Adaptive Control of Flexible Marine Risers with Varying Length

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Abstract: In this paper, a flexible marine riser with varying length undergoing the influence of the hydrodynamic force and an unknown disturbance to drillship is investigated. The governing equations describing the dynamics of the riser, the drillship, and the top hat are established based on the extended Hamilton principle. To suppress the lateral vibration of the riser, an adaptive boundary control algorithm is developed. Based on the Lyapunov method, the stability of the system under the boundary control and the adaptive law is proven. Finally, the efficiency of the proposed control algorithm is validated via the simulation results.

Keywords: marine riser, vibration suppression, axially moving system, varying length, boundary control, adaptive control, Lyapunov method.

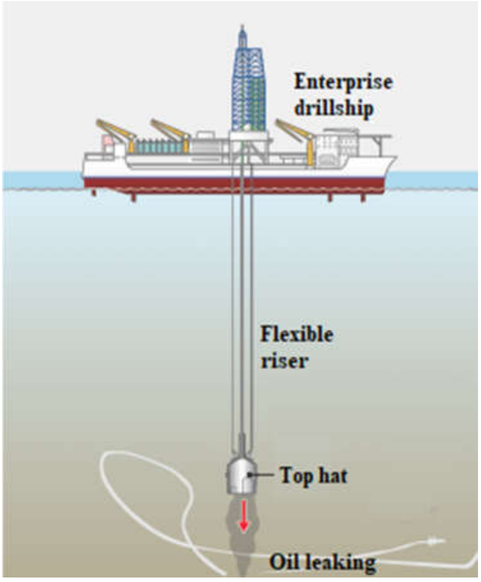
1. INTRODUCTION

An oil leak from a damaged oil line causes a serious problem on the seabed in the ocean environment [1, 2]. To stop such an oil leak, engineers proposed a temporary solution using a top hat containment system, as shown in Fig. 1 [3, 4]. The drillship carrying a top hat moves to the leak position, and then the top hat is lowered and placed over the broken riser on the seabed, thereby preventing the oil leak. During the process of the top hat containment system installation, the motion of drillship, the axial motion of the riser, and disturbances such as hydrodynamic forces can be lead to the residual vibrations along the riser. These vibrations significantly affect the exact positioning of the top hat; the vibration suppression of the flexible marine riser during the motions, therefore, is a paramount concern, which needs to be investigated.

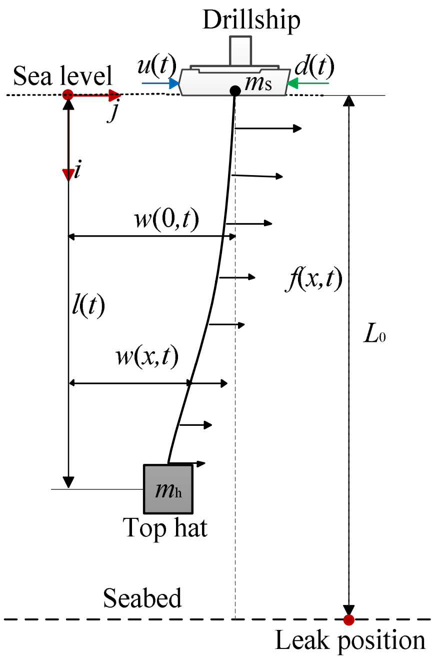
In dynamic analysis of the top hat installation system, the marine riser can be considered as an axially moving string with time-varying length due to the adequately large ratio between the length and the area of the riser. In this situation, the riser system is a gyroscopic distributed parameter system. In the literature [5-17], the dynamic models representing the lateral vibration of moving systems with varying length have been established through the extended Hamilton principle. Kotera [5] and Yamamoto et al. [6] performed pioneering studies on developing the equations of motion of moving strings with varying length. In [7, 8], axially moving strings with a mass-spring attached at the lower end were investigated, wherein the equation of motion of the system consisting of the dynamic of the string and the mass-spring. The governing equations of the hoisting cable and the trolley of a crane system are developed in [9, 10]. Besides studies on the lateral vibration, the longitudinal vibration behavior of axially moving strings has been considered in [18-21]. The influence of the hydrodynamic forces caused by the ocean current on distributed parameter systems has been investigated in [22-24]. Young et al. [22] analyzed the lateral displacement of a flexible marine riser subjected to the ocean current using the hydrodynamic force developed by Morison et al. [25]. Gou et al. [24] was introduced dynamics of a variable-length drill riser system undergoing the effect of the hydrodynamic force.

Vibration control of axially moving systems has attracted considerable attention in the past 25 years. Thus far, numerous studies concerning this aspect have carried out through diverse active control strategies [9-11, 26-45]. Fung et al. [26] is recognized as the first work in vibration control of axially moving systems with varying length. In this work, the variable structure control was implemented to eliminate the residual vibration of a time-variable length cable via a permanent magnet synchronous servo motor. Later, Dai et al. [31] designed a hybrid control method by synthesizing the fuzzy control and the sliding mode control for a flexible cable. According to the Lyapunov method, the boundary control for vibration reduction of axially moving systems was designed in [9-11, 33-45]. Furthermore, in [10, 39-41], the adaptive control that is applied for systems in the presence of uncertainties and unknown parameters [47-51] is also used to develop the controller for translating systems.

Thus far, to the authors' knowledge, vibration control of hybrid systems containing the dynamics of a variable-length riser and a drillship undergoing the hydrodynamic forces have not been addressed in the literature. This paper aims to investigate an active vibration control of a flexible variable-length marine riser subject to a distributed disturbance due to the ocean current and an unknown boundary disturbance. A PDE model representing the dynamics of the riser and drillship and boundary conditions are established using the extended Hamilton principle. Subsequently, an adaptive boundary control is implemented to mitigate the undesirable lateral vibration of the riser and to guarantee the stability of the system, in which the boundary disturbance is estimated. The vibration behavior of the marine riser and the control performance of the designed control law are shown via numerical



(a) [3]



(b) [45]

Fig. 1 A top hat containment system: (a) Schematic of installation, and (b) modeling.

simulations.

This paper is organized as follows. Section 2 introduces the governing equation of the system and the boundary conditions. In Section 3, an adaptive boundary control algorithms to suppress the lateral vibration of the riser is designed. In Section 5, the simulation results that prove the effectiveness of the proposed control law are provided. Finally, conclusions are given in Sections 6.

2. EQUATIONS OF MOTION

Fig. 1 depicts the schematic of the installation of a top hat containment system. The top hat is lowered from the drillship through a flexible riser with time-varying length $l(t)$ and mass density ρ. In this paper, it is assumed that the bending moment of the riser is negligible; and hence the dynamic of the riser is analyzed based on the axially moving string model. The lateral vibration $w(x, t)$ of the riser depends on both the spatial coordinate x and time t. The masses of the drillship and the top hat are m_s and m_h, respectively. A control force $u(t)$ is implemented to the drillship. Also, the drillship is affected by a boundary disturbance $d(t)$, whereas the riser is subjected to distributed disturbance $f(x, t)$ due to the influence of the ocean current.

The kinetic energy of the system consisting of the kinetic energy corresponding to the drillship, the riser, and the top hat is derived as follows:

$$T = \frac{1}{2}\rho\int_0^{l(t)}\left[\dot{l}(t)^2 + \left(w_t(x,t)+\dot{l}(t)w_x(x,t)\right)^2\right]dx$$
$$+\frac{1}{2}m_h\left\{\dot{l}(t)^2+\left[w_t(l,t)+\dot{l}(t)w_x(l,t)\right]^2\right\}$$
$$+\frac{1}{2}m_s w_t(0,t)^2 \tag{1}$$

where $\dot{l}(t)=dl(t)/dt$. Note that $(\bullet)_t$ denotes the partial derivative with respect to time t and $(\bullet)_x$ indicates the partial derivative with respect to the spatial variable x of the lateral vibration $w(x,t)$.

In Eq. (1), the domain of integration for spatial variable x in the second term (i.e., the kinetic energy of the riser) depends on time due to the time-varying length of the riser. The tension along the riser $P(x, t)$, which is spatiotemporal varying, is given as follows [11].

$$P(x,t)=\left[m_h+\rho\left(l(t)-x\right)\right]\left(g-\ddot{l}(t)\right) \tag{2}$$

where $\ddot{l}(t)=d^2l(t)/dt^2$ and g is the gravitational acceleration. The potential energy of the system due to the lateral vibration of the riser is derived as follows.

$$U=\frac{1}{2}\int_0^{l(t)}P(x,t)w_x(x,t)^2\,dx\,. \tag{3}$$

The total virtual work done on the system is given by

$$\delta W=-\int_0^{l(t)}c_r\left(w_t(x,t)+\dot{l}(t)w_x(x,t)\right)\delta w dx$$
$$-c_s w_t(0,t)\delta w(0,t)+\int_0^{l(t)}f(x,t)\delta w dx$$
$$+d(t)\delta w(0,t)+u(t)\delta w(0,t) \tag{4}$$

where c_r and c_s represent the damping coefficients of the riser and the drillship, respectively. In this paper, for simplicity, it is assumed that the oscillating effect by vortex shedding is neglected; therefore, the hydrodynamic force $f(x, t)$ is simplified and described through a mean drag force, namely,

$$f(x,t) = \frac{1}{2}\rho_w d_r C_D v_y(x,t)|v_y(x,t)| \tag{5}$$

where ρ_w indicates the density of the seawater, d_r denotes the outer diameter of the riser, C_D is the drag coefficient, and $v_y(x, t)$ represents the velocity of the ocean current. According to [23], the depth-dependent velocity of the ocean current $v_y(x, t)$ can be expressed as follow.

$$v_y(x,t) = \left(1 - \frac{x}{L_0}\right)\left[2 + 0.2\left(\sin(0.87t) + \sin(1.83t)\right)\right] \tag{6}$$

where L_0 is the depth of the seabed.

Substituting Eq. (1), (3), and (4) into the extended Hamilton principle and using the notations w, P, l, d, u instead of $w(x, t)$, $P(x, t)$, $l(t)$, $d(t)$ and $u(t)$, respectively; the dynamic model of the system is obtained as follows.

$$\rho\left(w_{tt} + 2\dot{l}w_{xt} + \ddot{l}w_x + \dot{l}^2 w_{xx}\right) - (Pw_x)_x + c_r\left(w_t + \dot{l}w_x\right) - f(x,t) = 0 \tag{7}$$

with the boundary conditions:

$$x = 0, \quad m_s w_{tt} + \left(\rho\dot{l}w_t + \dot{l}w_x\right) - Pw_x + c_s w_t + d = u\,, \tag{8}$$

$$x = l, \quad m_h\left(w_{tt} + 2\dot{l}w_{xt} + \ddot{l}w_x + \dot{l}^2 w_{xx}\right) + Pw_x = 0\,. \tag{9}$$

Eqs. (7) to (9) are the equations of motion describing the dynamics of the riser, the drillship, and the top hat, respectively.

3. CONTROL DESIGN

The control objectives are to move the drillship carrying the top hat to the leak position w_d and then suppress the top hat lateral vibration. It is assumed that the magnitude of the boundary disturbance $d(t)$ is an unknown positive constant d_m, and the disturbance direction is opposite to the direction of the drillship velocity. Namely, the disturbance is given as $d(t) = -\mathrm{sgn}(w_t(0,t))d_m$. Besides that, for suppressing the lateral vibration, the longitudinal motion of the riser can be ignored from the stability analysis point of view of the closed-loop system. In this paper, an adaptive boundary control law is designed via the Lyapunov method for positioning the top hat and handling the unknown disturbance. A Lyapunov function candidate associated with the mechanical energy of the riser, the drillship, and the top hat is considered as follows.

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{10}$$

where

$$V_1(t) = \frac{k_1}{2}\int_0^l \left[\rho\left(w_t + \dot{l}w_x\right)^2 + Pw_x^2\right]dx, \tag{11}$$

$$V_2(t) = \frac{1}{2}m_s w_t(0,t)^2 + \frac{k_2}{2}\left(w(0,t) - w_d\right)^2 + \frac{\tilde{d}_m^2}{2k_3}, \tag{12}$$

$$V_3(t) = \frac{k_1}{2}m_h\left(w_t + \dot{l}w_x\right)^2\Big|_{x=l}, \tag{13}$$

where $\tilde{d}_m = \hat{d}_m - d_m$, $\hat{d}_m$ is the estimated parameter of d_m, and k_1, k_2, and k_3 are positive constants. In this paper, for simplicity, the riser length is assumed as a second-order polynomial, therefore the jerk $\dddot{l}(t) = 0$. Therefore, the time derivative of the Lyapunov function is derived as follows.

$$\frac{\mathrm{D}V(t)}{\mathrm{D}t} = \frac{\mathrm{D}V_1(t)}{\mathrm{D}t} + \frac{\mathrm{D}V_2(t)}{\mathrm{D}t} + \frac{\mathrm{D}V_3(t)}{\mathrm{D}t}, \tag{14}$$

where

$$\frac{\mathrm{D}V_1(t)}{\mathrm{D}t} = -k_1 c_r\int_0^l \left(w_t + \dot{l}w_x\right)^2 dx + k_1 Pw_x\left(w_t + \dot{l}w_x\right)\Big|_{x=0}^{x=l} + k_1\int_0^l \left[f(x,t)\left(w_t + \dot{l}w_x\right)\right]dx, \tag{15}$$

$$\frac{\mathrm{D}V_2(t)}{\mathrm{D}t} = w_t\left[u + d + Pw_x - c_s w_t - \rho\dot{l}\left(w_t + \dot{l}w_x\right)\right]\Big|_{x=0} + k_2 w_t(0,t)\left(w(0,t) - w_d\right) + \frac{1}{k_3}\tilde{d}_m\dot{\hat{d}}_m, \tag{16}$$

$$\frac{\mathrm{D}V_3(t)}{\mathrm{D}t} = -k_1 Pw_x\left(w_t + \dot{l}w_x\right)\Big|_{x=l}. \tag{17}$$

Lemma 1. [46] Let $\phi_1(x,t), \phi_2(x,t) \in \mathbb{R}$ and δ is a positive constant. The following inequality holds.

$$|\phi_1\phi_2| \le \frac{1}{\delta}\phi_1^2 + \delta\phi_2^2. \tag{18}$$

The following inequality is obtained by using Lemma 1 for the third term in (15).

$$\frac{\mathrm{D}V_1(t)}{\mathrm{D}t} \le -k_1(c_r - \delta)\int_0^l \left(w_t + \dot{l}w_x\right)^2 dx + \frac{k_1}{\delta}\int_0^l f(x,t)^2\, dx + k_1 Pw_x\left(w_t + \dot{l}w_x\right)\Big|_{x=0}^{x=l}. \tag{19}$$

Substituting Eqs. (16) and (17) and inequality (19) into Eq. (14) yields

$$\frac{\mathrm{D}V(t)}{\mathrm{D}t} \le w_t\left[u + d + Pw_x - c_s w_t - \rho\dot{l}\left(w_t + \dot{l}w_x\right)\right]\Big|_{x=0} + Pw_x\left(w_t + \dot{l}w_x\right)\Big|_{x=0}^{x=l} - k_1 Pw_x\left(w_t + \dot{l}w_x\right)\Big|_{x=l}$$

$$+\frac{k_1}{\delta}\int_0^l f^2 dx - k_1\left(c_\mathrm{r}-\delta\right)\int_0^l\left(w_t+\dot{l}w_x\right)^2 dx$$

$$+k_1 w_t\left(0,t\right)\left(w\left(0,t\right)-w_\mathrm{d}\right)+\frac{1}{k_3}\tilde{d}_\mathrm{m}\dot{\hat{d}}_\mathrm{m}. \quad (20)$$

The inequality (20) can be rewritten as follows.

$$\frac{DV(t)}{Dt}\le w_t\left(0,t\right)\Big[u+d-\rho l\left(w_t\left(0,t\right)+\dot{l}w_x\left(0,t\right)\right)$$
$$-\left(k_1-1\right)P\left(0,t\right)w_x\left(0,t\right)+k_2\left(w_t\left(0,t\right)-w_d\right)\Big]$$
$$-k_1\left(c_\mathrm{r}-\delta\right)\int_0^l\left(w_t+\dot{l}w_x\right)^2 dx+\frac{1}{k_3}\tilde{d}_\mathrm{m}\dot{\hat{d}}_\mathrm{m}$$
$$-k_1\dot{l}P\left(0,t\right)w_x^{\,2}\left(0,t\right)+\frac{k_1}{\delta}\int_0^l f\left(x,t\right)^2 dx$$
$$-c_\mathrm{s}w_t\left(0,t\right). \quad (21)$$

To stabilize system (7) with the boundary conditions (8) and (9), a boundary control law is proposed as follows.

$$u\left(t\right)=c_\mathrm{s}w_t\left(0,t\right)+\rho l\left(w_t\left(0,t\right)+\dot{l}w_x\left(0,t\right)\right)-k_4\dot{w}_t\left(0,t\right)$$
$$-k_2\left(w_t\left(0,t\right)-w_\mathrm{d}\right)+\left(k_1-1\right)P(0,t)w_x\left(0,t\right)$$
$$+k_1\dot{l}P\left(0,t\right)\frac{w_x^2\left(0,t\right)}{w_t\left(0,t\right)+\alpha}-\mathrm{sgn}\left(w_t\left(0,t\right)\right)\hat{d}_\mathrm{m}$$
$$-\frac{k_1}{\delta\left(w_t\left(0,t\right)+\beta\right)}\int_0^l f\left(x,t\right)^2 dx, \quad (22)$$

where k_4 is a positive constant,

$$\alpha=0.5\,\mathrm{sgn}\left(\dot{l}w_t\left(0,t\right)\right)\left|w_t\left(0,t\right)\right|,$$
$$\beta=0.5\,\mathrm{sgn}\left(w_t\left(0,t\right)\right)\left|w_t\left(0,t\right)\right|.$$

The adaptive law is given as follows.

$$\dot{\hat{d}}_\mathrm{m}=-k_3\,\mathrm{sgn}\left(w_t\left(0,t\right)\right)w_t\left(0,t\right). \quad (23)$$

Substituting the control law (22) and the adaptive law (23) into inequality (21) yields

$$\frac{DV(t)}{Dt}\le -k_4 w_t^{\,2}\left(0,t\right)-k_1\left(c_\mathrm{r}-\delta\right)\int_0^l\left(w_t+\dot{l}w_x\right)^2 dx$$
$$w_t\left(0,t\right)d\left(t\right)+w_t\left(0,t\right)\mathrm{sgn}\left(w_t\left(0,t\right)\right)\hat{d}_m$$
$$+\frac{1}{k_3}\tilde{d}_\mathrm{m}\dot{\hat{d}}_\mathrm{m}-\frac{k_1}{2\delta}\int_0^l f\left(x,t\right)^2 dx$$
$$+k_1P\left(0,t\right)w_x^2\left(0,t\right)\left(\frac{\dot{l}}{w_t\left(0,t\right)+\alpha}-1\right)$$
$$\le -k_4 w_t^{\,2}\left(0,t\right)-k_1\left(c_\mathrm{r}-\delta\right)\int_0^l\left(w_t+\dot{l}w_x\right)^2 dx$$
$$-\frac{1}{\left|w_t\left(0,t\right)\right|}\left(\frac{0.5k_1P(0,t)w_x^2\left|\dot{l}w_t\left(0,t\right)\right|}{2+\mathrm{sgn}\left(\dot{l}w_t\left(0,t\right)\right)\mathrm{sgn}\left(w_t\left(0,t\right)\right)}\right)$$
$$-\frac{k_1}{2\delta}\int_0^l f\left(x,t\right)^2 dx. \quad (24)$$

Eq. (24) shows that $\mathrm{D}V/\mathrm{D}t\le 0$. It is noted that δ is chosen to satisfy the condition $\delta<c_\mathrm{r}$. According to the Lyapunov method, the closed-loop system (i.e., Eqs. (7) to (9)) under control law (22) and adaptive law (23) is stable.

Remark 1. The motion of the drillship can be utilized as the control input $u(t)$. In boundary control law (22), all the signals can be obtained through the sensors and algorithms. The boundary displacement $w(0,t)$ and velocity $w_t(0,t)$ can be determined based on the motion of the drillship; whereas, the slop angle $w_x(0,t)$ can be sensed by an inclinometer. The axial tension of riser and the hydrodynamic force can be calculated by using Eqs (2) and (5), respectively.

3. NUMERICAL SIMULATIONS

In order to validate the control performance of the proposed control law, the numerical simulations are presented in this section. The marine riser system with the following parameters is considered: d_r = 0.15 m, ρ = 90 kg/m, $m_\mathrm{s}=10^7$ kg, $m_\mathrm{h}=3\times10^3$ kg, c_r = 2 Ns/m^3, and c_d = 2 Ns/m^3. The hydrodynamic force is calculated by using Eqs. (5) and (6), where the drag coefficient C_D = 1.36 and seawater density ρ_w = 1024 kg/m^3. The drillship carrying the top hat transports from 0 m to the leaking position at 400 m; whereas the top hat is lowered from 200 m to 1,200 m; equivalently, the riser length $l(t)$ changes from 200 m to 1200 m.

Fig. 2 shows the lateral vibration of the top hat under adaptive boundary control law (22) in the case without the hydrodynamic force. In this situation, the vibration of the top hat converges to zero. The response of the system with the influence of the hydrodynamic force is shown in Fig. 3 and Fig. 4. The simulation results show that the lateral vibration of the top hat is suppressed significantly (Fig. 3) when the drillship reaches the desired position (Fig. 4). The lateral vibrations, however, do not converge to zero.

The comparison of the lateral vibration of the top hat of the proposed control law with a PD control law is presented in Fig. 5, wherein the PD control law is given by $u(t)=k_\mathrm{p}(w_\mathrm{d}-w_t(0,t))-k_\mathrm{d}w_t(0,t)$. As shown in this figure, the control performance of adaptive boundary control (22) is better than the control performance of the PD control law.

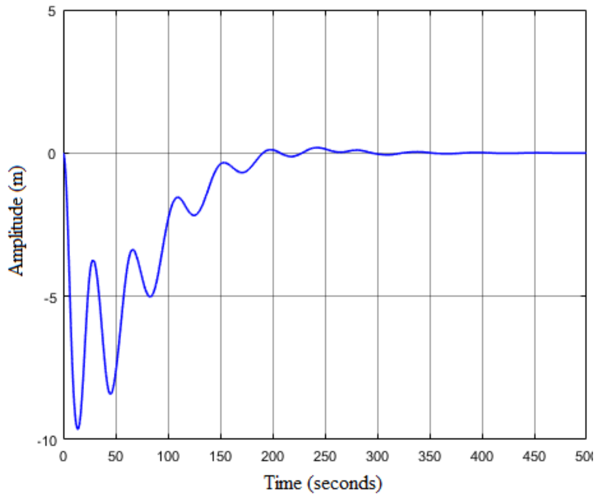


Fig. 2 Lateral vibration of the top hat in the case without the hydrodynamic force.

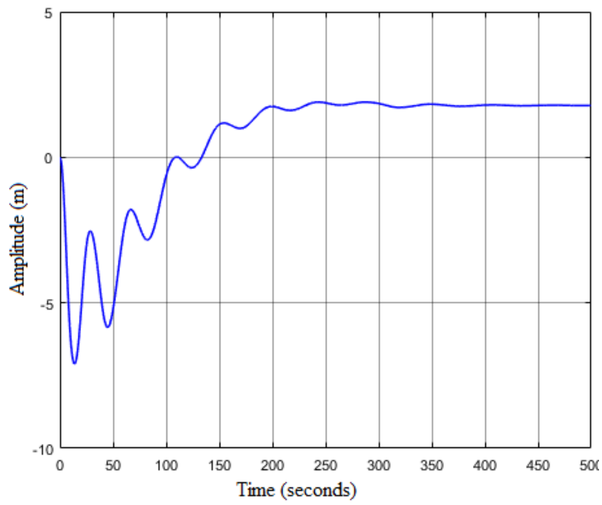


Fig. 3 Lateral vibration of the top hat in the case considering the hydrodynamic force.

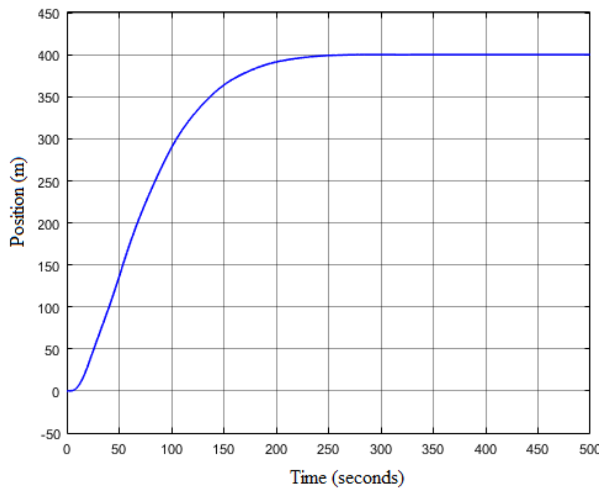


Fig. 4 Position of the top hat.

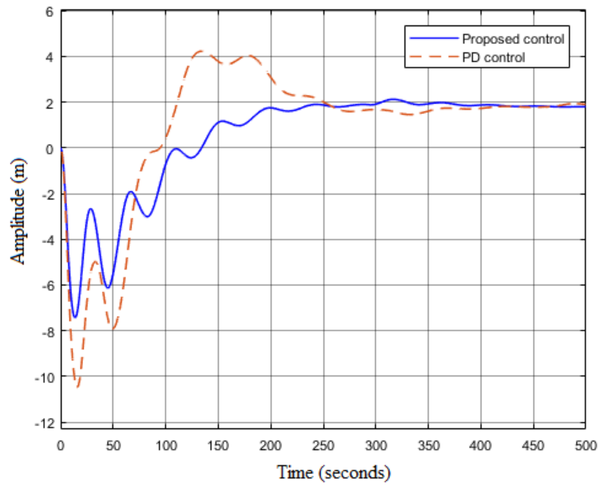


Fig. 5 Comparison of the proposed control law and the PD control law.

4. CONCLUSIONS

This paper investigated a flexible marine riser with varying length subjected to a distributed disturbance due to the ocean current and a boundary disturbance. The equations of motion of the system consisting of the riser, the drillship, and the top hat were developed by using the extended Hamilton principle. An adaptive boundary control law was designed to position the drillship and the top hat to their desired postions as well as to suppress the lateral vibration of the riser. The stability of the system under the proposed control law was proven by using the Lyapunov method. The control performance of the control law was also validated via numerical simulations.

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