Robust Labeled Multi-Bernoulli Filter with Inaccurate Noise Covariances

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***Abstract*—In this paper, a robust labeled multi-Bernoulli (RLMB) filter for the multi-target tracking (MTT) scenarios with inaccurate and time-varying process and measurement noise covariances is proposed. The process noise covariance and measurement noise covariance are modeled as inverse Wishart (IW) distributions, respectively. The state together with the predicted error and measurement noise covariances are inferred based on the variational Bayesian (VB) inference. Moreover, a closed-form implementation of the proposed RLMB filter is given for linear Gaussian system and the predictive likelihood function is calculated by minimizing the Kullback-Leibler (KL) divergence by the VB lower bound. Simulation results illustrate that the proposed RLMB filter outperforms the existing LMB filter in the tracking performance.**

***Index Terms*—variational Bayesian, labeled multi-Bernoulli filter, inverse Wishart distribution, inaccurate noise covariances, multi-target tracking**

I. Introduction

The finite set statistics (FISST) contributes a rigorous statistical theoretical framework for studying multi-target tracking problems. In this framework, the multi-target tracking problem is described as estimating the set values of the target state set and the measurement set, which are modeled as random finite sets (RFSs), respectively. FISST directly extends the Bayesian recursion formula for single-target tracking to multi-target tracking problems, unifying the mathematical forms of single-target tracking and multi-target tracking. Because of the complexity of the joint probability distribution and multiple integrals of multiple target states, some principled approximations have been proposed. Mahler proposed the probability hypothesis density (PHD) filter in [1], which is a recursive filter that propagating the first-order moments of multi-target. Subsequently, Mahler proposed the cardinality PHD (CPHD) filter [2], which is a generalization of the PHD filter. In the recursive process of the CPHD filter, the posterior PHD of states and posterior distributions of the number of targets are calculated and propagated simultaneously, so it has a better performance than the PHD filter. The multi-target multi-Bernoulli (MeMBer) filter utilizes independent Bernoulli process to represent and handle the multi-target tracking problem [3]. Vo *et al.* proved that the MeMBer filter has some problems in estimating the cardinality of targets, and an advanced approach named cardinality-balanced MeMBer (CBMeMBer) filter is proposed in [4].

Since the elements in the multi-target state set and the measurement set are out of order, the above-mentioned filters cannot directly obtain the multi-target tracks. To solve this problem, Vo *et al.* generalized the concept of RFS to labeled RFS by augmenting the target state with a label variable that can distinguish target identity, whats more, the rigorous δ-generalized labeled multi-Bernoulli (δ-GLMB) filter [5], [6] was derived under the framework of multi-target Bayesian filtering. Reuter *et al.* utilized a first-order approximate matching model to approximate the GLMB posterior distribution to the labeled multi-Bernoulli (LMB) distribution, and then proposed the LMB filter [7], which is an efficient implementation method of the δ-GLMB filter.

In the traditional MTT filters implementation method, the process noise covariance and measurement noise covariance are usually described as two given variables through heuristics, which are generally used to describe model uncertainty. However, in practical MTT applications, the accurate prior knowledge about process noise covariance and measurement noise covariance is usually unknown and time-varying. Heuristic process noise covariance and measurement noise covariance prior statistics are likely to cause large tracking errors or even filter divergence [8].

The VB approximation is not only used to compute the approximate posterior for which analytical solution does not exist, but also is employed in the situations, where the statistics of measurement noise is unknown [9], [10], [11], [12]. Therefore, VB approximation is introduced into RFS framework to deal with the MTT problem with unknown noise statistical characteristics. Adaptive PHD filters based on VB were proposed in [13], [14], [15], [16]. In [17] and [18], the VB approach was extended to the CBMeMBer filter. To improve performance in low signal-to-noise ratio situations, [19], [20] and [21] introduced the VB approach into the δ-GLMB filter and LMB filter framework, respectively.

However, the aforementioned VB multi-target filter is only suitable for MTT scenarios with unknown measurement noise covariance by choosing appropriate conjugate prior distributions. Multi-target filters for unknown process noise covariance still present a great challenge, because process noise covariance does not appear in a direct conjugate prior form like measurement noise. Ardeshiri *et al.* [22] presented a batch-processing VB algorithm for joint estimation of the dynamic system state, the measurement noise covariance and the process noise covariance, with the noise covariance matrices being identified off-line. Instead of estimating the process noise covariance matrix directly, Huang *et al.* [23] proposed a novel VB adaptive Kalman filter in which the inverse Wishart distribution was used as a prior for the predicted error covariance and measurement noise covariance. The approach of [23] is an on-line recursive method but it needs a nominal process noise covariance matrix at each time step as the parameter of the algorithm. Moreover, Ma *et al.* [24] developed a novel VB based adaptive Kalman filter for state estimation with unknown process noise covariance by introducing a new latent variable. To the best of our knowledge, it is always a challenge to design a MTT filter in the presence of unknown process noise covariance and unknown measurement noise covariance.

In this paper, a robust LMB (RLMB) filter with inaccurate process noise covariance and measurement noise covariance is proposed. Simulation experiments show that the proposed algorithm has stronger robustness and target tracking accuracy than the existing LMB filters.

The rest of this paper is organized as follows. Section II presents a brief description of the background for the LMB RFS and the RLMB filter. The closed-form implementation of the proposed RLMB filter is given in Section III. Numerical results and analyses are illustrated in Section IV, and finally, the conclusions are given in Section V.

II. Background

A. Problem Formulation

Consider the following discrete-time linear stochastic system as shown by the state-space model

$$x_k = F_{k|k-1}x_{k-1} + w_{k-1} \tag{1}$$

$$z_k = H_k x_k + v_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $z_k \in \mathbb{R}^m$ is the measurement vector, k is the discrete time index, $F_{k|k-1}$ is the state transition matrix, H_k is the measurement matrix; $w_{k-1} \sim \mathcal{N}(\cdot; 0, Q_{k-1})$ and $v_k \sim \mathcal{N}(\cdot; 0, R_k)$ are the Gaussian process and measurement noises with zero mean and covariances Q_{k-1} and R_k, respectively, and they are independent of each other. The prior distribution of the initial state vector x_0 is assumed to be a Gaussian distribution with mean vector $\hat{x}_0$ and covariance P_0, which can be denoted as $x_0 \sim \mathcal{N}(x_k; \hat{x}_0, P_0)$.

To illustrate the problem, the above dynamic model and measurement model can be expressed in the form of the probability density function (PDF) as shown below

$$p(x_k|x_{k-1}) = \mathcal{N}(x_k; F_{k|k-1}x_{k-1}, Q_{k-1}) \tag{3}$$

$$p(z_k|x_k) = \mathcal{N}(z_k; H_k x_k, R_k) \tag{4}$$

where $\mathcal{N}(\cdot; \mu, \Sigma)$ denotes the Gaussian PDF with mean μ and corresponding covariance Σ, and $p(x_k|x_{k-1})$ is the state transition PDF, $p(z_k|x_k)$ is the likelihood PDF. The one-step predicted PDF can be obtained by the following approximate method, and the result is also a Gaussian distribution, i.e

$$p(x_k|z_{1:k-1}) = \mathcal{N}(x_k|\hat{x}_{k|k-1}, P_{k|k-1}) \tag{5}$$

where $\hat{x}_{k|k-1} = F_{k|k-1}\hat{x}_{k-1}$ is the predicted state vector, $P_{k|k-1}$ is the prediction error covariance, which can be calculated by

$$P_{k|k-1} = F_{k|k-1}P_{k-1}F_{k|k-1}^T + Q_{k-1} \tag{6}$$

where $[\cdot]^T$represents the transpose operator, and $\hat{x}_{k-1}$ and P_{k-1} are respectively the state estimation vector and corresponding estimation error covariance at time $k-1$.

In target tracking applications, the true process noise covariance Q_{k-1} and measurement noise covariance R_k are usually unknown and time-varying. For unknown R_k, the approximate value is commonly obtained by VB inference. However, since Q_{k-1} appears in the prediction step of Kalman filter and the available information is very limited, similar method cannot be used to obtain approximation Q_{k-1}. It can be seen from formula (6) that an inaccurate Q_{k-1} will result in an inaccurate $P_{k|k-1}$. For the above reasons, the inverse Wishart (IW) distribution [26] [27] was selected as the conjugate prior distribution to model the prediction error covariance and measurement noise covariance in [23]. The target state x_k is augmented by $P_{k|k-1}$ and R_k to obtain the augmented state, which is estimated jointly via VB inference.

B. LMB RFS and RLMB Filter

Before introducing labeled RFS, some notations throughout this paper are defined:

- The inner product function
$$\langle f, g\rangle \triangleq \int f(x)g(x)dx$$
- The multi-target exponential function
$$h^X \triangleq \begin{cases} 1, & X = \varnothing \\ \prod_{x\in X} h(x), & \text{otherwise} \end{cases}$$
where $h(x)$ denotes a real-valued function.
- The generalized Kronecker delta function
$$\delta_Y(X) \triangleq \begin{cases} 1, \text{ if } & X = Y \\ 0, & \text{otherwise} \end{cases}$$
- The inclusion function
$$\mathbf{1}_Y(X) \triangleq \begin{cases} 1, \text{ if } & X \subseteq Y \\ 0, & \text{otherwise} \end{cases}$$

In order to incorporate target tracks in the framework of the multi-target Bayesian filtering, the concept of labeled RFS is introduced in [5]. For each target, the state x is augmented with a unique label $\ell = (k, i)$, where k is the time of birth and i is a unique index to distinguish targets born at time k. Then the labeled single target state is denoted as $\mathbf{x} = (x, \ell) \in \mathbb{X}\times\mathbb{L}$, where $\mathbb{X}$ is the dynamic state space and $\mathbb{L}$ is the discrete label space. Likewise, the labeled multi-target state is denoted as $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\} = \{(x_1, \ell_1), \ldots, (x_n, \ell_n)\} \subseteq \mathbb{X}\times\mathbb{L}$, where $\ell_1, \ldots, \ell_n$ are distinct from each other.

Define the function $\mathcal{L}$: $\mathbb{X}\times\mathbb{L}\rightarrow\mathbb{L}$. For a labeled single-target state $\mathbf{x}$, $\mathcal{L}(\mathbf{x})=\mathcal{L}((x,\ell))=\ell$. Likewise, for a labeled multi-target state $\mathbf{X}$, $\mathcal{L}(\mathbf{X})=\{\mathcal{L}(\mathbf{x}):\mathbf{x}\in\mathbf{X}\}$, and the labels must be distinct, i.e., $\delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|)=1$. For simplicity, the distinct label indicator is denoted as $\Delta(\mathbf{X})=\delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|)$.

An LMB RFS can be completely determined by a parameter set $\boldsymbol{\pi}=\{r^{(\ell)},p^{(\ell)}(x)\}_{\ell\in\mathbb{L}}$, where $r^{(\ell)}$ and $p^{(\ell)}(x)$ are the existence probability and the spatial distribution, respectively. Thus, the density of an LMB RFS $\boldsymbol{\pi}=\{r^{(\ell)},p^{(\ell)}(x)\}_{\ell\in\mathbb{L}}$ is given by

$$\boldsymbol{\pi}(\mathbf{X})=\Delta(\mathbf{X})w(L(\mathbf{X}))p^{\mathbf{X}} \tag{7}$$

where

$$w(L)=\prod_{i\in\mathbb{L}}(1-r^{(i)})\prod_{\ell\in}\frac{\mathbf{1}_{\mathbb{L}}(\ell)r^{(\ell)}}{1-r^{(\ell)}} \tag{8}$$

$$p(x,\ell)=p^{(\ell)}(x) \tag{9}$$

Further, the multi-target birth density is an LMB RFS with state space $\mathbb{X}$ and birth label space $\mathbb{B}$ which is given by

$$\boldsymbol{\pi}_B(\mathbf{X})=\Delta(\mathbf{X})w_B(L(\mathbf{X}))[p_B]^{\mathbf{X}} \tag{10}$$

where

$$w_B(I)=\prod_{i\in\mathbb{B}}\left(1-r_B^{(i)}\right)\prod_{\ell\in L}\frac{\mathbf{1}_{\mathbb{B}}(\ell)r_B^{(\ell)}}{1-r_B^{(\ell)}} \tag{11}$$

$$p_B(x,\ell)=p_B^{(\ell)}(x) \tag{12}$$

In the proposed RLMB filter, the augmented target state $\ddot{x}\triangleq(x,\tilde{P},R)$, where x is the kinematic state of a target including position and velocity, $\tilde{P}$ the predicted error covariance, R is the measurement noise covariance. The multi-target augmented state set can be given by

$$\ddot{\mathbf{X}}=\{(\ddot{x},\ell)\}_{i=1}^{N} \tag{13}$$

where i and N represent the target indexes and the number of targets, respectively.

Proposition 1: Suppose that the multi-target prior density and the multi-target birth density are both the LMB RFSs, that is $\boldsymbol{\pi}=\left\{\left(r^{(\ell)},p^{(\ell)}(\ddot{x})\right)\right\}_{\ell\in\mathbb{L}}$ and $\boldsymbol{\pi}_B=\left\{\left(r_B^{(\ell)},p_B^{(\ell)}(\ddot{x})\right)\right\}_{\ell\in\mathbb{B}}$, respectively, then the multi-target predicted density is also an LMB RFS

$$\boldsymbol{\pi}_+=\left\{\left(r_{+,S}^{(\ell)},p_{+,S}^{(\ell)}(\ddot{x}_+)\right)\right\}_{\ell\in\mathbb{L}}\bigcup\left\{\left(r_B^{(\ell)},p_B^{(\ell)}(\ddot{x})\right)\right\}_{\ell\in\mathbb{B}} \tag{14}$$

where

$$\ddot{x}_+\triangleq(x_+,\tilde{P}_+,R_+) \tag{15}$$

$$r_{+,S}^{(\ell)}=r^{(\ell)}\left(\int p_S(\ddot{x},\ell)p(\ddot{x},\ell)\,d\ddot{x}\right) \tag{16}$$

$$p_{+,S}^{(\ell)}(\ddot{x}_+)=\frac{\int p_S(\ddot{x},\ell)f(\ddot{x}_+|\ddot{x},\ell)p(\ddot{x},\ell)\,d\ddot{x}}{\int p_S(\ddot{x},\ell)p(\ddot{x},\ell)\,d\ddot{x}} \tag{17}$$

$$f(\ddot{x}_+|\ddot{x},\ell)=f(x_+|x,\ell)f(R_+|R,\ell)f(\tilde{P}_+|\tilde{P},\ell) \tag{18}$$

where $\ddot{x}_+$ is the predicted augmented state, $p_S(\ddot{x},\ell)$ is the probability of survival of the target, $f(\ddot{x}_+|\ddot{x},\ell)$ is the transition joint PDF of the augmented state, $f(x_+|x,\ell)$ denotes the target transition PDF, $f(R_+|R,\ell)$ and $f(\tilde{P}_+|\tilde{P},\ell)$ represent the transition PDFs of measurement noise covariance and prediction error covariance, respectively.

Before the update step, the LMB representation of the predicted multi-target density needs to be converted into a δ-GLMB representation. The predicted LMB can be converted to a single component δ-GLMB similar to (29) [7]. Then, we can obtain the update of the LMB filter with the augmented state.

Proposition 2: Suppose that the multi-target predicted density is an LMB RFS with parameter set $\pi_+=\left\{\left(r_+^{(\ell)},p_+^{(\ell)}(\ddot{x})\right)\right\}_{\ell\in\mathbb{L}_+}$, then the posterior density is

$$\pi\left(\ddot{\mathbf{X}}|Z\right)\approx\left\{\left(r^{(\ell)},p^{(\ell)}(\ddot{x})\right)\right\}_{\ell\in\mathbb{L}_+} \tag{19}$$

where

$$r^{(\ell)}=\sum_{(I_+,\theta)\in\mathcal{F}(\mathbb{L}_+)\times\Theta_{I_+}}w^{(I_+,\theta)}(Z)\cdot\mathbf{1}_{I_+}(\ell) \tag{20}$$

$$p^{(\ell)}(\ddot{x})=\frac{1}{r^{(\ell)}}\sum_{(I_+,\theta)\in\mathcal{F}(\mathbb{L}_+)\times\Theta_{I_+}}w^{(I_+,\theta)}(Z)\cdot\mathbf{1}_{I_+}(\ell)p^{(\theta)}(\ddot{x},\ell) \tag{21}$$

$$w^{(I_+,\theta)}(Z)\propto w_+(I_+)\left[\eta_Z^{(\theta)}\right]^{I_+} \tag{22}$$

$$p^{(\theta)}(\ddot{x},\ell|\mathrm{Z})=\frac{p_+(\ddot{x},\ell)\psi_Z(\ddot{x},\ell;\theta)}{\eta_Z^{(\theta)}(\ell)} \tag{23}$$

$$\eta_Z^{(\theta)}(\ell)=\int p_+(\ddot{x},\ell)\psi_Z(\ddot{x},\ell;\theta)d\ddot{x} \tag{24}$$

$$\begin{aligned}\psi_Z(\ddot{x},\ell;\theta)=&\delta_0(\theta(\ell))q_D(\ddot{x},\ell)\\&+(1-\delta_0(\theta(\ell)))\frac{p_D(\ddot{x},\ell)g(z_{\theta(\ell)}|\ddot{x},\ell)}{\kappa(z_{\theta(\ell)})}\end{aligned} \tag{25}$$

and Z is the set of measurements, $p_D(\ddot{x},\ell)$ denotes the detection probability associated with the augmented state and label, $q_D(\ddot{x},\ell)$ is the missed detection probability, Θ_{I_+} is the space of mappings θ: $I_+\rightarrow\{0,1,\ldots,|Z|\}$, such that $\theta(i)=\theta(i')$ means $i=i'$, $g(z|\ddot{x},\ell)$ is the single target likelihood function for z given $(\ddot{x},\ell)$, and $\kappa(\cdot)$ is the Poisson clutter density.

III. A Closed-form Solution

In the following, the proposed RLMB filter is implemented based on fixed-point iteration method. The joint PDF of the augmented state $\ddot{x}_k\triangleq(x_k,\tilde{P}_k,R_k)$ is modeled in the form of a Gaussian and IW distributions mixture as follow:

$$p_k(\ddot{x}_k)=\mathcal{N}(x_k;m_k,P_k)\mathcal{IW}(\tilde{P}_k;t_k,T_k)\mathcal{IW}(R_k;u_k,U_k) \tag{26}$$

Suppose that the survival probability and detection probability of the target are independent of the augmented state, i.e

$$p_S(\ddot{x},\ell)=p_S \tag{27}$$

$$p_D(\ddot{x},\ell)=p_D \tag{28}$$

Suppose that the spatial distributions of the birth tracks can be denoted as a mixture of Gaussian and IW distributions,

$$
\begin{aligned}
p_B^{(\ell)}(\ddot{x}) = & \sum_{j=1}^{J_B^{(\ell)}} w_{B,j}^{(\ell)} \mathcal{N}(x|m_{B,j}^{(\ell)}, P_{B,j}^{(\ell)}) \mathcal{IW}(\tilde{P}|t_{B,j}^{(\ell)}, T_{B,j}^{(\ell)}) \\
& \times \mathcal{IW}(R|u_{B,j}^{(\ell)}, U_{B,j}^{(\ell)})
\end{aligned} \tag{29}
$$

where $J_B^{(\ell)}$ is the number of the mixture of Gaussian and IW distributions, $w_{B,j}^{(\ell)}$, $m_{B,j}^{(\ell)}$ and $P_{B,j}^{(\ell)}$ represent the weight, the sate, and the according estimation error covariance of the jth mixed component, respectively. $t_{B,j}^{(\ell)}$ and $u_{B,j}^{(\ell)}$ represent the degrees of freedom of IW distributions, $T_{B,j}^{(\ell)}$ and $U_{B,j}^{(\ell)}$ are the corresponding scale matrices.

A. Prediction

Suppose that the multi-target posterior density is an LMB RFS with the augmented state space and parameter set $\boldsymbol{\pi}_{k-1} = \{r_{k-1}^{(\ell)}, p_{k-1}^{(\ell)}(\ddot{x})\}_{\ell \in \mathbb{L}}$, where the posterior probability densities p^{ℓ} of all labeled Bernoulli tracks $\ell \in \mathbb{L}$ are given by a mixture of Gaussian and IW distributions

$$
\begin{aligned}
p_{k-1}^{(\ell)}(\ddot{x}) = & \sum_{j=1}^{J_{k-1}^{(\ell)}} w_{j,k-1}^{(\ell)} \mathcal{N}(x|m_{j,k-1}^{(\ell)}, P_{j,k-1}^{(\ell)}) \\
& \times \mathcal{IW}(\tilde{P}|t_{j,k-1}^{(\ell)}, T_{j,k-1}^{(\ell)}) \times \mathcal{IW}(R|u_{j,k-1}^{(\ell)}, U_{j,k-1}^{(\ell)})
\end{aligned} \tag{30}
$$

then the predicted LMB distribution is represented by the union of surviving and new born tracks:

$$
\begin{aligned}
\boldsymbol{\pi}_{k|k-1} = & \left\{ \left(r_{k|k-1,S}^{(\ell)}, p_{k|k-1,S}^{(\ell)}(\ddot{x}_{k|k-1}) \right) \right\}_{\ell \in \mathbb{L}} \\
& \bigcup \left\{ \left(r_{k,B}^{(\ell)}, p_{k,B}^{(\ell)}(\ddot{x}) \right) \right\}_{\ell \in \mathbb{B}}
\end{aligned} \tag{31}
$$

The above parameters can be calculated as follows:

$$
r_{k|k-1,S}^{(\ell)} = r_{k-1}^{(\ell)} p_S \tag{32}
$$

$$
\begin{aligned}
p_{k|k-1,S}^{(\ell)}(\ddot{x}_{k|k-1}) = & \sum_{j=1}^{J_{k-1}^{(\ell)}} w_{j,k|k-1,S}^{(\ell)} \mathcal{N}(x|m_{j,k|k-1.S}^{(\ell)}, \tilde{P}_{j,k|k-1,S}^{(\ell)}) \\
& \times \mathcal{IW}(\tilde{P}|t_{j,k|k-1,S}^{(\ell)}, T_{j,k|k-1,S}^{(\ell)}) \\
& \times \mathcal{IW}(R|u_{j,k|k-1,S}^{(\ell)}, U_{j,k|k-1,S}^{(\ell)})
\end{aligned} \tag{33}
$$

where

$$
w_{j,k|k-1,S}^{(\ell)} = w_{j,k-1}^{(\ell)} \tag{34}
$$

$$
m_{j,k|k-1.S}^{(\ell)} = F_{k|k-1} m_{j,k-1}^{(\ell)} \tag{35}
$$

$$
\tilde{P}_{j,k|k-1,S}^{(\ell)} = F_{k|k-1} P_{j,k-1}^{(\ell)} F_{k|k-1}^T + \tilde{Q}_{k-1} \tag{36}
$$

$$
t_{j,k|k-1,S}^{(\ell)} = n + \tau + 1 \tag{37}
$$

$$
T_{j,k|k-1,S}^{(\ell)} = \tau \tilde{P}_{j,k|k-1,S}^{(\ell)} \tag{38}
$$

$$
u_{j,k|k-1,S}^{(\ell)} = \rho(u_{j,k-1}^{(\ell)} - m - 1) + m + 1 \tag{39}
$$

$$
U_{j,k|k-1,S}^{(\ell)} = \rho U_{j,k-1}^{(\ell)} \tag{40}
$$

where $\tilde{Q}_{k-1}$ is the nominal process noise covariance, $\tau \geq 0$ is a tuning parameter, $\rho \in (0, 1]$ is a forgetting factor, $\left\{ \left(r_{k,B}^{(\ell)}, p_{k,B}^{(\ell)}(\ddot{x}) \right) \right\}_{\ell \in \mathbb{B}}$ is the LMB RFS of the birth targets, which can be given by (29).

B. Update

Suppose that the multi-target predicted density is an LMB RFS with the predicted spatial distribution of a track ℓ is a mixture of Gaussian and IW distributions:

$$
\begin{aligned}
p_{k|k-1}^{(\ell)}(\ddot{x}) = & \sum_{j=1}^{J_{k|k-1}^{(\ell)}} w_{j,k|k-1}^{(\ell)} \mathcal{N}(x|m_{j,k|k-1}^{(\ell)}, \tilde{P}_{j,k|k-1}^{(\ell)}) \\
& \times \mathcal{IW}(\tilde{P}|t_{j,k|k-1}^{(\ell)}, T_{j,k|k-1}^{(\ell)}) \\
& \times \mathcal{IW}(R|u_{j,k|k-1}^{(\ell)}, U_{j,k|k-1}^{(\ell)})
\end{aligned} \tag{41}
$$

the posterior density is calculated by

$$
\begin{aligned}
p_k^{(\theta)}(\ddot{x}, \ell|Z) = & \sum_{j=1}^{J_k^{(\ell)}} w_{j,k}^{(\theta,\ell)} \mathcal{N}(x|m_{j,k}^{(\theta,\ell)}, P_{j,k}^{(\theta,\ell)}) \\
& \times \mathcal{IW}(\tilde{P}|t_{j,k}^{(\theta,\ell)}, T_{j,k}^{(\theta,\ell)}) \\
& \times \mathcal{IW}(R|u_{j,k}^{(\theta,\ell)}, U_{j,k}^{(\theta,\ell)})
\end{aligned} \tag{42}
$$

$$
w_{j,k}^{(\theta,\ell)} = \frac{w_{Z,j}^{(\theta,\ell)}}{\sum_{j=1}^{J_k^{(\ell)}} w_{Z,j}^{(\theta,\ell)}} \tag{43}
$$

If $\theta(\ell) = 0$,

$$
w_{Z,j}^{(\theta,\ell)} = w_{j,k|k-1}^{(\ell)}(1 - p_D) \tag{44}
$$

$$
m_{j,k}^{(\theta,\ell)} = m_{j,k|k-1}^{(\ell)}, P_{j,k}^{(\theta,\ell)} = \tilde{P}_{j,k|k-1}^{(\ell)} \tag{45}
$$

$$
t_{j,k}^{(\theta,\ell)} = t_{j,k|k-1}^{(\ell)}, T_{j,k}^{(\theta,\ell)} = T_{j,k|k-1}^{(\ell)} \tag{46}
$$

$$
u_{j,k}^{(\theta,\ell)} = u_{j,k|k-1}^{(\ell)}, U_{j,k}^{(\theta,\ell)} = U_{j,k|k-1}^{(\ell)} \tag{47}
$$

If a measurement is associated to track ℓ, i.e. $\theta(\ell) > 0$, the measurement likelihood function (25) is given by

$$
\begin{aligned}
\psi_Z(\ddot{x}, \ell; \theta) = & \frac{p_D g(z_{\theta(\ell)}|\ddot{x}, \ell)}{\kappa(z_{\theta(\ell)})} \\
= & \frac{p_D}{\kappa(z_{\theta(\ell)})} \mathcal{N}\left(z_{\theta(\ell)}; H_k x, R\right)
\end{aligned} \tag{48}
$$

Substituting (41) and (48) into the numerator of (23) ,

$$
\begin{aligned}
& p_{k|k-1}^{(\ell)}(\ddot{x}) \psi_Z(\ddot{x}, \ell; \theta) \\
& = \frac{p_D}{\kappa(z_{\theta(\ell)})} \sum_{j=1}^{J_{k|k-1}^{(\ell)}} w_{j,k|k-1}^{(\ell)} \mathcal{N}(x|m_{j,k|k-1}^{(\ell)}, \tilde{P}_{j,k|k-1}^{(\ell)}) \\
& \times \mathcal{IW}(\tilde{P}|t_{j,k|k-1}^{(\ell)}, T_{j,k|k-1}^{(\ell)}) \mathcal{IW}(R|u_{j,k|k-1}^{(\ell)}, U_{j,k|k-1}^{(\ell)}) \\
& \times \mathcal{N}\left(z_{\theta(\ell)}; H_k x, R_k\right)
\end{aligned} \tag{49}
$$

Since there is no analytical solution in (49), the approximate solution is obtained by VB method. Moreover, to obtain the measurement updated posterior distribution (23), the predictive likelihood function $\eta_Z^{(\theta)}(\ell)$ is required to have an analytical solution. However, it can be seen from (24) and (49) that the calculation of $\eta_Z^{(\theta)}(\ell)$ needs to solve multiple integrals. Fortunately, the above two problems can be solved by part (a) and part (b) of the following Lemma 1, respectively.

Lemma 1: Let x, $\tilde{P}$, R and z be random variables with the following joint density

$$p(x,\tilde{P},R,z) = p(z|x,R)p(x)p(\tilde{P})p(R) \tag{50}$$

and

$$p(z|x,R) = \mathcal{N}(z;Hx,R) \tag{51}$$

$$p(x) = \mathcal{N}(x;m_{k|k-1},\tilde{P}_{k|k-1}) \tag{52}$$

$$p(\tilde{P}) = \mathcal{IW}(\tilde{P};t_{k|k-1},T_{k|k-1}) \tag{53}$$

$$p(R) = \mathcal{IW}(R;u_{k|k-1},U_{k|k-1}) \tag{54}$$

Then,

(a) The posterior density $p(x,\tilde{P},R|z)$ can be approximated by minimizing the KL divergence by the product of densities $q(x)$, $q(\tilde{P})$ and $q(R)$, where

$$q(x) = \mathcal{N}(x;m_k,P_k) \tag{55}$$

$$q(\tilde{P}) = \mathcal{IW}(\tilde{P};t_k,T_k) \tag{56}$$

$$q(R) = \mathcal{IW}(R;u_k,U_k) \tag{57}$$

and the parameters m_k, P_k, t_k, T_k, u_k, U_k can be obtained by repeating the following iterations

$$A_k^{(i)} = P_k^{(i)} + \left(m_k^{(i)} - m_{k|k-1}\right)\left(m_k^{(i)} - m_{k|k-1}\right)^T \tag{58}$$

$$t_k^{(i+1)} = t_{k|k-1} + 1 \tag{59}$$

$$T_k^{(i+1)} = A_k^{(i)} + T_{k|k-1} \tag{60}$$

$$B_k^{(i)} = H_k P_k^{(i)} H_k^T + \left(z_k - H_k m_k^{(i)}\right)\left(z_k - H_k m_k^{(i)}\right)^T \tag{61}$$

$$u_k^{(i+1)} = u_{k|k-1} + 1 \tag{62}$$

$$U_k^{(i+1)} = B_k^{(i)} + U_{k|k-1} \tag{63}$$

$$\tilde{P}_k^{(i+1)} = \frac{T_k^{(i+1)}}{t_k^{(i+1)} - n - 1} \tag{64}$$

$$R_k^{(i+1)} = \frac{U_k^{(i+1)}}{u_k^{(i+1)} - m - 1} \tag{65}$$

$$K_k^{(i+1)} = \tilde{P}_k^{(i+1)} H_k^T (H_k \tilde{P}_k^{(i+1)} H_k^T + R_k^{(i+1)})^{-1} \tag{66}$$

$$m_k^{(i+1)} = m_{k|k-1} + K_k^{(i+1)}\left(z_k - H_k m_{k|k-1}\right) \tag{67}$$

$$P_k^{(i+1)} = \tilde{P}_k^{(i+1)} - K_k^{(i+1)} H_k \tilde{P}_k^{(i+1)} \tag{68}$$

where $a^{(i)}$ denotes the value of the variable a at the i^{th} iteration, and the initial conditions are given by $m_k^{(0)} = m_{k|k-1}$, $P_k^{(0)} = \tilde{P}_{k|k-1}$, $t_{k|k-1} = n + \tau + 1$, $T_{k|k-1} = \tau\tilde{P}_{k|k-1}$, $u_{k|k-1} = \rho(u_{k-1} - m - 1) + m + 1$, $U_{k|k-1} = \rho U_{k-1}$.

(b) The predictive likelihood function $p(z) = \int p(x,\tilde{P},R,z)dxd\tilde{P}dR$ can be approximated via minimizing the KL divergence by the variational lower bound

$$\begin{aligned}
\hat{q}(z) = \exp\{&-0.5m\log(2\pi) - 0.5\mathbb{E}[\log|R_k|] \\
&- 0.5tr\left(\mathbb{E}[R_k^{-1}]\mathbb{E}[(z_k - H_k x_k)(z_k - H_k x_k)^T]\right) \\
&- 0.5\mathbb{E}[\log|\tilde{P}_{k|k-1}|] + 0.5\mathbb{E}[\log|P_k|] \\
&- 0.5tr\left(\mathbb{E}[\tilde{P}_{k|k-1}^{-1}]\mathbb{E}[(x_k - m_{k|k-1})(x_k - m_{k|k-1})^T]\right) \\
&+ 0.5tr\left(\mathbb{E}[\log|P_k^{-1}|]\mathbb{E}[(x_k - m_k)(x_k - m_k)^T]\right) \\
&+ 0.5(t_{k|k-1} - n - 1)(\log|T_{k|k-1}| - n\log 2) \\
&- \log\Gamma[0.5(t_{k|k-1} - n - 1)] - 0.5t_{k|k-1}\log(\mathbb{E}[|\tilde{P}_k|]) \\
&- 0.5tr(\mathbb{E}[\tilde{P}_k^{-1}]T_{k|k-1}) + \log\Gamma[0.5(t_k - n - 1)] \\
&+ 0.5(t_k - n - 1)(n\log 2 - \log|T_k|) \\
&+ 0.5t_k\log\mathbb{E}[|\tilde{P}_k|] + 0.5tr\left(\mathbb{E}[\tilde{P}_k^{-1}]T_k\right) \\
&+ 0.5(u_{k|k-1} - m - 1)(\log|U_{k|k-1}| - m\log 2) \\
&- \log\Gamma[0.5(u_{k|k-1} - n - 1)] \\
&- 0.5u_{k|k-1}\log(\mathbb{E}[|R_k|]) - 0.5tr(\mathbb{E}[R_k^{-1}]U_{k|k-1}) \\
&+ 0.5(u_k - m - 1)(m\log 2 - \log|U_k|) \\
&+ \log\Gamma[0.5(u_k - m - 1)] \\
&+ 0.5u_k\log\mathbb{E}[|R_k|] + 0.5tr\left(\mathbb{E}[R_k^{-1}]U_k\right)\}
\end{aligned} \tag{69}$$

Proof: The proof of part (a) in Lemma 1 can be referred to [23], and the proof of part (b) can be found in Appendix A.

Corollary 1: The joint density can be approximated by approximate densities $q(x)$, $q(\tilde{P})$, $q(R)$ and $q(z)$ as follows

$$p(x,\tilde{P},R,z) \approx q(x)q(\tilde{P})q(R)q(z) \tag{70}$$

then

$$\begin{aligned}
&\mathcal{N}(x;m_{k|k-1},\tilde{P}_{k|k-1})\mathcal{N}(z;Hx,R) \\
&\times\mathcal{IW}(\tilde{P};t_{k|k-1},T_{k|k-1})\mathcal{IW}(R;u_{k|k-1},U_{k|k-1}) \\
&\approx\mathcal{N}(x;m_k,P_k)\mathcal{IW}(\tilde{P};t_k,T_k)\mathcal{IW}(R;u_k,U_k)q(z)
\end{aligned} \tag{71}$$

Using Lemma 1, Corollary 1 and (23), (42), (49), the posterior density is calculated by

$$w_{Z,j}^{(\theta,\ell)} = w_{j,k|k-1}^{(\ell)}\frac{p_D q_i(z_{\theta(\ell)})}{\kappa(z_{\theta(\ell)})} \tag{72}$$

where final values of these parameters are obtained by repeating the iterations as follows

$$\begin{aligned}
A_{j,k}^{(\theta,\ell)(i)} =& P_{j,k}^{(\theta,\ell)(i)} + \left(m_{j,k}^{(\theta,\ell)(i)} - m_{k|k-1}\right) \\
&\times\left(m_{j,k}^{(\theta,\ell)(i)} - m_{k|k-1}\right)^T
\end{aligned} \tag{73}$$

$$t_{j,k}^{(\theta,\ell)(i+1)} = t_{j,k|k-1}^{(\ell)} + 1 \tag{74}$$

$$T_{j,k}^{(\theta,\ell)(i+1)} = A_{j,k}^{(\theta,\ell)(i)} + T_{j,k|k-1}^{(\ell)} \tag{75}$$

$$B_{j,k}^{(\theta,\ell)(i)} = H_k P_{j,k}^{(\theta,\ell)(i)} H_k^T + \left(z_{\theta(\ell)} - H_k m_{j,k}^{(\theta,\ell)(i)}\right) \times \left(z_{\theta(\ell)} - H_k m_{j,k}^{(\theta,\ell)(i)}\right)^T \tag{76}$$

$$u_{j,k}^{(\theta,\ell)(i+1)} = u_{j,k|k-1}^{(\ell)} + 1 \tag{77}$$

$$U_{j,k}^{(\theta,\ell)(i+1)} = B_{j,k}^{(\theta,\ell)(i)} + U_{j,k|k-1}^{(\ell)} \tag{78}$$

$$\tilde{P}_{j,k}^{(\theta,\ell)(i+1)} = \frac{T_{j,k}^{(\theta,\ell)(i+1)}}{t_{j,k}^{(\theta,\ell)(i+1)} - n - 1} \tag{79}$$

$$R_{j,k}^{(\theta,\ell)(i+1)} = \frac{U_{j,k}^{(\theta,\ell)(i+1)}}{u_{j,k}^{(\theta,\ell)(i+1)} - m - 1} \tag{80}$$

$$K_{j,k}^{(\theta,\ell)(i+1)} = \tilde{P}_{j,k}^{(\theta,\ell)(i+1)} H_k^T \times \left(H_k \tilde{P}_{j,k}^{(\theta,\ell)(i+1)} H_k^T + R_{j,k}^{(\theta,\ell)(i+1)}\right)^{-1} \tag{81}$$

$$m_{j,k}^{(\theta,\ell)(i+1)} = m_{j,k|k-1}^{(\ell)} + K_{j,k}^{(\theta,\ell)(i+1)} \left(z_{\theta(\ell)} - H_k m_{j,k|k-1}^{(\ell)}\right) \tag{82}$$

$$P_{j,k}^{(\theta,\ell)(i+1)} = \tilde{P}_{j,k}^{(\theta,\ell)(i+1)} - K_{j,k}^{(\theta,\ell)(i+1)} \tilde{P}_{j,k}^{(\theta,\ell)(i+1)} \tag{83}$$

and the initial conditions are given by $m_{j,k}^{(\theta,\ell)(0)} = m_{j,k|k-1}^{(\ell)}$, $P_{j,k}^{(\theta,\ell)(0)} = \tilde{P}_{j,k|k-1}^{(\ell)}$, $t_{j,k|k-1}^{(\ell)} = n + \tau + 1$, $T_{j,k|k-1}^{(\ell)} = \tau \tilde{P}_{j,k|k-1}^{(\ell)}$, $u_{j,k|k-1}^{(\ell)} = \rho(u_{j,k-1}^{(\ell)} - m - 1) + m + 1$, $U_{j,k|k-1}^{(\ell)} = \rho U_{j,k-1}^{(\ell)}$.

The predicted likelihood function $q_i(z_{\theta(\ell)})$ can be obtained according to (69) using these final values.

IV. Numerical Simulation

The estimation performance of the proposed RLMB filter is illustrated in a multi-target tracking scenario with unknown and time-varying process noise covariance and measurement noise covariance (referred as RLMB-QR). We compare it with the LMB filter with nominal processes noise covariance and measurement noise covariance (nominal-LMB), as well as the existing RLMB filter considering only unknown measurement noise covariance (RLMB-R) [21] under the same simulation conditions. The cardinality statistics and the optimal subpattern assignment (OSPA) distance [25] are used to evaluate these filters estimation performance with $c = 100$ and $p = 2$.

Considering a two dimensional surveillance region $[-1000, 1000]m \times [-1000, 1000]m$, the kinematic state $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]^T$ of each target includes the position $(p_{x,k}, p_{y,k})$ and velocity $(\dot{p}_{x,k}, \dot{p}_{y,k})$, and sensor measurements are noisy vectors of the position. The state transition matrix $F_{k|k-1}$ and observation matrix H_k are given by

$$F_{k|k-1} = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, H_k = \mathbf{I}_2 \otimes \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{84}$$

where the sampling period is $T = 1$s and $\mathbf{I}_n$ is the n-dimensional identity matrix; $\otimes$ denotes the Kronecker product. The true process noise covariance and measurement noise covariance are given by

$$Q_k = \begin{cases} Q_0 & \text{if } k < k_p \\ 4Q_0 & \text{else} \end{cases} \tag{85}$$

$$R_k = \begin{cases} R_0 & \text{if } k < k_m \\ 4R_0 & \text{else} \end{cases} \tag{86}$$

where $k_p = 31$s and $k_m = 51$s denote the time step parameters of the true process noise covariance and measurement noise covariance change, respectively, and Q_0 and R_0 denote the nominal process noise covariance and measurement noise covariance, respectively.

$$Q_0 = {\sigma_w}^2 \mathbf{I}_2 \otimes \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \tag{87}$$

$$R_0 = {\sigma_v}^2 \mathbf{I}_2 \tag{88}$$

where $\sigma_w = 5$ and $\sigma_v = 5$ represent the standard deviations of the process noise and measurement noise, respectively.

The birth model is an LMB RFS with intensity $\pi_B = \{r_B^{(i)}, p_B^{(i)}(\ddot{x})\}_{i=1}^4$, where $r_B^{(i)} = 0.03$, and

$$p_B^{(i)}(\ddot{x}) = \mathcal{N}(x; m_B^{(i)}, P_B)\mathcal{IW}(\tilde{P}; t_B, V_B)\mathcal{IW}(R; u_B, U_B) \tag{89}$$

where $m_B^{(1)} = [0.1, 0, 0.1, 0]^T$, $m_B^{(2)} = [400, 0, -600, 0]^T$, $m_B^{(3)} = [-800, 0, -200, 0]^T$, $m_B^{(4)} = [-200, 0, 800, 0]^T$, $P_B = diag([10, 10, 10, 10]^T)^2$, $t_B = 10$, $T_B = 9Q_0$, $u_B = 10$, $U_B = 16R_0$.

The survival probability is $p_S = 0.99$ and the detection probability is $p_D = 0.98$. Clutter is a Poisson RFS with intensity $\kappa(z) = \lambda_c V u(z)$, where $\lambda_c = 2.5 \times 10^{-6} m^2$ denotes the average clutter intensity, $V = 4 \times 10^6 m^2$ is the volume of the surveillance region, and $u(\cdot)$ represents a uniform distribution over the region (giving an average of 10 clutter per scan). The tuning parameter $\tau = 3$, forgetting factor $\rho = 0.99$, and the number of iterations $N = 3$.

The true trajectories of targets are shown in Fig.1. Different colored lines represent different targets. And the circles and triangles are the starting and ending points of each target, respectively.

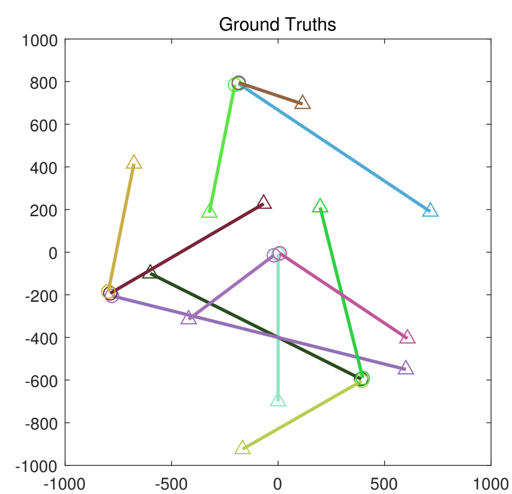

Fig. 1. True target trajectories.

To further verify the effectiveness of the proposed RLMB-QR filter, it is compared with other filters under different clutter densities over 100 Monte Carlo (MC) trials in the same simulation conditions, and the ideal-LMB filter with true noise covariances Q_k,R_k is used as a reference.

Fig.2 shows the cardinality statistics versus time for different filters. It can be seen that the target number estimation of the nominal LMB filter has serious target loss, and the existing RLMB-R filter also has a large estimation bias, especially after 50s. Since the 50th second, the measurement noise covariance has changed. However, the proposed RLMB-QR filter has a little performance loss compared to the ideal LMB filter.

Fig.3 shows the means of the OSPA distance versus time over 100 MC trials. It is noted that the OSPA distance of the four filters has increased to different degrees from the 50th second, but the proposed RLMB-QR filter still has the tracking performance comparable to that of the ideal filter. However, both the existing RLMB-R filter and the nominal-LMB filter have filtering divergence problems.

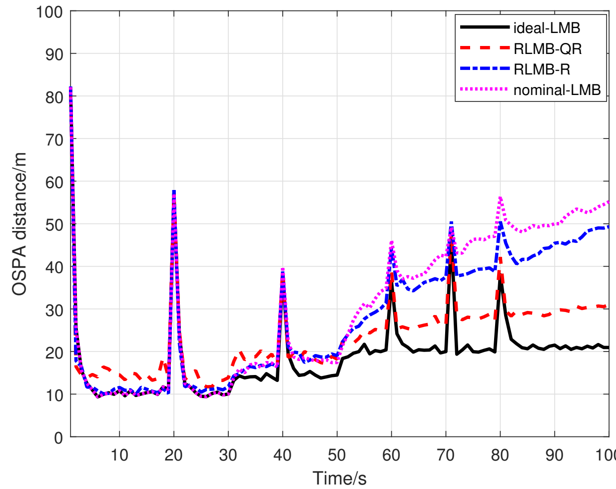

Fig. 3. Average OSPA distances for different filters.

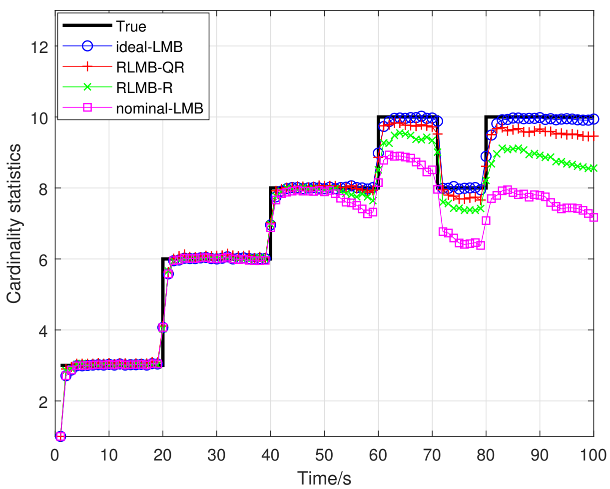

Fig. 2. Cardinality statistics for different filters.

V. Conclusion

Based on the LMB filter and VB approximation, a RLMB filter with adaptive ability to both process noise covariance and measurement noise covariance is proposed, In addition, a closed-form solution of the proposed RLMB filter is derived using Gaussian and inverse Wishart distributions mixtures. Simulation experiments show that the proposed RLMB filter still has desirable tracking performance in the presence of unknown and time-varying process noise covariance and measurement noise covariance, and it has comparable performance to the ideal LMB filter.

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References

[1] R. Mahler, Multitarget Bayes filtering via first-order multitarget moments. IEEE Transactions on Aerospace and Electronic Systems, 2003. 39(4): p. 1152-1178.

[2] R. Mahler, PHD filters of higher order in target number. IEEE Transactions on Aerospace and Electronic Systems, 2007. 43(4): p. 1523-1543.

[3] R. Mahler, Statistical multisource-multitarget information fusion. Vol. 31. 2007, Norwood, MA: Artech House.

[4] B.-T. Vo, B.-N. Vo, and A. Cantoni, The Cardinality Balanced Multi-Target Multi-Bernoulli Filter and Its Implementations. IEEE Transactions on Signal Processing, 2009. 57(2): p. 409-423.

[5] B.-T. Vo, and B.-N. Vo, Labeled Random Finite Sets and Multi-Object Conjugate Priors. IEEE Transactions on Signal Processing, 2013. 61(13): p. 3460-3475.

[6] B.-N. Vo, B.-T. Vo and D. Phung, Labeled Random Finite Sets and the Bayes Multi-Target Tracking Filter. IEEE Transactions on Signal Processing, 2014. 62(24): p. 6554-6567.

[7] S. Reuter, B.-T. Vo, B.-N. Vo and K. Dietmayer, The Labeled Multi-Bernoulli Filter. IEEE Transactions on Signal Processing, 2014. 62(12): pp. 3246-3260.

[8] A. Almagbile, J. Wang, W. Ding, Evaluating the performances of adaptive Kalman filter methods in GPS/INS integration. J. Global Position. Syst., 2010. 9(1): pp. 33-40.

[9] Särkkä S,Hartikainen J. Nonlinear noise adaptive Kalman filtering via variational Bayes. IEEE International Workshop on Machine Learning for Signal Processing. Southampton, UK, Sept 22-25 2013. IEEE Press; 2013. p. 1-6.

[10] Beal MJ. Variational algorithms for approximate Bayesian inference. University College London; 2003.

[11] H Zhu, H Leung, Z He. State Estimation in unknown non-Gaussian measurement noise using variational Bayesian technique. IEEE Trans Aerosp Electron Syst 2013; 49(4):2601-2614.

[12] P Dong, Z Jing, H Leung, K Shen. Variational bayesian adaptive cubature information filter based on Wishart distribution. IEEE Trans Autom Control 2017; 62(11):6051-6057.

[13] T. Ardeshiri , E. Özkan. An adaptive PHD filter for tracking with unknown sensor characteristics. In: IEEE international conference on information fusion. Istanbul, Turkey, July 9-12 2013. IEEE Press; 2013. p. 1-8.

[14] JL Yang, HW Ge. Adaptive probability hypothesis density filter based on variational Bayesian approximation for multi-target tracking. IET Radar Sonar Navigation. 2013;7(9):959-67.

[15] XH Wu, GM Huang, J Gao. Adaptive noise variance identification for PHD-based tracking algorithms using variational Bayesian approximations. IET Radar Sonar Navigation. 2013;7(8):895-903.

[16] XH Wu , GM Huang, J Gao. Particle filters for probability hypothesis density filter with the presence of unknown measurement noise covariance. Chin J Aeronaut 2013;26(6):1517-1523.
[17] JL Yang, HW Ge. An improved multi-target tracking algorithm based on CBMeMBer filter and variational Bayesian approximation. Signal Process. 2013;93(9):2510-2515.
[18] K Han, Z Qin, X Gao. Dynamic particle allocation for CBMeMBer filter. IEEE international conference on information, communication and signal processing. Singapore, Singapore, Dec 24 2015. IEEE Press; 2015. p. 1-5.
[19] C Yuan, J Wang, H Xiang. Adaptive -GLMB filtering algorithm based on VB approximation. Systems Engineering and Electronics 2017;39(2):237-243.
[20] H Qiu, GM Huang, J Gao. Variational Bayesian labeled multi-Bernoulli filter with unknown sensor noise statistics. Chin J Aeronaut 2016; 29(5): 1378-1384.
[21] J. Wang, Z. Jing, P. Dong and J. Cheng. A Variational Bayesian Labeled Multi-Bernoulli Filter for Tracking with Inverse Wishart Distribution. 2018 21st International Conference on Information Fusion, Cambridge, 2018, pp. 219-225.
[22] T. Ardeshiri, E. Özkan, U. Orguner, and F. Gustafsson. Approximate Bayesian smoothing with unknown process and measurement noise covariances. IEEE Signal Processing Letters. 2015; 22(12): 2450-2454.
[23] Y. Huang, Y. Zhang, Z. Wu, N. Li, and J. Chambers. A novel adaptive Kalman filter with inaccurate process and measurement noise covariance matrices. IEEE Transactions on Automatic Control. 2018; 63(2): 594-601.
[24] J. Ma, H. Lan, Z. Wang, X. Wang, Q. Pan and B. Moran. Improved Adaptive Kalman Filter with Unknown Process Noise Covariance. 2018 21st International Conference on Information Fusion, Cambridge, 2018, pp. 1-5.
[25] Schuhmacher D, Vo BT, Vo BN. A consistent metric for performance evaluation of multi-object filters. IEEE Trans Signal Process 2008; 56(8): 3447-3457.
[26] C. M. Bishop, Pattern Recognition and Machine Learning. Berlin, Germany: Springer, 2007.
[27] A. OHagan and J. J. Forster, Kendalls Advanced Theory of Statistics: Bayesian Inference. London, U.K.: Arnold, 2004.

APPENDIX A

For the proof of part (b) of the Lemma 1, the logarithm of the predictive likelihood function is decomposed as follows [26]

$$\log p(z) = \mathcal{C} + KL\left[q(x)q(\tilde{P})q(R)\,\middle\|\,p(x,\tilde{P},R\,|z)\right] \quad (90)$$

where

$$\begin{aligned}\mathcal{C} &= \int q(x)q(\tilde{P})q(R)\log\frac{p(x,\tilde{P},R,z)}{q(x)q(\tilde{P})q(R)}dxd\tilde{P}dR \\ &= \mathbb{E}[\log(p(x,\tilde{P},R,z))] - \mathbb{E}[\log(q(x)q(\tilde{P})q(R))]\end{aligned} \quad (91)$$

Since the first term on the right hand side of (90) can be minimized by the variational inference, the predictive likelihood function $p(z)$ can be approximated by [26]

$$p(z) \approx \exp(\mathcal{C}) \quad (92)$$

Substituting (50)-(57) into (91) respectively, we can get

$$\begin{aligned}\mathcal{C} &= \mathbb{E}[\log\mathcal{N}(z_k;H_kx_k,R_k)] \\ &+ \mathbb{E}[\log\mathcal{N}(x_k;m_{k|k-1},\tilde{P}_{k|k-1})] \\ &+ \mathbb{E}[\log\mathcal{IW}(\tilde{P}_k;t_{k|k-1},T_{k|k-1})] \\ &+ \mathbb{E}[\log\mathcal{IW}(R_k;u_{k|k-1},U_{k|k-1})] \\ &- \mathbb{E}[\log\mathcal{N}(x_k;m_k,P_k)] \\ &- \mathbb{E}[\log\mathcal{IW}(\tilde{P}_k;t_k,T_k)] \\ &- \mathbb{E}[\log\mathcal{IW}(R_k;u_k,U_k)]\end{aligned} \quad (93)$$

using the definition and properties of Gaussian density and IW density, (93) can be calculated as

$$\begin{aligned}\mathcal{C} &= -0.5m\log(2\pi) - 0.5\mathbb{E}[\log|R_k|] \\ &- 0.5tr\left(\mathbb{E}[R_k^{-1}]\mathbb{E}[(z_k-H_kx_k)(z_k-H_kx_k)^T]\right) \\ &- 0.5\mathbb{E}[\log|\tilde{P}_{k|k-1}|] + 0.5\mathbb{E}[\log|P_k|] \\ &- 0.5tr\left(\mathbb{E}[\tilde{P}_{k|k-1}^{-1}]\mathbb{E}[(x_k-m_{k|k-1})(x_k-m_{k|k-1})^T]\right) \\ &+ 0.5tr\left(\mathbb{E}[\log|P_k^{-1}|]\mathbb{E}[(x_k-m_k)(x_k-m_k)^T]\right) \\ &+ 0.5(t_{k|k-1}-n-1)(\log|T_{k|k-1}| - n\log 2) \\ &- \log\Gamma[0.5(t_{k|k-1}-n-1)] - 0.5t_{k|k-1}\log(\mathbb{E}[|\tilde{P}_k|]) \\ &- 0.5tr(\mathbb{E}[\tilde{P}_k^{-1}]T_{k|k-1}) + \log\Gamma[0.5(t_k-n-1)] \\ &+ 0.5(t_k-n-1)(n\log 2 - \log|T_k|) \\ &+ 0.5t_k\log\mathbb{E}[|\tilde{P}_k|] + 0.5tr\left(\mathbb{E}[\tilde{P}_k^{-1}]T_k\right) \\ &+ 0.5(u_{k|k-1}-m-1)(\log|U_{k|k-1}| - m\log 2) \\ &- \log\Gamma[0.5(u_{k|k-1}-n-1)] \\ &- 0.5u_{k|k-1}\log(\mathbb{E}[|R_k|]) - 0.5tr(\mathbb{E}[R_k^{-1}]U_{k|k-1}) \\ &+ 0.5(u_k-m-1)(m\log 2 - \log|U_k|) \\ &+ \log\Gamma[0.5(u_k-m-1)] \\ &+ 0.5u_k\log\mathbb{E}[|R_k|] + 0.5tr\left(\mathbb{E}[R_k^{-1}]U_k\right)\end{aligned} \quad (94)$$

The predicted likelihood function $\hat{q}(z)$ shown in (69) can be obtained by substituting (94) into (92).