is given as an example.

I. FUZZY LOGIC

Sade[10] has introduced fuzzy set as a model to deal with imprecise, inconsistent and inexact information. Fuzzy set is a class of objects with a continuum of grades of membership.

The set A of X is characterized by its membership function $\mu_A(x)$ and ranging values in the unit interval [0, 1]

$\mu_A(x)$: X →[0, 1], x Є X, where X is Universe of discourse.

A = $\mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(a_n)/x_n$, where "+" is union

For example, the fuzzy proposition "x is young"

young ={.95/10+0.9/20+0.8/30+0.6/40+0.4/50+0.3/60+0.2/70+0.15/80+0.1/90 }

not young ={ 0.05/10 + 0.1/20 + 0.2/30+0.4/40 +0.6/50 + 0.8/60+0.7/70 +0.95/80+0.9/90 }

For instance "Rama is young" with Fuzziness 0.8 The Graphical representation of young and Not young is shown in fig.1

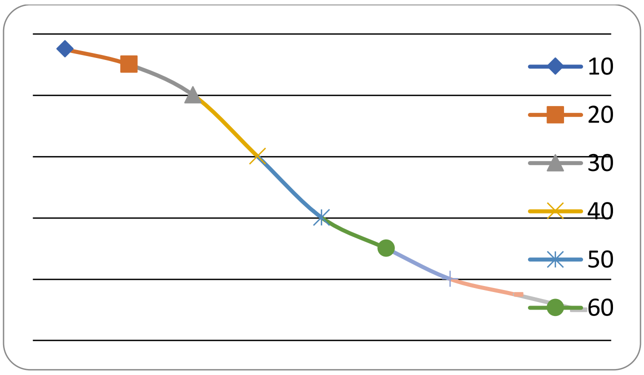

Fig.1: Fuzzy membership function

The fuzzy logic is defined as combination of fuzzy sets using logical operators[9]. Some of the logical operations are given below.

A, B and C are fuzzy sets..The operations on fuzzy sets are given as

If x is not A

A'=1-$\mu_A(x)$/x   Negation

x is A and y is B→ (x, y) is A x B

A x B=min($\mu_A(x)$), $\mu_B(y)$}(x,y)

If x=y

x is A and y is B→ (x, y) is AΛB

AΛB=min($\mu_A(x)$), $\mu_B(y)$}/x   Conjunction

x is A or y is B→ (x, y) is A' x B'

A' x B' =max($\mu_A(x)$), $\mu_B(y)$}(x,y)

If x=y

x is A and x is B→ (x, x) is A V B

AVB=max($\mu_A(x)$), $\mu_B(y)$}/x   Disjunction

if x is A then y is B=

A→B = min{1, 1- $\mu_A(x)$) +$\mu_B(y)$}/(x,y)   Implication

If x is A then y is B else y is C= A x B + A' x C

A→B= min {1, 1-μA(x)+ μB(y)}/(x,y) if A

A'→ C = min {1, 1- $\mu_A(x)) + \mu_C(y)\}/(x,y)$ if not A

if x=y

if x is A then x is B =

$A \to B = \min\{1, 1-\mu A(x)+ \mu B(x)\}/x$

Zadeh[9] fuzzy conditional inference is given for "If x is A then x is B else z is C" as

If x is A then x is B else z is C= A x B + A' x C

The fuzzy conditional inference is given for "If x is A then x is B else z is C" as[7]

If "A", i.e., A is positive

$A \to B = \min\{1, 1- \mu_A(x)) + \mu_B(y)\}/x$

If "not A", i.e., A is negative

$A' \to C = \min\{1, 1- \mu_A(x)) + \mu_B(y)\}/x$

A o R= A x B=min{ $\mu_A(x)), \mu_R(x,y)\}/(x,y)$

If x = y A o B==min{ $\mu_A(x)), \mu_B(y)\}/x$ Composition

The fuzzy propositions may contain quantifiers like "Very", "More or Less" . These fuzzy quantifiers may be eliminated as

$\mu_{\text{Very } A}(x) = \mu_A(x)^2$ Concentration

$\mu_{\text{More or Less } A}(x) = \mu_A(x)^{0.5}$ Diffusion

II. GENERALIZED FUZZY LOGIC

Since formation of the generalized fuzzy set simply as two fold fuzzy set. The fuzzy logic is defined as combination of fuzzy sets using logical operators[8].

Some of the logical operations are given below

Suppose A, B and C are fuzzy sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

AVB={ $\max(\mu_A^{true}(x), \mu_A^{true}(y)), \max(\mu_B^{false}(x), \mu_B^{false}(y))\}(x,y)$ Disjunction

AΛB={ $\min(\mu_A^{true}(x), \mu_A^{true}(y)), \min(\mu_B^{false}(x), \mu_B^{false}(x))$ } Conjunction

$A' = \{1- \mu_A^{true}(x), 1- \mu_A^{false}(x)\}/x$ Negation

$A \to B = \{\min(1, 1- \mu_A^{true}(x) + \mu_B^{true}(y), \min(1, 1- \mu_A^{false}(x) + \mu_B^{false}(y)\}(x,y)$ Implication

A o R= $\{\min_x(\mu_A^{true}(x), \mu_A^{true}(x)), \min_x(\mu_R^{false}(x), \mu_R^{false}(x))\}$ Composition

The rule "If x is A then y is B else y is C" is given as "If x is A then y is B or if x is not then y is C" [7]

If x is A then y is B else y is C =

$A \to B = \{\min(1, 1- \mu_A^{true}(x) + \mu_B^{true}(y), \min(1, 1- \mu_A^{false}(x) + \mu_B^{false}(y)\}$

$A' \to C = \min(1, \mu_A^{true}(x) + \mu_B^{true}(x), \min(1, \mu_A^{false}(y) + \mu_B^{false}(y)\}$

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

For the proposition

"x is very A

Concentration

$\mu_{\text{Very } A}(x) = \{\mu_A^{true}(x)^2, \mu_A^{false}(x)\mu_A(x)^2\}$ Concentration

For proposition "x is more or less A" } Diffusion

$\mu_{\text{More or Less } A}(x) = (\mu_A^{true}(x)^{0.5}, \mu_A^{false}(x)\mu_A(x)^{0.5}$

A = { $0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5$, $0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 + 0.6/x_5$}

B = { $0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0..5/x_4 + 0.6/x_5$, $0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.7/x_5$}

A V B = { $0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 + 0.6/x_5$, $0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.7/x_5$}

A Λ B = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5$, $0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5$}

A′ = not A= { $0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5$, $0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5$}

A→ B = { $1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 + 1/x_5$, $1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5$}

A o B = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5$, $0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5$}

$\mu_{\text{Very } A}(x) = \{\mu_A^{true}(x)^2, \mu_A^{false}(x)\mu_A(x)^2\}$ Concentration

= { $0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 + 0.25/x_5$, $0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 + 0.36/x_5$

$\mu_{\text{More or Less } A}(x) = (\mu_A^{true}(x)^{1/2}, \mu_A^{false}(x)\mu_A(x)^{1/2}\}$ Diffusion

= { $0.89/x_1 + 0.94/x_2 + 0.83/x_3 + 0.77/x_4 + 0.70/x_5$, $0.63/x_1 + 0.54/x_2 + 0.63/x_3 + 0.83/x_4 + 0.77/x_5$}

III. FUZZY RISK SETS

The Fuzzy Certainty Factor may be defined as

$\mu_A^{FCF}(x) = \mu_A^{true}(x) - \mu_A^{false}(x)$, where

$\mu_A^{FCF}(x) = \mu_A^{true}(x) - \mu_A^{false}(x)$ >0

$\mu_A^{FCF}(x) = \mu_A^{true}(x) - \mu_A^{false}(x)$ =0 and

$\mu_A^{FCF}(x) = \mu_A^{true}(x) - \mu_A^{false}(x)$ <0

The above are interpreted as redundant, insufficient and sufficient incomplete information respectively.

The Fuzzy Risk sets defined by

R= $\mu_A^R(x) = 1$ $\mu_A^{FCF}(x) \geq \alpha$, α-cut

0 $\mu_A^{FCF}(x) < \alpha$ is hole

where α€[0,1]

The fuzzy decision making is defined by

if Fuzzy risk set R of the proposition" x is A" is

R ≥α, the decision is Yes

the decision is No otherwise

For instance,

Demand ={ 0..8/x1+0.7/x2+0.86/x3+0.75/x4+0.88/x5, 0.2/x1+0.3/x2+0.25/x3+0.3/x4+0.2/x5 }

$\mu_{\text{Demand}}^{FCF}(x)$ = 0.6/x1+0.4/x2+0.61/x3+0.45/x4+0.68/x5

The Generalized fuzzy set with two fold fuzzy set for Demand for the Items and Fuzzy certainty factor is shown in Fig.1

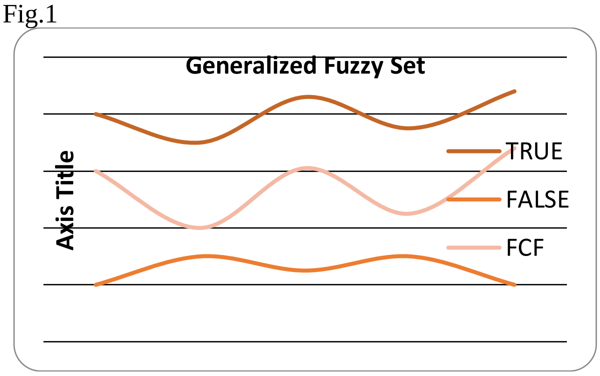

Fig.1

Suppose Fuzzy risk set is defined

$\mu_{Demand}^{R}(x) = 1 \quad \mu_{Demand}^{FCF}(x) \geq 0.5$

$\quad\quad 0 \quad \mu_{Demand}^{FCF}(x) < 0.5$

$=1/x1+0/x2+1/x3+0/x4+1/x5$

Decision may be taken to increase the Price under risk Yes for x1, x3 and x5 and, No for x2, x4 is shown in Fig.2

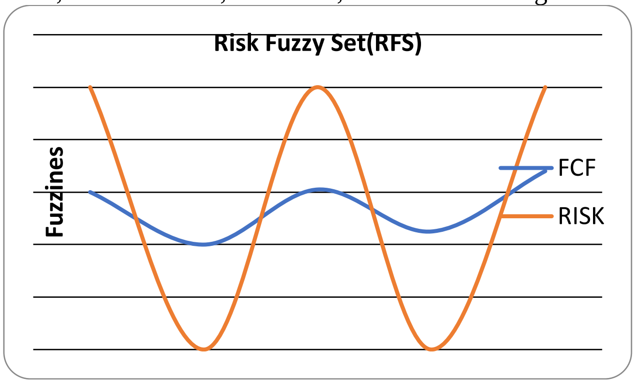

Fig.2

Fuzzy propositions may contain quantifiers like "very", "more or less" . These Fuzzy quantifiers may be eliminated as

For the proposition

"x is very A Concentration

$\mu_{A}^{FCF}(x)^{2}$

"x is not very A"

$\mu_{A}^{FCF}(x)^{2}$

For proposition "x is more or less A" Diffusion

$\mu_{A}^{FCF}(x)^{1/2}$

"x is not more or less A"

$1-\mu_{A}^{FCF}(x)^{0.5}$

The fuzzy logic is combination of logical operators.

Consider the logical operations on Fuzzy risk sets

x is A or B

$\mu_{A\vee B}^{FCF}(x)=\max\{\mu_{A}^{R}(x), \mu_{B}^{R}(x)\}$ Disjunction

x is A and B

$\mu_{A\wedge B}^{FCF}(x)=\min\{\mu_{A}^{R}(x), \mu_{B}^{R}(x)\}$ Conjunction

If x is not A

$\mu_{\neg A}^{FCF}(x)=1-\mu_{A}^{R}(x)$ Negation

if x is A then y is B

$\mu_{A\to B}^{FCF}(x)=\min\{1, 1-\mu_{A}^{FCF}(x) + \mu_{B}^{FCF}(x)\}$ Implication

x is A

if x is A then y is B

---

y is A o (A→B) Composition

For instance,

If x is Demand Then y is high sales

x is very Demand

---

y is very Demand o Demand → high sales

IV. FUZZY INFRENCE IN DECISION MAKING

Risk management is usually happens in Decision Support Systems[1].

Consider for the fuzzy rule " if x is A then x is B "

EXAMPLE1

Consider Business rule

If x is Demand of the product then x is High Price

Let x1, x2, x3, x4, x5 be the Items.

The Generalized fuzzy set

Demand ={ 0.56/x1+0.48/x2+0.86/x3+0.36/x4+0.88/x5, 0.06/x1+0.04/x2+0.07/x3+0.03/x4+0.2/x5 }

$\mu_{Demand}^{FCF}(x) =$

0.5/x1+0.44/x2+0.79/x3+0.33/x4+0.68/x5

High Price = 0.49/x1+0.52/x2+0.35/x3+0.4/x4+0.3/x5, 0.09/x1+0.02/x2+0.06/x3+0.02/x4+0.1/x5 }

$\mu_{High\ Price}^{FCF}(x) =$

0.4/x1+0.5/x2+0.29/x3+0.38/x4+0.2/x5

With the inference rule $A\to B=\min\{1, 1-\mu_A(x) + \mu_B(x)\}$

$\mu_{Demand \to High\ Price}^{FCF}(x) =$

0.9/x1+1/x2+0.5/x3+1/x4+0.52/x5

$\mu_{Demand \to High\ Price}^{R}(x) = 1 \quad \mu_{Demand \to High\ Price}^{FCF}(x) \geq 0.6$

$\quad\quad 0 \quad \mu_{Demand \to High\ Price}^{FCF}(x) < 0.6$

= 1/x1+1/x2+0/x3+1/x4+0/x5

The investment decision may be

Yes for x1, x2, x4

No for x3 and x5.

The fuzzy reasoning Risk management in Decision Support Systems is given as

Consider for the fuzzy rule and fuzzy fact

if x is A then x is B

x is very A

---

x is very A o A→B

If x is Demand of the product then x is High Price

x is very Demand

---

x is very Demand o Demand→Price

Demand ={ 0.56/x1+0.48/x2+0.86/x3+0.36/x4+0.88/x5, 0.06/x1+0.04/x2+0.07/x3+0.03/x4+0.2/x5 }

$\mu_{Demand}^{FCF}(x) =$

0.5/x1+0.44/x2+0.79/x3+0.33/x4+0.68/x5

High Price = 0.49/x1+0.52/x2+0.35/x3+0.4/x4+0.3/x5, 0.09/x1+0.02/x2+0.06/x3+0.02/x4+0.1/x5 }

$\mu_{High\ Price}^{FCF}(x) =$

0.4/x1+0.5/x2+0.29/x3+0.38/x4+0.2/x5

With the inference rule $A\to B=\min\{1, 1-\mu_A(x) + \mu_B(x)\}$

$\mu_{Demand \to High\ Price}^{FCF}(x) =$

0.9/x1+1/x2+0.5/x3+1/x4+0.52/x5

$\mu_{very\ Demand}^{FCF}(x) =$

0.25/x1+0.19/x2+0.62/x3+0.1/x4+0.46/x5

x is very Demand o Demand→High Price

=0.25/x1+0.19/x2+0.5/x3+0.1/x4 + 0.46/x5

Suppose Fuzzy risk set for $\alpha \geq .5$, the decision is Yes for x3 and No for x1, x2, x4 and x5.

V. CONCLUSION

Sometimes the decision has to be taken under risk with incomplete information in many applications like Control systems, Business and Medicine The fuzzy logic is used to deal with the incomplete information because the information available to these applications is belief rather than probability. The fuzzy logic with two membership functions "true" and "false" will give more information than the single membership function. The two fold fuzzy logic is studied. The FCF is defined as difference between "true" and "false" to make as single membership function. The fuzzy risk set and fuzzy decision making are defined. The Business application is studied for fuzzy decision making. The fuzzy decision making may be applicable to other systems like Fuzzy Control Systems and Fuzzy Medical Systems where the decision making is required under incomplete information.

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