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# Analysis of Mixed Reality Tools for Learning Math in Primary and Secondary School

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**Abstract.** In our study, we provide a state of the art on Mixed Reality (MR) learning tools for teaching math in primary and secondary school. Through a detailed analysis of eight representative applications, we provide an overview of the MR applications currently used, their educational objectives, the augmentations and interactions they offer, the technologies they use, their advantages and their limitations. We conclude by identifying several remaining challenges that need to be addressed in order to benefit from the full educational potential of MR for teaching math in schools.

**Keywords:** Learning, augmented reality, mixed reality, math, serious game.

## 1 Mixed Reality to Help Children Learn

Teaching methods have evolved a lot in the recent decades. The use of digital tools has become widespread because they are essential in modern professional and non-professional life, but also because they have many educational benefits. Among these digital innovations, we will focus on Mixed Reality (MR).

As defined by Drascic and Milgram [1], “MR refers to the incorporation of virtual computer graphics objects into a real three-dimensional scene, or alternatively the inclusion of real world elements into a virtual environment. The former case is generally referred to as Augmented Reality (AR), and the latter as Augmented Virtuality.” The augmentations are displayed on a screen, in glasses or directly on real objects using a video projector. It is possible to interact with these digital objects while keeping their link with the real world.

The educational potential of MR comes from several factors. First, **the manipulation of real objects** has an impact on **embodied cognition** and would allow to significantly reduce mental load [2]. Object manipulation also motivates learners and encourages them to carry out their activities [3]. In addition, Chandler and Tricot’s study [4] demonstrates the positive impact of **physical activity** that accompanies this object manipulation, especially for young children. Physical movement seems to be especially relevant for mathematical cognition. It is through the explanation of mathematical concepts that one can notice different types of gestures (pointing, representation and metaphorical gestures) which make it possible to externalize information and improve

memory management [5]. Finally, MR makes it possible to **create multimodal (visuo-haptic) activities**, which have a superior pedagogical potential over unimodal (visual) activities for children [6]. MR allows for example, displaying different types of contextualized information directly on physical objects (*e.g.* 3D animated model of the solar system, organ names). MR can also give students more autonomy by displaying information to guide them (*e.g.* contextual information or step-by-step guide on the actions to be performed) or even validate the activities once they have been completed (*e.g.* validation of the objects position in geometry).

Several studies show that MR has its place in classrooms. Some studies [7] show, for example, that students using AR have better understanding of the course and memorization, compared to those using only the books. Kun-Hung Cheng's study of 267 middle school students also shows that AR increases students' motivation in science. However, these previous studies also revealed some negative points. Some students think MR is responsible for reducing imagination and obstructing their reading skills [8]. In addition, the equipment used can be expensive but also complicate the activities [9]. This equipment can also be tedious to set up for teachers but also to use for students (*e.g.* wearing glasses for a long time).

Despite the undeniable potential of MR, its integration into schools therefore raises a certain number of questions related to the type of activities and equipment that should be used to maximize the educational potential of MR without constraining teachers and students. In the rest of this article, we focus on MR for math. As we will present, this is an area which has a strong impact on other scientific fields and which could particularly benefit from the advantages of MR.

### 1.1 Learning Math with Mixed Reality

At school, all subjects are important, but math represents the knowledge from which most other sciences derive. According to the study of Watts *et al.* [10], the skills of children aged four to five would predict their scientific skills in adolescence. Another study [11] shows that succeeding in math generally implies future success in other fields such as reading. In addition, many primary school students find science to be a masculine, elitist and consider math as a difficult subject. Using a motivating medium like MR could be a good solution to avoid blockage [12]. MR interactions are also very well suited to convey notions of geometry and algebra by displaying virtual 3D shapes or showing 2D information directly on 3D figures. MR applications for math are numerous and varied by the interactions they offer (display of information, help, validation), the targeted educational objectives (*e.g.* additions, fractions, 3D geometry), but also by the equipment they require (*e.g.* tablets, projectors, markers, glasses). In this article, we offer an analysis of existing MR applications, in order to understand their different characteristics and their impact on learning.

## 2 Analysis of Mixed Reality Applications for Math

To our knowledge, even though there are a considerable number of MR applications for learning math, there is still no state of the art on this subject. We therefore propose

to analyze a representative selection of these applications. We searched on *Google Scholar*, which indexes the articles from the main publishers (*Springer*, *IEEE*, *HAL archives ouvertes*, etc.) by combining the following keywords: (*teaching or Learning and (augmented reality or mixed reality)*) or (*math or fraction or geometry*), (*Learning or Teaching*) and (*object manipulation or physical movement*) or (*digital*) or (*preschool or primary*) Since MR technologies evolve very quickly, we only selected applications after 2016. We also selected papers from several countries (Asian, European, American countries) to see the different uses of MR in math according to the programs and cultural aspects, we also took into consideration the different fields of application, the different means of interaction and the various materials used.

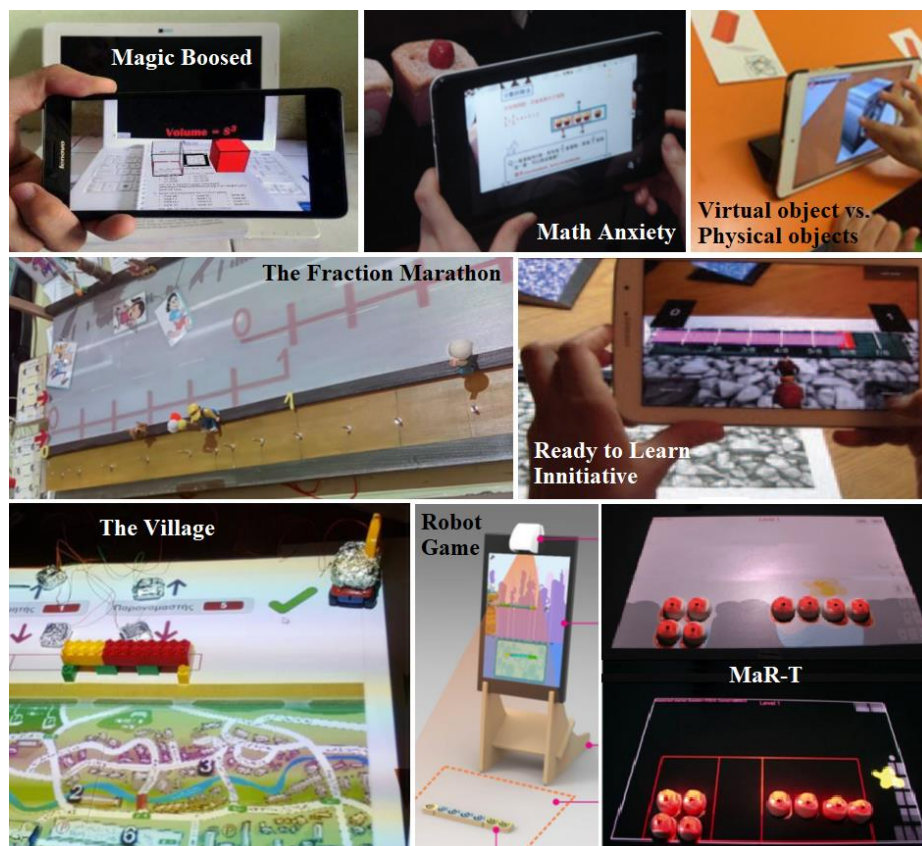


Figure 1. Eight representative Mixed Reality applications for math

Among the applications found, we will detail only eight, because they provide a good overview of the existing applications. In particular, we chose applications with different educational objectives. We also selected applications that require different types of hardware to discuss the advantages and disadvantages of each for schools.

In the next section, we analyze the eight applications according to a six-point grid: the educational objectives, the activities it offers, the functionalities offered by MR, the

equipment necessary to use the application and finally, the context in which the system was evaluated and the advantages and limitations identified by the authors.

### 2.1 Magic Boosed

The Magic Boosed [13] app comes from Indonesia.

Educational objectives: Improve the spatial perception of volumes and surfaces of geometric shapes for children aged 7 to 12.

Mathematical exercises: The students need to answer basic geometry questions (*e.g.* what is the surface of this 3D figure?).

MR functionalities: The application provides AR information to help resolve the problems (*e.g.* 3D objects, height, formulas). These elements offer no interaction.

Equipment: paper textbook and smartphone

Experimentation: According to the authors, the experimentations lead with two teachers and eight students showed that the application increases the motivation and interest of the students and facilitates exchanges between them and the teacher.

### 2.2 Math anxiety

This application is the result of a research project on anxiety in Taiwan [14].

Mathematical exercises: Eight quizzes are scattered across the classroom to review subjects studied in class such as fractions and geometry.

MAR functionalities: In addition to displaying 3D objects in AR, the height and radius of real objects, the application also triggers videos when it recognizes real objects to help students answer the quizzes. These virtual items are not interactive.

Equipment: Printed out sheets of paper, real objects and tablet.

Experimentation: Based on the experience lead with 137 students, the AR application decreases or eliminates anxiety in math, increases attention, motivation, confidence and student satisfaction compared to the mobile application without AR. The authors identify several perspectives such as allowing students to interact directly with the virtual 3D objects and using tools that are more efficient than HP Reveal and Augment because they could not personalize the content and interactions according to their needs.

### 2.3 Virtual object vs. Physical object

This study, which comes from Turkey, aims at measuring the differences between the use of AR virtual objects and physical objects for learning geometry [15].

Educational objectives: Teach 5 to 6-year-old children how to recognize geometric shapes, including 2D shapes (*e.g.* triangle, square) and 3D shapes (*e.g.* sphere, cube).

Mathematical exercises: Students need to classify the cards with the geometric shapes.

MR functionalities: The AR application displays the 3D objects on the cards. The students can manipulate these objects, change their size, position and orientation.

Equipment: Cards with markers and tablet.

Experimentation: the experimentation, lead with 72 children, showed that the application seemed to have effectively supported the learning process, and created excitement.

The authors argue that it would be beneficial to add educational feedback to help children understand the type of each object based on the choices they make.

#### 2.4 The Fraction Marathon

The Fraction marathon uses number lines to teach fractions to Greek children [16].  
Educational objectives: teach additions and subtractions of fractions to 11-year-olds.  
Mathematical exercises: The game scenario features several runners interrupted in their 2km race due to rain. The students must place the runners back where they stopped. Their position is given as a fraction relative to the finish point (*e.g.*  $\frac{2}{3}$  of the finish), the position of other runners or elements of the scene. If the students make a mistake, they lose points and are prompted to click on the help button. This triggers a video with voice instructions that guides them on how to handle the lines and solve the problem.  
MR functionalities: Students can measure the size between two objects by resize and moving the MR number line. They can also update the fractional unit with a physical button. The MR also automatically validates the position of the runners.  
Equipment: miniature wooden stadium, projector, laptop, two *Makey Makey* boards (is an electronic invention kit that allows you to connect "everyday objects" to the computer program without having any technical knowledge).  
Experimentation: After a study lead with 28 students, the authors argue that the game immerses and amuses the students. The feedback and help mechanisms were particularly effective for empowering them. The authors also believe that it would be more effective to present the game without mentioning fractions to reduce stress at the beginning of the game. The authors also wish to enlarge the play space so that more children can play at the same time and find a less expensive solution.

#### 2.5 Ready To Learn Initiative

Ready to Learn initiative [17] aims to study the potential of AR for learning certain mathematical themes (Geometry, Fraction, Counting, etc.) in the US. Among other applications, they offer an AR application for teaching fractions.  
Educational objectives: Introduce fractions to children aged 6 to 9.  
Mathematical exercises: Represent a fraction by placing a real object on a number line.  
MR functionalities: The application automatically measures the position of the object and displays it as a fraction. Students can change the denominator of the fraction and the application automatically updates the numerator.  
Equipment: Paper with marker and tablet.  
Experimentation: Three teachers evaluated the prototype without students. They noted a good educational potential and noted that the presence of interactions with the 3D objects would be likely to capture the attention of their students.

#### 2.6 The Village

The objective of this Greek project is to teach fractions to elementary students [18].  
Educational objectives: To teach fractions to children from 6 to 12 years old.

Mathematical exercises: The scenario of the game describes a child who visits his/her grandparents in their partially destroyed village. With the help of two virtual characters, the player's goal is to repair the village's infrastructure (e.g. pipe, bridge represented by legos), by measuring the damaged part (in red) with the number line.

MR functionalities: The players can adjust the size, the numerator and the denominator of the MR number line by manipulating screwdrivers and buttons. The game automatically detects if the damaged part of the infrastructure is measured correctly.

Equipment: *Makey Makey* board, projector, table and accessories.

Classroom tests: This game was not tested in an educational context. The authors' perspectives are to use mobile devices in order to simplify the game.

## 2.7 Robot Game

The Robot Game, developed in Uruguay, aims to teach additions [19].

Educational objectives: Help 5 to 6-year-old children to learn additions.

Mathematical exercises: The children need to choose enough blocks (lengths 1 to 5) to extend a robot's arm so it can reach a screw.

MAR functionalities: The system provides continuous feedback, since the children control the size of the virtual arm by placing physical blocks in front of the tablet.

Equipment: Tablet, mirror, a stand for the tablet and a set of wooden blocks.

Experimentations: After testing the game with 19 students, the authors argue that it provides cognitive offload, increases commitment and joy and empowers students through the feedback system. Their perspectives are to use a markerless technology to directly detect the shape and color of the physical objects, improve the feedback by making it fluid and add animations and advice from the robot.

## 2.8 MaR-T

This Turkish study aims to help children understand non-symbolic numbers [20].

Educational objectives: Help children from 3 to 5 years old to compare amounts.

Mathematical exercises: Momo, a fictional character, asks the children for help getting home. First, they must place objects on the designated locations to allow Momo to cross obstacles (e.g. river, ditch). Momo then asks them to point to the side where there are the most objects or to put their hand in the middle if there are as many on both sides. If the children give the correct answer, they are praised, otherwise Momo asks questions such as "Why do you think this side has more elements than the other side?".

MR functionalities: MR is used to validate the activities and to tell Momo's story.

Equipment: Camera and projector.

Experimentations: After testing the game with ten children, the authors argue that the interactions with Momo helps children keep focused. The feedback and reward systems also seemed to help the children complete the activities.

### 3 Global Analysis

#### 3.1 Analysis of Educational Objectives

It is possible to group the applications in three mathematical themes. The first category deals with problems linked to the **basics of math** such as the *Robot Game* and *MaR-T*. There are many other MR applications that teach additions and subtractions. For example, the *Counting With Paula* AR app [7] or the *AR Flashcards Addition* application [21]. The effectiveness of MR for teaching the basics of math appears to be due to its capacity to immerse students in a fun environment and arouse their curiosity [3] which leads to a better understanding and motivation [22].

The second category of applications, in which we find *Magic Boosed*, *Math anxiety and Virtual object vs Physical object*, deals with **geometry**. There are many MR applications for this topic, certainly due to the fact MR technology improves spatial intuition of students, which is an effective way to learn geometry [23].

The last category, in which we find *The Fraction Marathon*, *Ready To Learn Initiative* and *The Village*, deals with **fractions**. Often seen as the scourge of students, fractions often discourage them and push them to dislike math. The use of MR can be particularly effective for this complex and unpopular matter, because it transforms it into a fun and captivating experience.

#### 3.2 Analysis of Mixed Reality Interactions

The applications analyzed above present two types of MR interactions. The first group offers **very light or non-existent AR interactions**, such as *Ready To Learn Initiative*, *Virtual Object vs Physical Object* and *Magic Boosed*. These applications only use AR to display virtual information on real objects. At best, the students can manipulate, turn and change the size of the virtual augmentations. These applications have the advantage of being easy to set up since they only require a tablet or smartphone and printed out markers on paper or cardboard.

The second group of applications offer **rich MR interactions**, in which we find *MaR-T*, *The Village*, *The Fraction Marathon*, *Robot Game* and *Math anxiety*. In addition to the AR augmentations, the manipulation and the position of the real objects have an impact on the applications. In *MaR-T* for example, it is the position of the cubes that triggers the next level in the game. Some of these applications also offer personalized feedback and help. Such rich interactions have undeniable advantages for different areas of learning, for example studies in the medical field show that the presence of feedback contributes to the development of psycho-motor and cognitive skills. Studies in the medical field show that the presence of feedback contributes to the development of psycho-motor and cognitive skills [24]. Another study [25] shows that students appreciate this type of rich interaction, and in particular the feedback, which promotes self-regulated learning, at their own pace. However, the applications mentioned above all require specific equipment such as projectors, 3D objects, as well as space, thus making their use in a school complicated.



### 3.3 Analysis of Mixed Reality Hardware and Software

Three types of hardware can support MR applications. **Smartphone and tablets** are the most common. According to a study in Switzerland, on more than 1,000 students, 78% from age 6 to 13 use mobile phones regularly and 3/4 of students aged 12 to 13 already owned one [26]. The figures for tablets are similar. The fact that the majority of children are already familiar with this equipment and own one is an important advantage to using them in class. On the other hand, tablets or smartphones do not allow to have both hands free to handle objects. In addition, some devices do not support advanced technologies. For example, Vuforia's Ground Plane technology, which allows placing digital content on a table, requires fairly modern mobile devices as well as specific operating system updates [27]. Some cameras may also stay out of focus temporarily and therefore take some time to function properly.

**Projectors** have the advantage of being moderately expensive and users have their hands free to manipulate objects. In addition, projectors have a wide field of vision, and allow group work [28]. However, they are complex to set up and take up a lot of room.

Finally, the last type of hardware for MR applications is **AR glasses and headsets**. These have the advantages of offering particularly good perception of depth [29]. Their mobility is also a major asset since the devices can be transported everywhere and leave the user's hands free. However, they are mostly used for professional training due to their very high cost. They can also cause visual and mental fatigue and nausea.

There are several types of software to design MR applications. Teachers can use **AR application editors** that allow them to create AR applications without any development skills such as *Augment* [30] and *Aurasma* [31]. Thanks to simple interfaces, they can record their markers (object, image or QR Code) and associate them with different 3D models or documents. Applications, created with these editors, only support very light AR interactions, which consist in displaying digital content on the detected marker.

There are several **open source technologies** such as *Artoolkit* [32] or *OpenCv* for example, used to develop *MaR-T* and *Robot Game*. However, these open source and free technologies require advanced expertise in image processing.

Finally, there are several **paying technologies**, generally offering a free version, which allow developers to create MR applications, without being an expert in image recognition. The most popular is *Vuforia* [27] which offers good quality, stable and efficient services. *Wikitude* [33] and *Kudan* [34] are other alternatives.

## 4 Conclusion and Discussion

Through the analysis of eight Augmented Reality (AR) and Mixed Reality (MR) applications for teaching math in primary and secondary schools, we show that this new technology offers several educational benefits such as cognitive offloading, captivating students' attention and making them learn while having fun.

The experimentations, led by the authors of these applications, show that, even more than the AR augmentations (virtual information and object projected on real objects),

it is the rich interactions, the custom feedback and help provided by the MR applications that have the highest impact on engagement and learning. As we stand, the only technology capable of creating such rich MR interactions are paying technologies such as *Vuforia*, that requires solid programming skills. If we want MR to be accessible in schools, is it important to provide similar open-source technologies or MR editors that could allow teachers to create their own complex MR applications.

The only way to develop the use of MR in schools is to provide applications that function with inexpensive equipment that is suitable for children. In terms of hardware, our state of the art show that the best option is clearly tablets. Most schools are usually equipped with enough for an entire class and they are simple to set up since teachers and students are used to manipulating them. However, their use deprives children for using their hands at the same time. This constraint therefore needs to be taken into account when designing activities by clearly identifying when the children should be manipulating real objects and when they should pick up the tablet to get feedback or validate the exercises. Another method would be to design collaborative activities in which the children take turns in holding the tablet, while the other manipulates the objects. The experimentation also show that the MR applications should also function with markers that teachers can easily print out on paper or cardboard. Another interesting perspective would be to help them create augmentation on material they already have in their class (*e.g.* cubes, globe) by using custom marker stickers.

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