A Nonlinear Credit Rating Optimization Methodology for Resolving the Mismatch Between Credit Ratings and Loss Given Default

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***ABSTRACT*:** This paper investigates the mismatch phenomenon that the loss given default (LGD) caused by borrowers with high credit rating is not low. To address this problem, we develop a nonlinear credit rating optimization methodology that the credit rating increases with the decreasing LGD. It forces the LGD strictly decreasing according to the credit rating from C rating to AAA rating, which avoids the unreasonable phenomena as higher rating with higher LGD. Furthermore, the method is validated using three actual microfinance data samples from Chinese commercial banks. The empirical results show that the proposed method indeed guides the way to solve the mismatch issue between credit ratings and LGDs. Moreover, the results derived from this paper provide valuable information for the bankers, for the society, and for the bond investors to manage credit risk.

***Keywords*:** Credit rating; credit risk; loss given default; microfinance loan

JEL: C61, G32, E51, H81.

1. Introduction

Credit rating plays crucial roles for investors and corporations (Benbouzid et al., 2017; Chi et al., 2017; Abedin et al., 2018; Abedin et al., 2019; Medina-Olivares et al., 2021). Accurate rating system would avoid the misleading for investors and benefit for corporations as well as help regulators to manage (Ogut et al., 2012; Doumpos and

Figueira, 2019; Sun et al., 2022 Abedin et al., 2022). It is important for commercial banks to price accordingly for various loans of debtors with possibly different credit ratings to manage the default risk. Estimating the LGD with different credit rating would be extremely crucial for the creditors to manage the default risk from the debtors with marginal ratings (Loterman et al., 2012; Zamore et al., 2018). Hence, the issue of credit rating has been concerned by many researchers (Altman, 1968; Karlan et al., 2011; Marques et al., 2013; Benbouzid et al., 2018; Bai et al., 2019). However, through the test of the actual data of commercial banks, we found the existing credit rating results have an unreasonable phenomenon. The LGD caused by borrowers with high credit rating is not low. More specifically, when we explored the credit rating results using actual farmer loan data from a state-owned commercial bank in China, it was found that the LGD of borrowers in the fifth level (i.e. BB rating) is higher than the LGD of borrowers in the sixth level (i.e. B rating), as shown in the last Column of Table 1. The credit rating in Table 1 is the horizontal axis and the corresponding LGD_k is the vertical axis, we can obtain the LGD distribution corresponding to the credit rating result, as shown in Fig. 1. Drawn from Table 1 and Fig. 1, LGD of the fifth credit level is more than that of the sixth, i.e. $LGD_{\mathrm{BB}}=1.037\%>LGD_{\mathrm{B}}=0.898\%$, causing the unreasonable phenomenon that the LGD of a higher credit rating is more than that of a lower one. Since the credit rating results cannot fully compensate the credit risk of borrowers of different levels in loan pricing, it will increase the loan loss of banks. So it is an important and prominent problem to explore the mismatch between credit ratings and LGD. But in reality, due to the data confidentiality of commercial banks, the actual loan data is hard to get. The mismatch between credit ratings and default losses has not been well addressed.

Table 1. Information of the 2,044 farmers' microfinance

(1) Name	(2) Credit score	(3) Proportion	(7) Sample size m_k	(5) Credit ratings k	(6) Credit score interval S_i	(7) The owed debt principal and interest L_{ik}	(8) The receivable debt principal and interest R_{ik}	(9) LGD_k
SMG	96.54					0	52 613.75	
…	…	8%	164	1 AAA	$93.44 \leq S_i \leq 100$	…	…	0.007%
GHL	93.44					0	52 610.00	
HDH	93.42					0	52 610.00	
…	…	16%	327	2 AA	$92.12 \leq S_i < 93.44$	…	…	0.452%
ZQK	92.12					0	52 684.00	
RZB	92.11					0	51 870.00	
…	…	30%	613	3 A	$89.38 \leq S_i < 92.12$	…	…	0.864%
LSQ	89.38					109.78	31 293.60	
SJ	89.30					0	12 412.38	
…	…	16%	327	4 BBB	$87.44 \leq S_i < 89.38$	…	…	0.983%
TAX	87.44					0	51 891.25	
HJT	87.42					0	52 618.00	
…	…	10%	204	5 BB	$86.02 \leq S_i < 87.44$	…	…	**1.037%**
ZYF	86.02					0	31 566.00	
YTH	86.01					0	15 600.30	
…	…	8%	164	6 B	$83.98 \leq S_i < 86.02$	…	…	**0.898%**
HB	83.98					0	52 566.50	
LCJ	83.97					0	5 265.35	
…	…	6%	122	7 CCC	$81.02 \leq S_i < 83.98$	…	…	2.427%
YLH	81.02					0	51 892.25	
PQX	81.00					22.08	51 976.25	
…	…	4%	82	8 CC	$76.20 \leq S_i < 81.02$	…	…	3.964%
HQY	76.20					0	30 267.75	
WXL	76.16					53.60	31 581.00	
…	…	2%	41	9 C	$0 \leq S_i < 76.20$	…	…	9.481%
LC	59.90					55 330.36	55 505.00	

Note: This table reports the actual loan information of 2,044 farmers' microfinance.

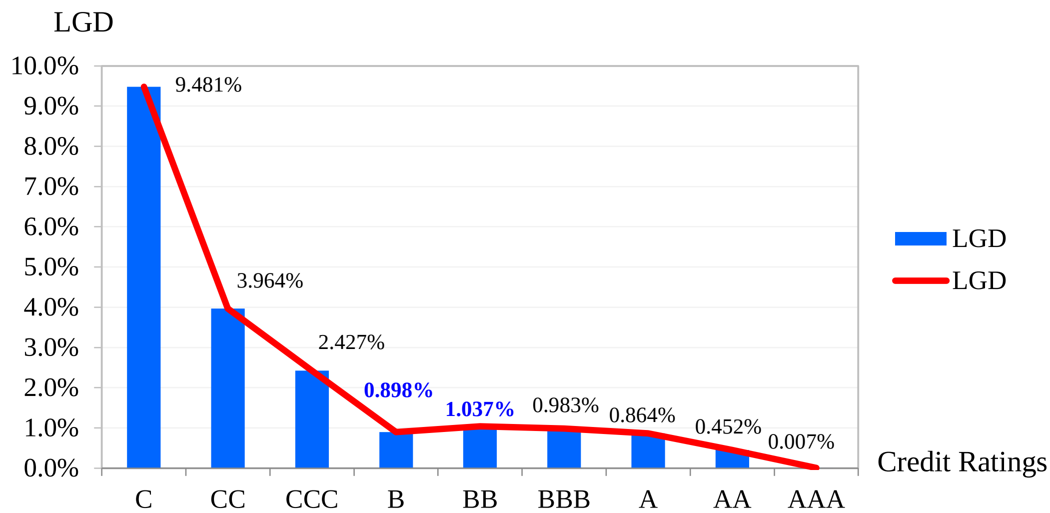

Fig. 1. Distribution of LGDs of credit rating for 2,044 farmers.

This paper makes three contributions to the literature. First, we provide a novel credit risk evaluation method according to the LGD strictly decreasing order, and our method guarantees that the distribution of LGDs follows the credit risk-rating match-up standard to eliminate the unreasonable phenomenon in the existing research that the higher credit rating the more LGD. Furthermore, the empirical and robustness analysis on the method developed are carried out from three actual bank data sets, i.e. the microfinance data of 2,044 farmers, the microfinance data of 2,157 small private businesses and the microfinance data of 3,111 small and medium enterprises (SMEs). The result shows that the proposed method can precisely find the credit rating result satisfying the credit risk-rating match-up standard, which indeed guides the way to solve the mismatch problem between credit ratings and LGD. Finally, the method is accessible and easy to implement in many similar situations. It provides valuable information and references for the bankers, for the society, and for the bond investors to manage credit risk.

The paper is organized as follows. Section 2 is the literature review. Section 3 introduces the credit risk-rating match-up standard to divide credit ratings. Section 4 presents the data and empirical analysis. Section 5 gives the robustness analysis. A conclusion with future research directions and limitations is presented in Section 6.

2. Related Literature

Much previous literature has conducted studies about the way to measure the credit risk of loan applicants. The main proposed references to evaluate credit risk can be divided into two categories. The first category focuses on evaluating applicants' credit risk by estimating the probability of default (PD), and the second category of studies concentrate on the measurement of the LGD to depict the credit risk of a loan client. As early as in 1960s, the famous Z-score model of five variables to assess firms' PD was established based on the observation of bankrupt and non-bankrupt firms in the United States (Altman, 1968). After that, methods to measure customer credit risk by PD have emerged in large numbers (Gupton et al., 1997; CSFB, 1997). KMV Company uses the probability of asset value less than debt value to measure the PD of the corporation (Crosbie and Bohn, 2003). Although study on modeling credit risk has surged in the past decades, few researchers have focused on the impact of macroeconomic variables on customer credit risk. Aimed at this issue, Carling et al. (2007) build a continuous time model with macroeconomic factors such as the GDP growth rate and the unemployment rate to modify the debtors' credit rating transition probabilities. Shi et al. (2019) developed a credit rating framework that considers the impact of macroeconomic variables on the financial institutions' credit decisions. This framework can help to ease the financing constraints of small and micro credit entities. By using the data of 25,000 customers' default payments in Taiwan, Yeh and Lien (2009) proposed a novel 'Sorting Smoothing Method' to measure the customers' PD. The results showed that the credit rating method of neural network is superior to the other five methods. Twala (2010) explores the predictive behavior of five classifiers on four real-world datasets in terms of credit risk prediction accuracy, and shows that the accuracy could be improved by classifier ensembles. In terms of the ability to correctly classify loan applicants as good or bad credit risks, Finlay (2011) compared the performance of several multiple classifiers and found that the ETB method (Error Trimmed Boosting) outperformed all other multiple classifiers on UK credit data. As non-performing loans and loan volumes increase around the world, it becomes more and more important for commercial banks to distinguish the credit risk of loan applicants. To do that, Akkoc (2012) proposed the three-stage hybrid adaptive neuro fuzzy inference system credit scoring model to predict the probability of customer default. By using the credit card data of 2,000 loan applicants in Turkey, the empirical results showed that the novel method performs better than other three methods. In order to analyze the relationships between the measurement method of banks' PD considering the Basel regulatory requirement and the traditional risk monitoring indicators, Gomez-Fernandez-Aguado et al. (2018) computed the PD of loan clients utilizing the WSYstemic model. Niu and Hua (2019) developed a credit risk evaluation approach considering moral risk and rollover risk. Numerical simulation results indicate that moral risk can be used to interpret the credit spread puzzle. In addition to the above studies, the methods of using PD to measure credit risk have been widely applied in reality. Derviz and Podpiera (2008) described that the Federal Financial Institutions Examination Council (FFIEC) released CAMELS credit rating system in November 1979. The CAMELS system predicts debtors' probability of default by using the debtors' Capital Adequacy, Asset Quality, Management, Earning and Liquidity and has successfully applied in many situations. Moody's (2009), Standard & Poor's (2012), Fitch Ratings (2013) established their own credit risk prediction systems by using applicants' financial data. Although the methods of using PD to measure credit risk have made great progress, the theoretical assumptions and parameters setting of credit rating models are mainly derived from listed companies in developed countries. It makes the application of these models have some limitation.

In the meanwhile, many scholars have made useful explorations in terms of the measurement of the LGD (Frontczak and Rostek, 2015; Kruger and Rosch, 2017; Maciag et al., 2018; Tanoue et al., 2019). Qi and Zhao (2011) compared six modeling approaches for predicting LGD. The empirical results showed that the regression tree (RT) and neural network (NN) non-parametric methods perform better than the other four parametric methods, like Fractional response regression (FRR), Inverse Gaussian regression (IGR), Ordinary least squares regression (OLS) and Inverse Gaussian regression with beta transformation (IGR-BT). In order to improve LGD forecasts, Gurtler and Hibbeln (2013) proposed a two-step approach for modeling LGDs of non-defaulted loans and defaulted loans. By applying this approach to a data set of 69,985 retail loans of a large German bank, the empirical results showed that the predictive performance of the proposed approach is significantly improved in comparison to the direct regression approach. Misankova et al. (2015) made the detailed theoretical analysis of four types of LGD measurement approaches: the Market LGD, the Workout LGD, the Implied Market LGD and the Accounting LGD. The studyed shows that each of the four types of LGD methods has its advantages and disadvantages. In order to improve the prediction accuracy of LGD, Yao et al. (2017) proposed a two-stage modelling methodology for loss given default with support vector machine technique. By empirical comparison of credit card data from a UK retail bank, they found that the proposed model performed better than statistical regression methods. In addition, with the development of financial technology (fin-tech) and mathematical analysis technology, new credit rating methods emerge one after another. For example, Kim et al. (2008) proposed a random effected multivariate regression model to estimate transition probabilities of credit ratings. Chen et al. (2009) developed a two-stage decision tree credit assessment model to evaluate applicants' credit level. Hwang et al. (2010) established an ordered semi-parametric Probit model of credit rating by using the ordered semi-parametric function instead of the linear regression function. Yao et al. (2015) developed an improved support vector regression technique to predict LGD of customers. In order to improve the accuracy of corporate bond default loss prediction, Nazemi et al. (2017) proposed a novel fuzzy decision fusion method. In the enterprise financing ability evaluation, Shi et al. (2018) bulit a credit risk evaluation indicator system through the empirical analysis on 713 small enterprises in China. Hurlin et al. (2018) create a LGD prediction model based on loss functions defined in terms of regulatory capital charge. By using a sample of almost 10,000 observations provided by an international bank, Hurlin et al. compared the proposed model with six LGD models, i.e. the regression tree, the fractional response regression model, the gradient boosting, the random forest, the least squares support vector regression and the artificial neural network model. The empirical results showed that the proposed model had higher prediction accuracy. Chai et al. (2019) propose a multicriteria credit rating model combining TOPSIS together with Fuzzy C-Means methods and calculate the credit scores of Chinese 687 small wholesale and retail enterprises.

The above researches mainly focus on the prediction of PD or LGD, seldom consider the mismatch between credit ratings and LGD. It is very common in reality that many credit rating systems, performing well on the evaluation indicators, may have the unreasonable phenomenon that LGD caused by borrowers with high credit rating is not low. This paper focuses on this issue to conduct the detailed analysis.

3. Methodology

In this section, we introduce the nonlinear credit rating optimization model. First of all, we introduce the credit risk-rating match-up standard that the credit rating increases with the decreasing LGD. Then we propose a nonlinear programming approach to establish the credit evaluation model satisfying the credit risk-rating match-up standard. Finally, we show how to determine in each step toward an optimal credit rating result with the credit risk-rating match-up standard.

3.1. The credit risk-rating match-up standard

In this subsection, we introduce the credit risk-rating match-up standard for the credit rating model in order to get consistent credit rating and its corresponding LGD.

The credit risk-rating match-up standard:

- The LGD of lower credit rating should be bigger than that of previous higher credit rating, i.e., $LGD_{k-1} < LGD_k$ for all k.
- The total difference of the LGD on adjacent credit ratings should be minimal, i.e., $\min \sum_k (LGD_k - LGD_{k-1})^2$.

The economic implications of these two constraints are as follows. The first condition requires the strictly decreasing property for LGD on its corresponding credit rating from C rating to AAA rating, and this will refer to be the first constraint condition for the credit rating model in next section. It can eliminate the unreasonable phenomena that higher credit rating may have higher LGD and avoid the misleading for investors. The second condition shows that all the difference from adjacent credit ratings reaches the minima among all possible divisions with first constraint conditions, and the credit rating results would reach the best. The function

$\min \sum_k (LGD_k - LGD_{k-1})^2$ will be called the objective function in the next section for the credit rating model.

We present an example on how to divide the credit rating to satisfy the credit risk-rating match-up standard. Table 2 comes from the empirical analysis using the real data. It illustrates from the first Column on credit ratings from higher to lower and from the third Column on LGD for the corresponding credit rating. We have nine credit ratings with different LGD_k and first Column k. It is clear that $LGD_1 < LGD_2 < \cdots < LGD_7 < LGD_8 < LGD_9$, and the first condition satisfies. In the following empirical analysis, we show that the objective function value for Table 2 reaches 0.000067846 the smallest value, hence the second condition is verified. Table 2 gives an example of the credit rating result matching up with its corresponding LGD. The credit rating modeling ideas can be illustrated as Fig. 2. It gives a framework to create a credit evaluation method in order to achieve the credit rating result satisfying the credit risk-rating match-up standard.

Table 2. The global optimal credit rating results.

(1) Credit rating		(2) Sample size	(3) Global optimal LGD_k	(3) Trend of LGD_k
1	AAA	81	0.007%	LGDs strictly decreasing (from C rating to AAA rating)
2	AA	310	0.419%	
3	A	190	0.628%	
4	BBB	130	0.667%	
5	BB	310	0.724%	
6	B	130	0.929%	
7	CCC	160	1.269%	
8	CC	190	1.658%	
9	C	543	2.047%	
Objective function values			0.000 067 846	

Note: This table reports the credit rating results obtained from the proposed credit rating model.

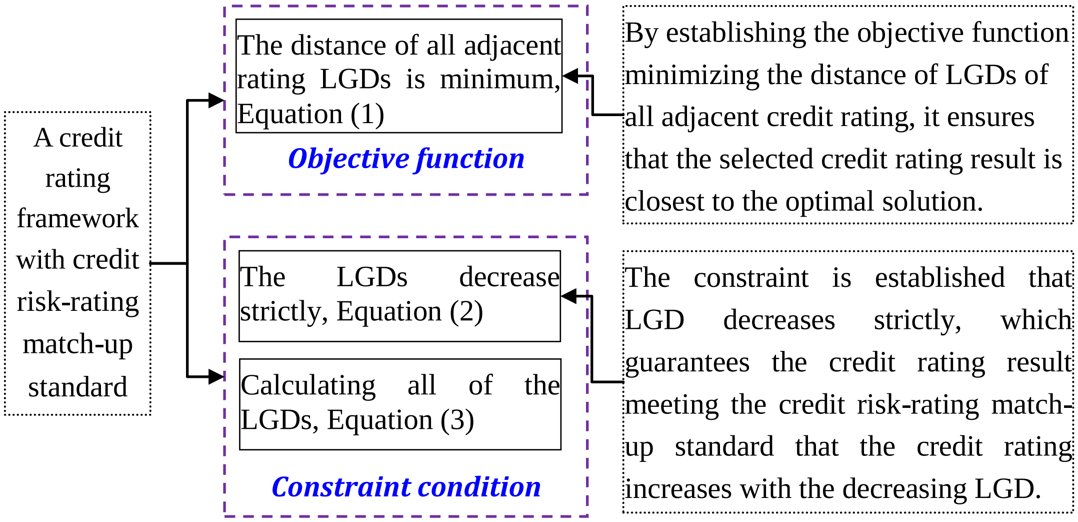

Fig. 2. A modeling framework for matching up credit ratings with LGDs.

3.2. Establishment of credit rating model

Let LGD_k be the LGD of the k^{th} credit rating for k = 1, 2, ⋯, 9 corresponding to AAA, AA, ⋯, C. We set the objective function to be the minimal distance of all adjacent rating LGDs, as shown in Equation (1) (Shi et al., 2020).

$$Obj:\ \min f = \min \sum_{k=2}^{9} (LGD_k - LGD_{k-1})^2 \tag{1}$$

The procedure to search for the minimal of distance of all adjacent ratings LGDs guarantees that the selected credit rating result is the closest to the optimal solution. Note that LGD=1 means that all of the credit rating customers are default, and LGD=0 may refer to some countries' sovereign debt. So we set the first constraint condition (the LGD_k is strictly decreasing in k) as follows.

$$0 < LGD_1 < \cdots < LGD_8 < LGD_9 \le 1 \tag{2}$$

Let L_{ik} be the owed debt principal and interest of the k^{th} credit rating and the i^{th} customer. Let R_{ik} be the receivable debt principal and interest of the k^{th} credit rating and the i^{th} customer. Thus the LGD of the k^{th} credit rating is given by

$$LGD_k = \frac{\sum_i L_{ik}}{\sum_i R_{ik}} \tag{3}$$

The value LGD_k reflects the total loss given default from all customers owed debts comparing with all customers receivable debts. It does measure the banks' real loss. For instance, we evaluate the value LGD_1 from the sample size m_1 of the credit rating AAA of the first level as follows.

$$LGD_1 = \frac{\sum_{i=1}^{m_1} L_{i1}}{\sum_{i=1}^{m_1} R_{i1}} \tag{4}$$

3.3. Solving the credit rating model

Now we outline the steps to solve the credit rating model. The first step is to determine the sample size m_1 in the credit rating AAA, and then one can rank customers' credit scores. The second step is to determine the sample size m_2 in the credit rating AA of second level and its corresponding LGD_2. Repeat the procedure until the last credit rating m_9. Under the assumption that all customer numbers of credit rating satisfy a normal distribution (Zhang et al., 2018), we have the sample size m_1 of the first credit rating AAA should less than $N/9$, where N is the total sample number. Based on the bell-shaped normal distribution of the customer numbers of all credit ratings, the first sample size is about 8% of total sample N. Therefore $m_1 = 0.08 * N$. In an example of next section, the total empirical sample N = 2,044, and $m_1 = 0.08 * N = 164 < 227 = N/9$. The sample size $m_1 = 0.08N$ is chosen in order to apply the empirical data we analyze in the next section. In general, the sample size m_1 is not unique as long as $m_1 < N/9$. However, if the model runs with no solution under the assumption $m_1 < N/9$, the sample size of the first grade can exceed $N/9$ (Yu et al., 2019).

The result of credit scoring in descending order can be arranged from stepwise discriminant analysis and logistic regression equation (PSBC and DUT, 2014). The second Column of Table 1 is the credit scoring from top to bottom in a descending order. Now we use the 7-th Column and 8-th Column of Table 1 to compute LGD_1 by Equation (4). Once we determine m_1=164 and calculate LGD_1= 0.007%, the 8-th and 9-th Columns of Table 1 can be used to computed LGD_2 by Equation (3). In Table 1, we start the sample size m_2=1 in the m_1+1 customer on the 165 row. Using this simple one size and Equation (3), we evaluate LGD_2=0 which is less than LGD_1=0.007% from Equation (2) for k = 2. This breaks the first constraint condition in the credit risk-rating match-up standard. So we have to increase the sample size m_2=2 including the 165-th and 166-th customers until LGD_2>LGD_1=0.007%. With the empirical data in Table 1, we have m_2=250 sample size so that the credit rating of second level refers to customers from 165 to 414 since LGD_2=0.534%>LGD_1=0.007% (note that for this case if m_2=249 then $LGD_2 < LGD_1$. This procedure determines m_2=250 and LGD_2=0.534%, as shown in Table 3.

Table 3. Determine the sample size m_2 and LGD_2.

(1) No.	(2) Customer name	(3) The owed debt principal and interest L_{ik} (Yuan)	(4) The receivable debt principal and interest R_{ik} (Yuan)	(5) LGD_k
1	SMG	0	52 613.75	0.007%
…	…	…	…	
164	GHL	0	52 610.00	
165	HDH	0	52 610.00	0.534%
166	HWF	0	52 631.75	
…	…	…	…	
414	WCT	0	36 755.60	
415	SMJ	0	26 133.00	0.000%
…	…	…	…	…

Note: This table reports the calculation process of the second sample size (m_2) and the corresponding LGD (LGD_2).

Inductively we repeat the procedure above. We determine the sample size m_3 from m_2+1 customer until $LGD_3>LGD_2$. If the sample size exhausts the total sample size N, then one has to stop the process. Go back to adjust the sample size m_2. Now we have to increase the sample size for the second credit rating by choosing $m_2=251$ as long as $LGD_2>LGD_1$. There are more than one solution for the second sample size m_2, denoted by $m_2^{(i)}$ (i=1, 2, 3…). Similarly, $m_j^{(i)}$ for j = 3, 4, …, 9. With the new $m_2^{(i)}$, the sample size m_3 and LGD_3 are determined. Based on the searching sample sizes, there are multi-vector sample sizes $(m_1, m_2^{(i2)}, m_3^{(i3)}, \cdots, m_8^{(i8)}, m_9)$ for credit rating satisfying both first and second constraint conditions of the credit risk-rating match-up standard, where the last sample size for the lowest credit rating of level nine is determined by $N-m_1-m_2^{(i2)}-m_3^{(i3)}-\cdots-m_8^{(i8)}$. With the sample sizes determined by $LGD_9>LGD_8>\cdots>LGD_2>LGD_1$, there are various way to assign the credit ratings. Now we evaluate the objective function for all those finitely many solutions to find the least solution or the optimal credit rating model for the given sample size N and the debt information. The optimal credit rating result may not be unique, but it always exists. There are more than one choice for the first sample size of the credit rating AAA, hence, there are more than one possible optimal credit rating model with the credit risk-rating match-up standard.

If $m_1=0.08N$ and that there is no optimal credit rating model, then one has to adjust $m_1=0.08N-1, 0.08N-2, \cdots, 1$ to search the optimal credit rating model to determine a sample size vector $(m_1, m_2^{(i2)}, m_3^{(i3)}, \cdots, m_8^{(i8)}, m_9)$. Table 4 illustrates the adjustment process of the first sample size m_1 from empirical analysis in the next section. First to fourth Columns in Table 4 gather from Table 1. We present a method how to search for a global optimal credit rating model which satisfies the credit risk-rating match-up standard.

Step 1: determine m_1=0.08N=164 and LGD_1=0.007% from the third and fourth Columns and Equation (4).

Step 2: look for new LGD_1 which is close to 0.007%. Based on the choice of m_1 satisfying $m_1 < {}^{N}\!/_{9}$=227, so there are 226 possibilities to choose m_1 (m_1= 1, 2, ⋯; 226). One can compute the corresponding LGD_1 for each m_1 from the third and fourth Columns and Equation (4) and list into the fifth Column. From the fifth Column of Table 4, we search for the m_1 with LGD_1 close to 0.007%.

Step 3: choose a new m_1. One may pick m_1=81 from Table 4. Similarly, we have five new choices for m_1 in this real data, i.e., $m_1^{(1)}=81$, $m_1^{(2)}=82$, $m_1^{(3)}=117$, $m_1^{(4)}=118$, $m_1^{(5)}=165$ with $m_1^{(i)} < {}^{N}\!/_{9}$=227 ($i$=1, 2, 3, 4, 5).

Step 4: repeat the previous method to find an optimal credit rating model for each $m_1^{(i)}$, i=1, 2, 3, 4, 5.

Step 5: evaluate the objective function for every optimal credit rating model from Step 4.

Step 6: find the minimal value among the computations in Step 5. It is the global optimal credit rating result.

Table 4. Adjust the initial sample size m_1.

(1) No.	(2) Customer name	(3) The owed debt principal and interest L_{ik} (Yuan)	(4) The receivable debt principal and interest R_{ik} (Yuan)	(5) LGD of the first credit rating (LGD_1)
1	SMG	0	52 613.75	0.000%
…	…	…	…	…
81	WGL	0	51 725.00	0.007%
…	…	…	…	…
164	GHL	0	52 610.00	0.007%
…	…	…	…	…
226	XYR	0	31 592.10	0.541%
227	CYK	0	31 592.10	——
…	…	…	…	
2044	LC	55 330.36	55 505.00	

Note: This table reports the adjustment process of the first sample size (m_1).

4. Results and Discussion

For this empirical study data are collected from a Chinese government owned commercial bank that deals with 2,044 individual farmers from 28 provinces, as illustrated in Table 1, Column 1. The 28 provinces includes Tianjin, Shanxi province, Hebei province and Inner Mongolia province in the north region; Liaoning province, Jilin province and Heilongjiang province in the north-east region; Shanghai, Shandong province, Jiangsu province, Jiangxi province, Zhejiang province and Anhui in the east region; Fujian province, Hainan province and Guangdong province in the south region; Shaanxi province, Gansu province, Qinghai province, Ningxia province and Xinjiang province in the north-west region; Henan province, Hubei province and Hunan province in the middle region; Chongqing, Sichuan province, Guizhou province and Guangxi province in the south-west region. The total 28 provincial administrative regions exclude Beijing, Yunnan province, Tibet province, Taiwan, Hong Kong and Macao out of the 34 provincial administrative regions of China. The data collected from the 28 provincial administrative regions covers not only the east coast regions as Liaoning province and Shandong province etc, but also the inner regions as Shanxi province and Henan province etc, from the economic relatively developed regions as Shanghai and Guangdong province etc, also from the economic developing regions as Sichuan, Heilongjiang province etc, some minority nationals as Guangxi province and Xinjiang province etc included. It guarantees the reliability and applicability of the empirical analysis derived in this paper. Table 1 illustrates the information of microlending system of a Chinese state-owed commercial bank headquarters (see also PSBC and DUT, 2014). The second Column of Table 1 is the credit scores obtained by applied logistic regression model ranked from high to low, and data is used without any modification or adjustment. The nonlinear programming refers to calculate Equation (1), Equation (2) and Equation (3) from the 8-th and 9-th Columns and finds a nonlinear optimal credit rating result. Due to the dynamic behavior of LGD_k (k=2, 3, $\cdots$; 9), Equation (3) cannot be evaluated uniquely from various choices one can make.

We set m_1=164 and calculate LGD_1 from Table 1 to input into the tenth Column. Then we use Step 2 in the previous section to determine the sample size m_2. Running Step 3 and Step 4 to search for a local optimal credit rating model with the credit risk-rating match-up standard, i.e., we obtain a local optimal solution $(164, m_2^{(i2)}, m_3^{(i3)}, \cdots, m_8^{(i8)}, m_9)$, with corresponding vector on the loss given default $(0.007\%, LGD_2^{(i2)}, LGD_3^{(i3)}, \cdots, LGD_8^{(i8)}, LGD_9)$, as shown in Table 5. Then with various choices of m_1 in Table 6, repeat the previous Step 1 to Step 4 to search for more local optimal credit rating results. Following Step 5 and Step 6, we can obtain the global optimal credit rating result, as shown in Table 7.

Table 7 shows that there are six local optimal credit rating results, and the third and fourth Columns of Table 7 show that the global optimal credit rating results is for the initial sample size $m_1^{(1)}$=81 (See also in Table 2). The corresponding distribution of LGDs of the global optimal credit rating results is shown in Fig. 3. From Table 2, the third Column shows that the LGD is strictly decreasing from C rating to AAA rating, hence, the first constraint condition of the credit risk-rating match-up standard follows. Remind of the hypothesis on the normal distribution of all credit rating sample sizes, the basic requirement for $m_1 < N/9$ provides various choices of the sample size for the first credit rating AAA. In fact, this initial choice on m_1 derives the sample size for later credit ratings to avoid the extremely unbalanced sizes for other credit ratings and to eliminate the unreasonable phenomena that higher credit rating may have higher LGD. From the practical purpose, the method we develop so far always reaches the local and global optimal credit rating results with the credit risk-rating match-up standard. By comparing the global optimal credit rating results (see Table 2 and Fig. 3) with the non-optimal one (see Table 1 and Fig. 1), we obtain the LGDs of each credit ratings decrease strictly from C rating to AAA rating, meeting the credit risk-rating match-up standard. However, known from Fig. 1 and the ninth Column of Table 1, LGD of the fifth rating is more than the sixth one, i.e. LGD_{BB}=1.037%>LGD_{B}=0.898%. There is an unreasonable phenomenon that the

LGD of a higher credit rating is more than of a lower one, not meeting the credit risk-rating match-up standard.

It should be noted that they apply collinear analysis and stepwise discriminant analysis to reduce indicators to establish the credit rating indicator system for farmers' microfinance, and use the logistic regression model to calculate the credit scores via various weighted indicators (PSBC and DUT, 2014). We refer the details to PSBC and DUT (2014), and omit the computations in this paper to focus on our purpose.

Table 5. The local optimal credit rating results with m_1=164.

(1) Credit ratings		(2) Sample size m_i	(3) LGD_k
1	AAA	164	0.007%
2	AA	250	0.534%
3	A	160	0.740%
4	BBB	370	0.837%
5	BB	130	0.875%
6	B	160	0.899%
7	CCC	160	1.077%
8	CC	130	1.602%
9	C	520	2.149%

Note: This table reports the obtained local optimal credit rating results of 2,044 farmers when the first sample size $m_1=164 < N/9 = 2044/9 \approx 227$.

Table 6. The inference of sample size for the first credit rating AAA.

(1) No.	(2) The first credit rating sample size $m_1^{(i)}$	(3) LGD_1
1	81	0.007%
2	82	0.007%
3	117	0.007%
4	118	0.007%
5	165	0.007%

Note: This table reports the adjusted results of the first credit rating sample size m_1 and the corresponding LGD_1.

Table 7. All local optimal credit rating results and their objective function values.

(1) Credit ratings		**Initial value** $m_1^{(1)}=81$		Initial value $m_1^{(2)}=82$		Initial value $m_1^{(3)}=117$		Initial value $m_1^{(4)}=118$		Initial value $m_1=164$		Initial value $m_1^{(5)}=165$	
		(2) Sample size	(3) LGD_k	(4) Sample size	(5) LGD_k	(6) Sample size	(7) LGD_k	(8) Sample size	(9) LGD_k	(10) Sample size	(11) LGD_k	(12) Sample size	(13) LGD_k
1	AAA	81	0.007%	82	0.007%	117	0.007%	118	0.007%	164	0.007%	165	0.007%
2	AA	310	0.419%	310	0.419%	280	0.472%	280	0.472%	250	0.534%	250	0.535%
3	A	190	0.628%	190	0.628%	190	0.623%	190	0.626%	160	0.740%	160	0.737%
4	BBB	130	0.667%	130	0.667%	130	0.671%	130	0.666%	370	0.837%	370	0.838%
5	BB	310	0.724%	310	0.724%	310	0.725%	310	0.760%	130	0.875%	130	0.873%
6	B	130	0.929%	130	0.929%	130	0.926%	130	0.836%	160	0.899%	160	0.898%
7	CCC	160	1.269%	160	1.269%	160	1.269%	160	1.270%	160	1.077%	160	1.082%
8	CC	190	1.658%	190	1.659%	190	1.666%	190	1.670%	130	1.602%	130	1.592%
9	C	543	2.047%	542	2.052%	537	2.071%	536	2.074%	520	2.149%	519	2.155%
Objective function values		**0.000067846**		0.000068237		0.000072393		0.000076773		0.000093811		0.000094256	

Note: This table reports all of the local optimal credit rating results for farmers' microfinance.

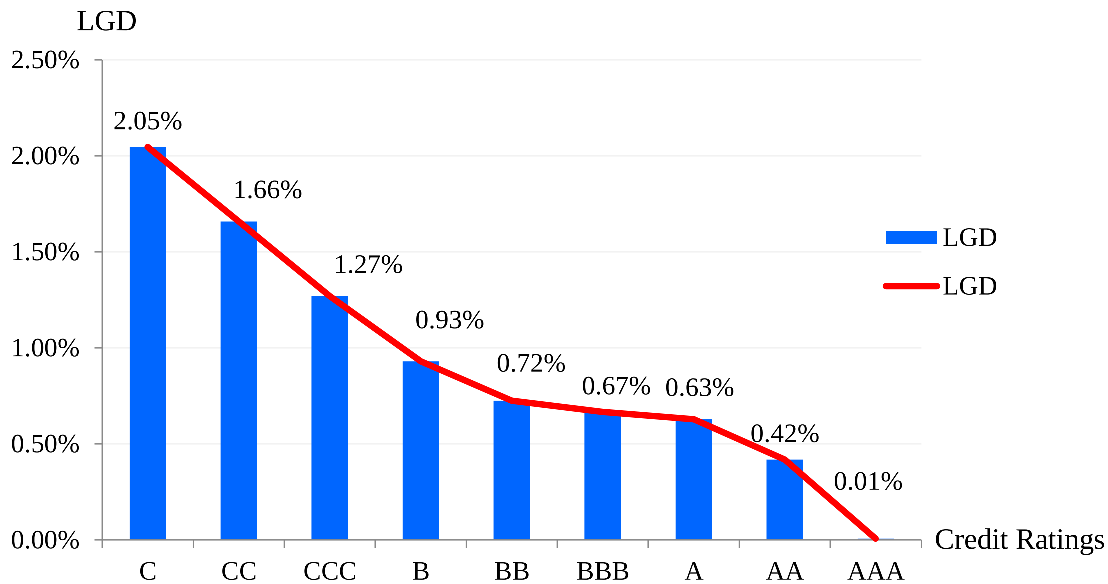

Fig. 3. Distribution of LGDs of credit rating obtained by the proposed model for 2,044 farmers.

5. Robustness Analysis

In order to test the robustness of the proposed credit rating method, this paper uses another two samples to conduct the empirical analysis. First of all, this paper collects the credit data of 2,157 small private businesses from a state-owed commercial bank in China (Shi et al., 2016) and the credit data of 3,111 SMEs from a regional commercial bank in China (BD and DUT, 2015). The credit data for each customer contains their credit score, the owed debt principal and interest, the receivable debt principal and interest. Secondly, we substitute the data of 2,157 small private businesses' credit score, the owed debt principal and interest, the receivable debt principal and interest into Equations (1) to (4). By repeating the solving process as mentioned in subsection 3.3, the credit rating result of 2,157 small private businesses can be obtained (See Table 8). Similarly, we substitute the data of 3,111 SMEs' credit score, the owed debt principal and interest, the receivable debt principal and interest into Equations (1) to (4). The credit rating result of 3,111 SMEs can be obtained (See Table 9). Table 8 reports the obtained global optimal credit rating results of 2,157 small businesses when the first sample size $m_1\text{=}127 < N/9 = 2157/9 \approx 240$. Table 9 reports the obtained global optimal credit rating results of 3,111 SMEs when the first sample size $m_1\text{=}307 < N/9 = 3111/9 \approx 346$.

Known from the last Column of Table 8 and Table 9, the LGDs of nine ratings from C rating to AAA rating are strictly decreasing, which satisfy the credit risk-rating match-up standard. It means that the proposed method indeed guides the way to explore the credit rating result that the credit rating increases with the decreasing LGD. By comparing Table 7, Table 8 and Table 9, another interesting finding is that different types of loan customers with the same level (or the same credit rating) have different credit risks. For example, the LGD of first-level farmers is 0.007%, the LGD of first-level small private businesses is 0.074%, and the LGD of first-level SMEs is 0.340%. In the same level, the default risk of SMEs is the biggest, the default risk of small private businesses is the middle, and the default risk of farmers is the smallest. Derived from the investigation, it can be obtained that the financial institutions should establish different credit risk evaluation systems for different loan customers in credit decisions.

Table 8. The credit rating result of 2,157 small private businesses

(1) Credit ratings		(2) Sample size m_i	(3) Credit score interval S_i	(4) LGD_k
1	AAA	127	$95.240 \le S_i \le 100$	0.074%
2	AA	518	$92.815 \le S_i < 95.240$	0.598%
3	A	756	$87.329 \le S_i < 92.815$	0.627%
4	BBB	312	$84.011 \le S_i < 87.329$	0.771%
5	BB	256	$79.233 \le S_i < 84.011$	1.030%
6	B	81	$76.591 \le S_i < 79.233$	1.520%
7	CCC	56	$72.625 \le S_i < 76.591$	2.360%
8	CC	22	$68.964 \le S_i < 72.625$	5.580%
9	C	29	$0 \le S_i < 68.964$	8.920%

Note: This table reports the obtained global optimal credit rating results of 2,157 small private businesses when the first sample size $m_1 = 127 < N/9 = 2157/9 \approx 240$.

Table 9. The credit rating result of 3,111 SMEs

(1) Credit ratings		(2) Sample size m_i	(3) Credit score interval S_i	(4) LGD_k
1	AAA	307	$88.81 \le S_i \le 100$	0.340%
2	AA	1768	$85.03 \le S_i < 88.81$	6.161%
3	A	278	$83.18 \le S_i < 85.03$	9.792%
4	BBB	616	$60.98 \le S_i < 83.18$	12.793%
5	BB	47	$59.67 \le S_i < 60.98$	15.110%
6	B	14	$59.43 \le S_i < 59.67$	18.261%
7	CCC	24	$57.99 \le S_i < 59.43$	20.824%
8	CC	15	$55.10 \le S_i < 57.99$	23.950%
9	C	42	$0 \le S_i < 55.10$	27.823%

Note: This table reports the obtained global optimal credit rating results of 3,111 SMEs when the first sample size $m_1 = 307 < N/9 = 3111/9 \approx 346$.

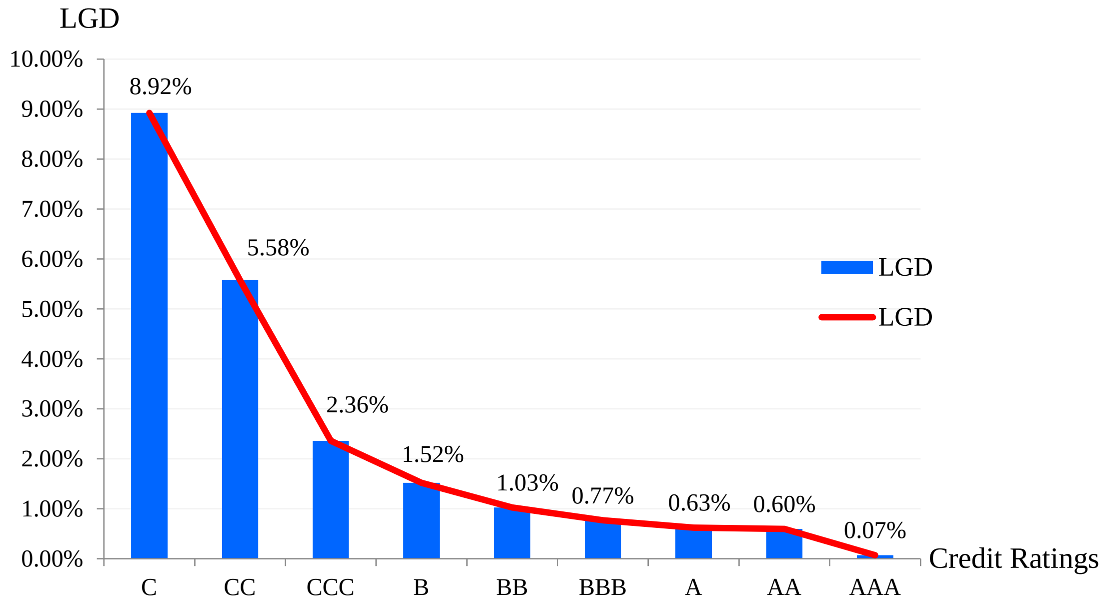

Fig. 4. Distribution of LGDs of credit rating obtained by the proposed model for 2,157 small private businesses.

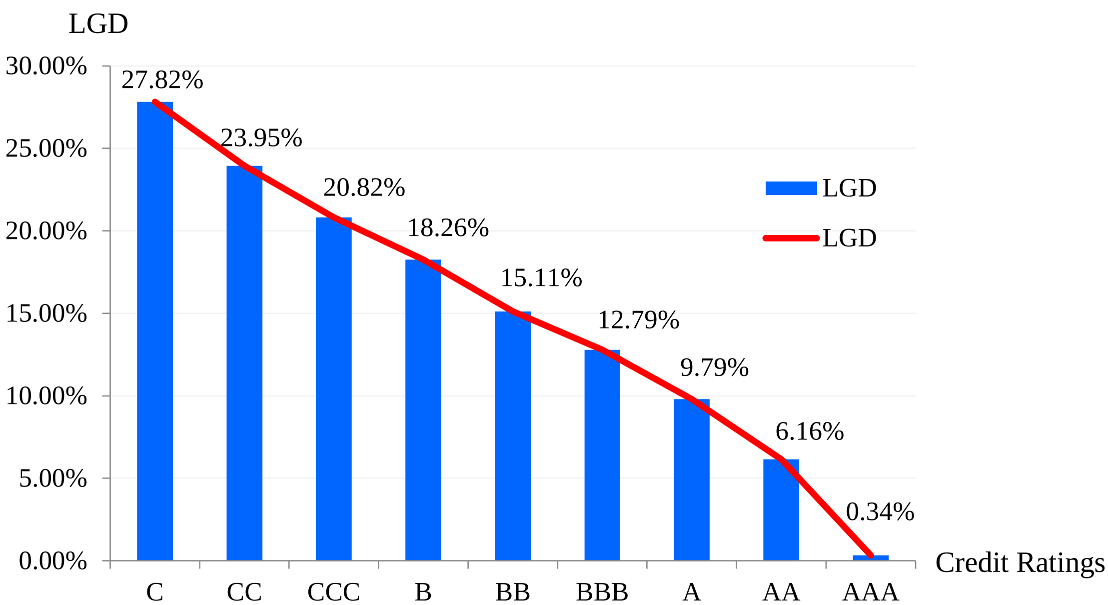

Fig. 5. Distribution of LGDs of credit rating obtained by the proposed model for 3,111 SMEs.

6. Conclusion

Credit rating is important in the financial economics. That credit rating model accurately reflects the corporations or individuals capital and debt would be extremely helpful for investors and debtors as well as banks. Usual credit rating model may have higher credit rating as well as higher loss given default. Resolve this issue is one of the goals in this paper.

First, this paper introduces the credit risk-rating match-up standard to force the LGD strictly decreasing according to the credit rating from C rating to AAA rating. The constraint condition avoids the unreasonable phenomena as higher rating with higher LGD. Second, we provide a detailed nonlinear programming method to explicitly derive the credit rating and the LGD. Based on the normal distribution assumption on the customer numbers of the sample, one can determine the first sample size of the AAA rating and calculate its corresponding LGD_{AAA}. Then we show how to derive the second sample size and adjust the model, etc to find an optimal credit rating result with the credit risk-rating match-up standard. There are possible many local optimal credit rating results with the credit risk-rating match-up standard. Therefore we use the objective function from the distance square of the adjacent LGDs to spot the global optimal credit rating result. Third, the empirical and robustness analysis on the method developed are carried out from three actual bank data sets, i.e. the microfinance data of 2,044 farmers, the microfinance data of 2,157 small private businesses and the credit data of 3,111 SMEs. The analysis result shows that the proposed method can precisely find the credit rating result satisfying the credit risk-rating match-up standard. It indeed guides the way to solve the mismatch problem between credit ratings and LGD. Moreover, different types of loan customers with the same level (or the same credit rating) have different credit risks. In the same level, the default risk of SMEs is the biggest, the default risk of small private businesses is the middle, and the default risk of farmers is the smallest. Finally, the approach is accessible and easy to implement in many similar credit risk evaluation. It provides valuable information and references for the bankers, for the society, and for the bond investors to manage credit risk.

It is clear that the method we develop in this paper has more applicable spaces and easier to access and implement in practice. We encourage researchers, credit rating operator and policy practitioners to use this methodology to further explore the relationship between credit ratings and the corresponding default loss by using different actual bank data. In addition, based on testing various credit rating systems, the proposed method can rectify some rating systems and further adjust the existing rating results to a better result satisfying the credit risk-rating match-up standard.

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