Generalized Fuzzy Logic and Fuzzy Decision Set for Incomplete Information

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***Abstract*—The information available to the system is incomplete in many applications particularly in Decision Support Systems. The fuzzy logic deals incomplete information with belief rather than likelihood(probability). Some times decision has to be taken with the fuzzy information. In this paper, fuzzy Decision set is defined with two fold fuzzy set. The fuzzy inference and reasoning are studied for fuzzy Decision sets. Business and Medical applications are given.**

I. Introduction

The information available to many applications like Business, Medical, Geological, Control Systems etc are incomplete or unceryain. The fuzzy logic will deal incomplete information with belief rather than likelihood (probable). Zadeh formulated uncertain information as fuzzy set with single membership function**.** The fuzzy set with two membership function will give more evidence than single membership function . The two fold fuzzy set is with fuzzy membership functions "Belief " and" false. Usually , in Mdical and Business applications, there are two opinions like " Belief " and "Disbelief " about the information and decicision has to be taken under risk. For instance, in Mycin[1], The medical information is defined with belief and disbelief i.e CF[h,e]=MB[h,e] – MD[h.e], whe "e" is the evidence for given hypothesis "h". The fuzzy set is used instead of Probability to define fuzzy certainty factor.

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In the following fuzzy logic[23] and Geralized fuzzy logic[19] are studied briefly. The fuzzy Certainty Factor is studied and fuzzy Decision set is proposed. The fuzzy inference and fuzzy reasoning are studied for fuzzy Decision set. The Business and Medical applications are studed as applications of fuzzy Decision set.

II. FUZZY LOGIC

Various theories are studied to deal with imprecise, inconsistent and inexact information and these theories deal with likelihood where as fuzzy logic will dels with belief. Zadeh[23] has introduced fuzzy set as a model to deal with uncertain information as single membership function.. The fuzzy set is a class of objects with a continuum of grades of membership. The set A of X is characterized by its membership function $\mu A(x)$and ranging values in the unit interval [0, 1]

$\mu A(x): X \rightarrow [0, 1]$, $x \in X$, where X is Universe of discourse.

$A = \mu A(x1)/x1 + \mu A(x2)/x2 + \ldots + \mu A(xn)/xn$, "+" is union

For example, the fuzzy proposition "x is young"

Young ={.95/10+0.9/20+0.8/30+0.6/40+0.4/50+0.3/60+0.2/70+0.15/80+0.1/90 }

not young ={ 0.05/10 + 0.1/20 + 0.2/30+0.4/40 +0.6/50 + 0.8/60+0.7/70 +0.95/80+0.9/90 }

For instance "Rama is young" and the fuzziness of "young" is 0.8 The Graphical representation of young and Not young is shown in fig.1

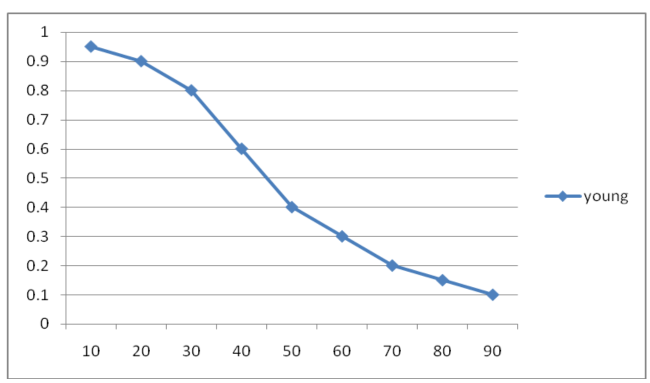

Fig.1: fuzzy membership function

The fuzzy set of tye 2 "Headache" is defined as

Headache = { 0.4/mild + 0.6/moderate+ 0.8/Serious}

For example, Consider the fuzzy proposition "x has mild Headache"

. For instance "Rama has mild headache" with Fuzziness 0.4

The fuzzy logic is defined as combination of fuzzy sets using logical operators]21]. Some of the logical operations are given below

Let A, B and C are fuzzy sets. The operations on fuzzy sets are

Negation

If x is not A

$A'=1-\mu_A(x)/x$

Conjunction

x is A and y is B→ (x, y) is A x B

$A \times B=\min(\mu_A(x)), \mu_B(y)\}(x,y)$

If x=y

x is A and y is B→ (x, y) is AΛB

$A\Lambda B=\min(\mu_A(x)), \mu_B(y)\}/x$ x is A or y is B→ (x, y) is A' x B'

$A' \times B' =\max(\mu_A(x)), \mu_B(y)\}(x,y)$

If x=y

x is A and x is B→ (x, x) is A V B

$AVB=\max(\mu_A(x)), \mu_B(y)\}/x$ Disjunction

Implication

if x is A then y is B $=A\rightarrow B = \min\{1, 1- \mu_A(x)) +\mu_B(y)\}/(x,y)$

if x= y

$A\rightarrow B= \min \{1, 1- \mu_A(x)) +\mu_B(y)\}/x$

If x is A then y is B else y is C= A x B + A' x C

The fuzzy propostion "If x is A then y is B else y is C" may be devided into two clause "If x is A then y is B " and "If x is not A then y is C" [15]

If x is A then y is B else y is C $=A\rightarrow B= \min \{1, 1-\mu A(x)+ \mu B(y)\}/(x,y)$

If x is not A then y is B else y is C $=A'\rightarrow C = \min \{1, 1- \mu_A(x)) +\mu_C(y)\}/(x,y)$

Composition

$A \text{ o } B= A \times B=\min\{ \mu_A(x)), \mu_B(y)\}/(x,y)$

If x = y

$A \text{ o } B==\min\{ \mu_A(x)), \mu_B(y)\}/x$ Composition

The fuzzy propositions may contain quantifiers like "Very", "More or Less" . These fuzzy quantifiers may be eliminated as

Concentration

x is very A

$\mu_{\text{very } A}(x)), =\mu_A(x)^2$

Difusion

x is very A

$\mu_{\text{more or less } A}(x) =\mu_A(x)^{0.5}$

III. GENERALIZED FUZZY LOGIC WITH TWO FOLD FUZZY SET

Since formation of the generalized fuzzy set simply as two fold fuzzy set and is extension Zadeh fuzzy logic[23].

The fuzzy logic is defined as combination of fuzzy sets using logical operators. Some of the logical operations are given below

Suppose A, B and C are fuzzy sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Since formation of the generalized fuzzy set simply as two fold fuzzy set, Zadeh fuzzy logic is extended to these generalized fuzzy sets.

Negation

$A'= \{1- \mu_A^{\text{Belief}}(x), 1- \mu_A^{\text{Disbelief}}(x) \}/x$

Disjunction

$AVB=\{ \max(\mu_A^{\text{Belief}}(x), \mu_A^{\text{Belief}}(y)), \max(\mu_B^{\text{Disbelief}}(x), \mu_B^{\text{Disbelief}}(y))\}(x,y)$

Conjunction

$A\Lambda B=\{ \min(\mu_A^{\text{Belief}}(x) ,\mu_A^{\text{Belief}}(y)), \min(\mu_B^{\text{Disbelief}}(x) , \mu_B^{\text{Disbelief}}(y)) \}/(x,y)$

Implication

$A\rightarrow B= \{\min (1, 1- \mu_A^{\text{Belief}}(x) + \mu_B^{\text{Belief}}(y) , \min (1, 1- \mu_A^{\text{Disbelief}}(x) + \mu_B^{\text{Disbelief}}(y)\}(x,y)$

If x is A then y is B else y is C = A x B + A' x C

If x is A then y is B else y is C $=A\rightarrow B = \{\min (1, 1- \mu_A^{\text{Belief}}(x) + \mu_B^{\text{Belief}}(y) , \min (1, 1- \mu_A^{\text{Disbelief}}(x) + \mu_B^{\text{Disbelief}}(y)\}/(x,y)$

If x is not A then y is B else y is C $=A'\rightarrow C = \min (1, \mu_A^{\text{Belief}}(x) + \mu_C^{\text{Belief}}(y) , \min (1,\mu_A^{\text{Disbelief}}(x) + \mu_C^{\text{Disbelief}}(y)\}(x,y)$

Compositin

$A \text{ o } R= \{\min_x (\mu_A^{\text{Belief}}(x), \mu_A^{\text{Belief}}(x)), \min_x(\mu_R^{\text{Disbelief}}(x), \mu_R^{\text{Disbelief}}(x))\}/y$

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

Concentration

"x is very A

$\mu_{\text{very } A}(x) = \{ \mu_A^{\text{Belief}}(x)^2, \mu_A^{\text{Disbelief}}(x)\mu_A(x)^2 \}$

Diffusion

"x is more or less A"

$\mu_{\text{more or less } A}(x) = (\mu_A^{\text{Belief}}(x)^{0.5}, \mu_A^{\text{Disbelief}}(x)\mu_A(x)^{0.5}$

For instance, Let A, B and C are

$A = \{ 0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 +0.5/x_5 ,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 +0.6/x_5\}$

$B = \{ 0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0..5/x_4 +0.6/x_5 ,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 +0.7/x_5\}$

$A \text{ V } B = \{ 0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 +0.6/x_5 ,$
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 +0.7/x_5\}$

$A \Lambda B = \{ 0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 +0.5/x_5 ,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 +0.6/x_5\}$

$A' = \text{not } A= \{ 0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 +0.5/x_5 ,$
$0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 +0.4/x_5\}$

$A\rightarrow B = \{ 1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 +1/x_5 ,$
$1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 +1/x_5\}$

$A \text{ o } B = \{ 0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 +0.5/x_5 ,$
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 +0.6/x_5\}$

$\mu_{\text{Very } A}(x) = \{ \mu_A^{\text{Belief}}(x)^2, \mu_A^{\text{Disbelief}}(x)\mu_A(x)^2 \}$
$= \{ 0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 +0.25/x_5 ,$
$0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 +0.36/x_5$

$\mu_{\text{More or Less } A}(x) = (\mu_A^{\text{Belief}}(x)^{1/2}, \mu_A^{\text{Disbelief}}(x)\mu_A(x)^{1/2} \}$
$= \{ 0.89/x_1 + 0.94/x_2 + 0.83/x_3 + 0.77/x_4 +0.70/x_5 ,$
$0.63/x_1 + 0.54/x_2 + 0.63/x_3 + 0.83/x_4 +0.77/x_5\}$

IV. FUZZY DECISION SET

Zadeh[22] proposed fuzzy set to deal with incomplete information. Generalized fuzzy set with two fold membership function $\mu_A(x) = \{ \mu_A^{Belief}(x), \mu_A^{Disbelief}(x) \}$ is studied [18]

The fuzzy Certainty Factor may be defined as (FCF)

$\mu_A^{FCF}(x) = \mu_A^{Belief}(x) - \mu_A^{Disbelief}(x)$, where

$$\mu_A^{FCF}(x) = \mu_A^{Belief}(x) - \mu_A^{Disbelief}(x) \qquad \mu_A^{Belief}(x) \geq \mu_A^{Disbelief}(x)$$
$$= 0 \qquad \mu_A^{Belief}(x) < \mu_A^{Disbelief}(x)$$

fuzzy Decision set Ris defined based on convex fuzzy set [10]

R= A $\mu_A^{FCF}(x) \geq \alpha$, where $\alpha€[0,1]$

For instance,

Demand ={ 0..8/x1+0.7/x2+0.86/x3+0.75/x4+0.88/x5, 0.2/x1+0.3/x2+0.25/x3+0.3/x4+0.2/x5 }

$\mu_{Demand}^{FCF}(x) =$

0.6/x1+0.4/x2+0.61/x3+0.45/x4+0.68/x5

The Generalized fuzzy set for Demand for the Items and fuzzy certainity factor is ahown in Fig2.

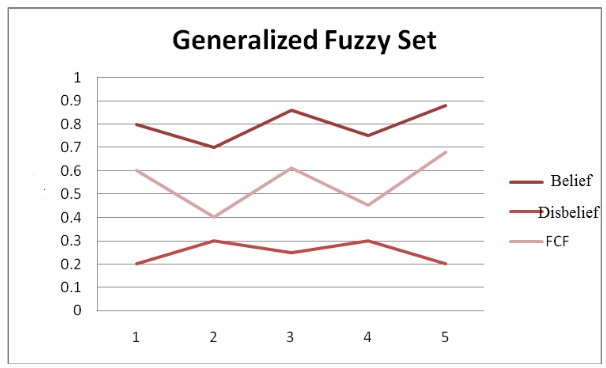

Fig2: Generalized fuzzy set

Suppose fuzzy Decision set is defined

$\mu_{Demand}^{FCF}(x) \geq 0.5$

=1/x1+0/x2+1/x3+0/x4+1/x5

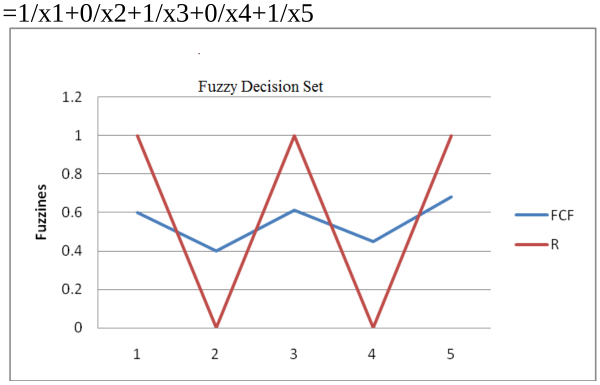

Fig3: fuzzy Decision set

The fuzzy logic is combination of logical operators. Consider the logical operations on fuzzy Decision sets r1, R2 and R3

Negation

If x is not R1

$R1' = 1-\mu_{R1}(x)/x$

Conjunction

x is R1 and y is R2→ (x, y) is R1 x R2

R1 x R2=min(μ_{R1}(x)), μ_{R2}(y)}(x,y)

If x=y

x is R1 and y is R2→ (x, y) is R1ΛR2

R1ΛR2=min(μ_{R1}(x)), μ_{R2}(y)}/x x is R1 or y is R2→ (x, y) is R1' x R2'

R1' x R2' =max(μ_{R1}(x)), μ_{R2}(y)}(x,y)

If x=y

x is R1 and x is R2→ (x, x) is R1 V R2

R1VR2=max(μ_{R1}(x)), μ_{R2}(y)}/x Disjunction

Implication

if x is R1 then y is R2 =R1→R2 = min{1, 1- μ_{R1}(x)) +μ_{R2}(y)}/(x,y)

if x= y

R1→R2= min {1, 1- μ_{R1}(x)) +μ_{R2}(y)}/x

If x is R1 then y is R2 else y is R3= R1 x R2 + R1' x R3

The fuzzy propostion "If x is R1 then y is R2 else y is R3" may be devided into two clause "If x is R1 then y is R2 " and "If x is not R1 then y is R3" [15]

If x is R1 then y is R2 else y is R3 =R1→R2= min {1, 1-μR1(x)+ μR2(y)}/(x,y)

If x is not R1 then y is R2 else y is R3 =R1'→ R3 = min {1, 1-μ_{R1}(x)) +μ_{R3}(y)}/(x,y)

Composition

R1 o R2= R1 x R2=min{ μ_{R1}(x)), μ_{R2}(y)}/(x,y)

If x = y

R1 o R2==min{ μ_{R1}(x)), μ_{R2}(y)}/x

The fuzzy propositions may contain quantifiers like "Very", "More or Less" . These fuzzy quantifiers may be eliminated as

Concentration

x is very R1

$\mu_{very\ R1}$(x)), =$\mu_{R1}(x)^2$

Difusion

x is very R1

$\mu_{more\ or\ less\ R1}(x) = \mu_{R1}(x)^{0.5}$

V. FUZZY INFRENCE IN DECISION MAKING

Decision management is usually happens in Decision Support Systems.

EXAMPLE1

Consider Business rule

If x is Demand of the product then x is High Price

Let x1, x2, x3, x4, x5 be the Items.

The Generalized fuzzy set

Demand ={ 0.56/x1+0.48/x2+0.86/x3+0.36/x4+0.88/x5, 0.06/x1+0.04/x2+0.07/x3+0.03/x4+0.2/x5 }

$\mu_{Demand}^{FCF}(x) =$

0.5/x1+0.44/x2+0.79/x3+0.33/x4+0.68/x5

High Price = 0.49/x1+0.52/x2+0.35/x3+0.4/x4+0.3/x5, 0.09/x1+0.02/x2+0.06/x3+0.02/x4+0.1/x5 }

$\mu_{Hgh\ Price}^{FCF}(x) =$

0.4/x1+0.5/x2+0.29/x3+0.38/x4+0.2/x5

With the inference rule A→B= min{1, 1-μ_A(x) + μ_B(x)}

$\mu_{Demand \rightarrow High\ Price}^{FCF}(x) =$

0.9/x1+1/x2+0.5/x3+1/x4+0.52/x5

$\mu_{Demand \rightarrow High\ Price}^{FCF}(x) \geq 0.6$

= 1/x1+1/x2+0/x3+1/x4+0/x5

EXAMPLE2

Consider Medical Diagnosis

If x has infection in the leg then surgery

Let x1, x2, x3, x4, x5 are the Patients.

The fuzzy set

$\mu_{Infection}^{FCF}(x)$

= 0.76/x1+0.78/x2+0.46/x3+0.86/x4+0.58/x5, 0.16/x1+0.12/x2+0.06/x3+0.14/x4+0.05/x5 }

= 0.6/x1+0.64/x2+0.4/x3+0.72/x4+0.53/x5

$\mu_{Surgery}^{FCF}(x)$

= 0.59/x1+0.26/x2+0.55/x3+0.24/x4+0.35/x5, 0.09/x1+0.06/x2+0.05/x3+0.04/x4+0.03/x5 }

= 0.5/x1+0.2/x2+0.5/x3+0.2/x4+0.32/x5

Using inference rule A→B= min{1, 1-μ_A(x) + μ_B(x)}

$\mu_{Infection \rightarrow Surgery}^{FCF}(x)$

= 0.9/x1+0.56/x2+0.9/x3+1/x4+1/x5

$\mu_{Infection \rightarrow Surgery}^{FCF}(x) \geq 6$

0 $\mu_{Infection \rightarrow Surgery}^{FCF}(x) < 6$

The fuzzy Decision set is

1/x1+0/x2+1/x3+1/x4+1/x5

VI. CONCLUSION

The decision has to be taken under incomplete information in many applications like Business, Medicine ect.. The fuzzy logicis used to deal with incomplete informayion The fuzzy Decision set is defined with two fold fuzzy set. The fuzzy logic is discussed with two fold fuzzy set. The fuzzy Decision set, inference and reasoning are studied. The Business and Medical applications are discussed for fuzzy Decision set.

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