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№ 522

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September 25, 2018

XXI ENMC
ENCONTRO NACIONAL DE MODELAGEM COMPUTACIONAL
IX ECTM
ENCONTRO DE CIÊNCIA E TECNOLOGIA DE MATERIAIS

08 a 11 de Outubro de 2018
Instituto Federal Fluminense
Búzios - RJ

PREDATOR-PRAY SWARM INTERACTION IN 2D:NUMERICAL SIMULATION

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Abstract. In the present work is described an ordinary differential system of equations for simulating the swarming behavior of preys in the presence of a predator. Preys and predator are represented by a set of ODEs taking in account the Newtonian attraction-repulsion forces. The predator interact with the preys through a Newtonian force, which is a nonconservative force (includes friction) that acts in the same direction for both agents. A perturbing force is introduced for the predator dynamics in order to simulate its behavior among preys. The resulting system of ordinary differential equations is solved numerically by Runge-Kutta of fourth order and the dynamics are discussed in the present work as the swarm's ability to realistically avoid the predator. The main goal is to reproduce swarm behavior that has been observed in nature with the minimal and simple possible model of ODE system.

Keywords: Prey-Predator swarm interaction, Runge-Kutta 4 order, Numerical simulation, Dynamical system.

1. INTRODUCTION

In nature it is observed for a long time that animal aggregation is part of the set of individuals behavior (Perrish & Edelstein-Keshet, 1999),(Moussad et al, 2009),(Vicsk et al., 1995),(Roner & Tu, 1998). In general is observed fish schooling (shoaling), bird flocking, mammal herding, insect/bacterial swarming, and human crowding dynamics. It is also observed that even predators have been known to hunt in group in the form of packs. It is well known that all these aggregations shares similarities, such as the fact that the group of organisms act in unison and reacting rapidly to obstacles or threats. The generality of such features leads to realize models for its simulation. In the present work, swarming will refer to any such behavior in which individuals come together and act in a reasonably coordinated manner to produce an aggregate set in dynamical motion. Swarming has been studied in an extensive manner by computer simulation (Reynolds, 1987),(Olfati-Saber, 2006). In several sources, the models are taken as individual-based, where swarm individuals are represented as a set of individuals that interact with other as a function of their positions (Topaz & Bertozzi, 2005),(Lett & Mirabet, 2008). The use of Newtonian force law and variations of it has been a general approach for this dynamical system.

In these models, the designed forces consist of a long-range attractive force that makes the individuals to approach and form the swarm typical geometry, coupled with a short-range repulsive force so that they try do not collide with each other (Liu et al, 2008),(Duan et al, 2005),(Gazi & Passino, 2004). A self-propulsive force that pushes each individual forward toward some preferred velocity is also often added (D'Orsogna et al, 2006),(Nishmura & Ikegani, 1997),(Levine et al, 2000). A model designed to align the individuals with each other is present in flocking simulation (Lee, 2006),(Inada & Kawachi, 2001). All these models successfully reproduced main behavior aspects of swarming. The well known predator behavior called confusion (Krause & Ruxton, 2002), which occurs when the predator is confused related to which individual to pursue is simulated in the present work. Predator confusion acts mainly decreasing its ability to hunt their prey.

2. MODEL

A simple but yet robust model is developed in order to approach swarm dynamics. It is designed to represent each prey by a particle that moves based on its interactions with other prey and its interaction with the predator. There is a large material available out there about particle models in biology science, mainly they have been designed to model biological individuals aggregation in general (Mogilner & Edelstein-Keshet, 1999) also locusts (Bernoff & Topaz, 2011) or fish schooling populations (Zheng et al, 2005). The model is established as following (Chen et al, 2014). It is assumed that there are N preys whose positions are given by $P(x_j, y_j) \in \mathrm{R}^2$, $j = 1,2,...,N$, *N* is the size of the individual population whereas (x, y) are function of time (t). Taking Newton's law so that $m\, {d^2 P_j}/{dt} + \mu\, {dP_j}/{dt} = F_{j,prey-prey} + F_{j,prey-predator}$ where $F_{j,prey-prey} + F_{j,prey-predator}$ is the total force acting on the *j-th* particle, μ is the strength of friction force and *m* is its mass. Simplification as the mass *m* is negligible compared with the friction force μ is applied. After rescaling to set $\mu = 1$ the model is then simplified as: ${dP_j}/{dt} = F_{j,prey-prey} + F_{j,prey-predator}$. This reduces the second-order ordinary differential system model to a first-order model system, so that the prey moves in the direction of the total force. The prey–prey interaction of the form $F_{j,prey-prey} = {1}/{N} \sum_{k=1,k\neq j}^{N} \left({1}/{\left|P_j - P_k\right|^2} - a \right)(P_j - P_k)$: The term $(P_j - P_k)/|P_j - P_k|^2$ represents Newtonian-type of short-range repulsion that acts in the direction from P_j to P_k, whereas $a(P_j - P_k)$ is a linear long-range attraction in the same direction. The model for prey–predator interactions can be established by a similar manner. In order to deal with more realistic model assume that there is a single predator; however it is possible to have more. The predator position is given as $Pz(x, y, t)$. It is considered that the predator acts on the individual's preys as a repulsive particle, it is taken as $F_{j,prey-predator} = b(P_j - Pz)/|P_j - Pz|^2$ with b being the strength of the repulsion. Following, the model for the predator–prey interactions as an attractive force given in a similar way such as, $dPz/dt = F_{j,prey-predator}$. In this case is considered a very simple scenario which $F_{j,prey-predator}$ is the average over all predator–prey interactions and each individual interaction is a power law, which decays at large distances, as consequence the prey moves in the direction of the average force. Once these assumptions are put together the following system can be written:

$$\begin{cases} \dfrac{dP_j}{dt} = \dfrac{1}{N} \displaystyle\sum_{k=1,k\neq j}^{N} \left(\dfrac{(P_j - P_k)}{|P_j - P_k|^2} - a(P_j - P_k) \right) - b\dfrac{(P_j - Pz)}{|P_j - Pz|^2} \\ \dfrac{dPz}{dt} = \dfrac{c}{N} \displaystyle\sum_{k=1}^{N} \dfrac{(P_k - Pz)}{|P_k - Pz|^p} \end{cases} \tag{1}$$

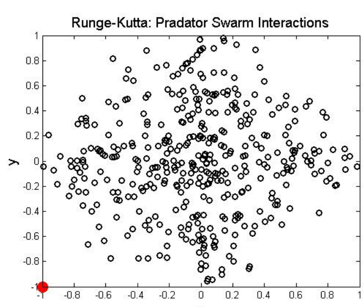
As stated before, *a* is the linear long-range attraction parameter, *b* is the predator repulsive parameter and *c* is the predator-prey attraction control parameter. The system of ordinary differential equations given by eq(1) is solved numerically by means of Runge-Kutta of fourth order, and it is need to know the *tini* for initial time, the end simulation time *tend* and the number of steps *m*. This model is also modified making use of a perturbation function added to the predator in order to simulate its decision as the dynamical system evolve in time. It is chosen two perturbation functions which are given by:

$$\begin{cases} a) F_{j,pert} = \varepsilon e^{\lambda t} \\ b) F_{j,pert} = \varepsilon(\cos(P_j), \sin(P_j)) \end{cases} \tag{2}$$

In eq(2a) and eq(2b) $\varepsilon = (0.3;0.2), \lambda = 0.26$.

3. NUMRICAL SIMULATION

The numerical simulation are performed taking the following parameters as constant: *p=2.4, a=1, b=0.5, tini=0.0, tend=12.0, N=400(particles), m=480(time steps)*. The first simulation is without perturbation function.



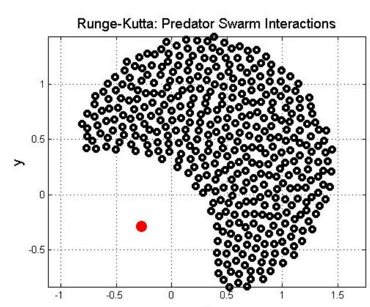


Figure 1: c=2.8, t=0.0s

Figure 2: c=2.8, t=2.0s

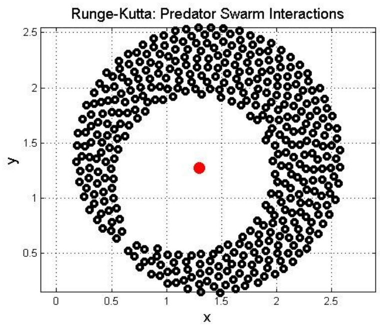


Figure 3: c=0.8, time=8.0s

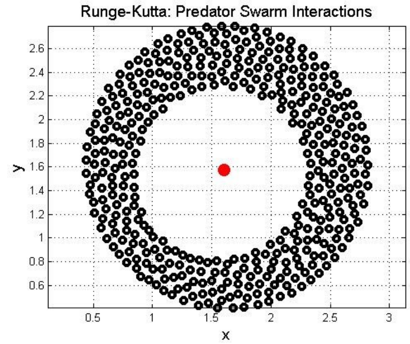


Figure 4: c=0.8, time=12.0s

In the figure 1 is depicted the initial distribution of the 400 individuals. Figure 2 shows an enlapsed time of 2.0s showing the natual repulsive presence of (the red dot) as the predator and the protective action of the preys as the predator moves towards the population. In figure 3 the predator is surrounded by the prey and its movement slow down leading to a confused situation as depicted in figure 4 after 12.0s. In this stage the system becomes stable and the predator is kept in confused position (Chen et al, 2014).

The following numerical simulation is performed using the perturbation function given by eq(2a). Its is considered the model parameters as constant: *p=2.4, a=1, b=0.5, tini=0.0, tend=6.0, N=400(particles), m=480(time steps)*.

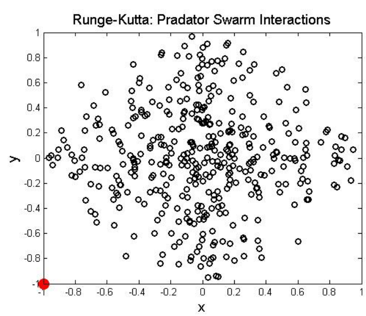


Figure 5: c=2.8, time=0.0s

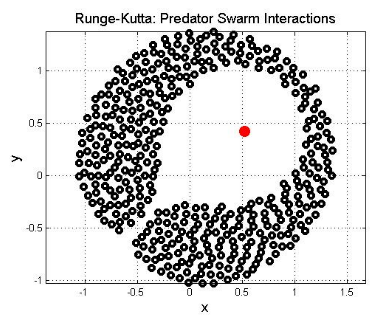


Figure 6: c=2.8, time=2.0s

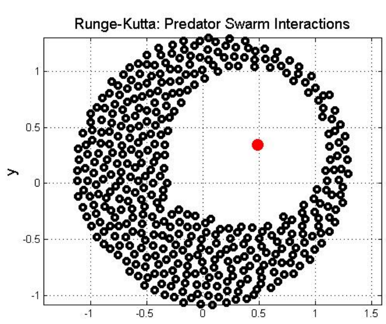


Figure 7: c=2.8, time=3.0s

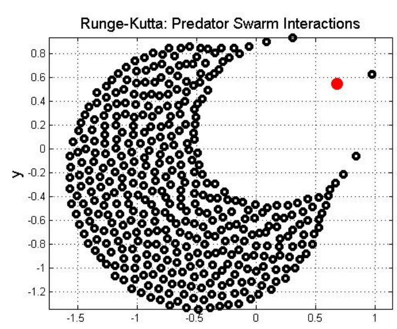


Figure 8: c=2.8, time=6.0s

In the figure 5 is depicted the initial distribution of the 400 individuals. Figure 6 shows an enlapsed time of 2.0s showing the invasive movement of the predator towards the preys its natual repulsive presence of (the red dot) as the predator and the protective action of the preys forming a circle leading the predator to a completed surrounded situation. In figure 7 the predator is already surrounded by the preys and its attack movement is kept leading to a final chasing as depicted in figure 8 after 6.0s ending up to cath up. In this stage the system as propagated in time tends to regroup the prey population.

The numerical simulation using the perturbation function given by eq(2b) is performed taking the parameters as used for eq(2a).

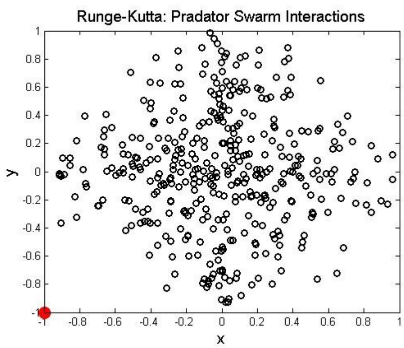


Figure 9: c=2.8, time=0.0s

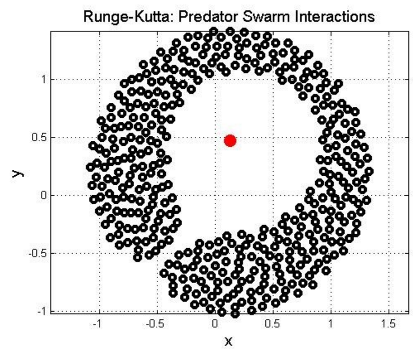


Figure 10: c=2.8, time=2.0s

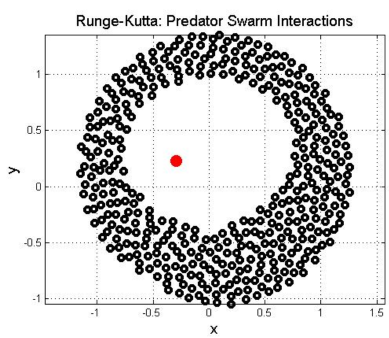


Figure 11: c=2.8, time=3.0s

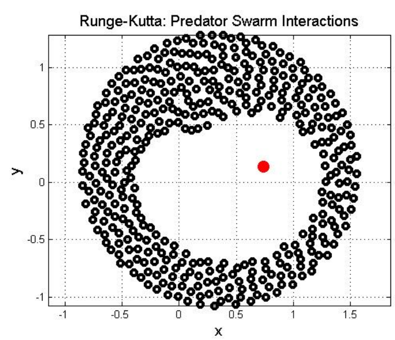


Figure 12: c=2.8, time=6.0s

In the figure 9 is depicted the initial distribution of the 400 individuals. Figure 10 shows an enlapsed time of 2.0s showing the invasive movement of the predator towards the preys population. In this case the perturbation function simulates a more aggressive behaviour of the predator leading it to an oscilationg movement approaching the preys faster than for the perturbation function given by eq(2a). Daynamiclly in real time processing is visible the preys evasive actions as time evolves. In figure 11 the predator is already surrounded by the preys and its attack movement is continuos as depicted in figure 12 after 6.0s where the preys system becomes more unstable due a more aggressive predator behaviour. Qualitatively the results shown here is in aggrement with those of presented by (Chen et al, 2014).

4. CONCLUSION

The numerical simulation is well done even for this large system of ordinary differential equations, the RK-4 is robust enough to deal with it, since the time step used is small enough to follow the predator-prey engagement dynamics. One must care for this fact because the predator dynamics is more sensitive to the time step used than preys dynamics. The present model is completely able to predict the swarm dynamics and through these simulations became clear that the *b* and *c* parameter has a deep influence in this dynamics system. Lower *c* values (< 2.8) also acts in lower down the predator movement towards the prey population. The adoption of the perturbation function for the predator also revealed that the function given by eq(2b) makes the predator to move towards the preys in a irregular path causing then to regroup in protective shape. This causes the prey population take a fast evasive movement always keeping the safe population shape of protection which tray to involve the predator in a confused position. But this perturbation function leads the preys to change rapidly in evasive movements. However, the perturbation function given by eq(2a) implies a smooth path movement for the predator leading to a stable prey response. So, the eq(2b) seems to be more realistic.

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