Fuzzy Temporal Predicate Logic for Incomplete Information

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***Abstract*— The fuzzy temporal logic is fuzzy logic to deal time constraints of incomplete information. The Fuzzy Temporal logic is to solve independent time constraints problems of incomplete information in AI. The Knowledge Representation (KR) is the main component to solve the problems in Artificial Intelligence (AI). In this paper, fuzzy predicate temporal logic is studied for incomplete information with time constraints. Fuzzy Temporal Agent is discussed for Fuzzy Automated Reasoning. The Logic Programming Prolog is discussed for Fuzzy Temporal Predicate Logic**

Keywords*— *fuzzy logic; fuzzy reasoning; fuzzy agent; fuzzy modules; temporal logic fuzzy temporal predicate logic, logic programming

I. INTRODUCTION

The problem solving in the system may be viewed as a collection of intelligent agents [3]. The Intelligent agent is the agent which deals with independent component. There are intelligent agents for automated reasoning systems like TMS[6] to deal with incomplete information, which are non fuzzy based.

The fuzzy agent is fuzzy based automated system to deals with incomplete information which consists of fuzzy modulations The fuzzy agent is fuzzy based automated system to deals with incomplete information The Knowledge Representation (KR) is key component of Intelligence Agent to solve the problems. There are KR methods like predicate logic for complete information [6]. The information available to the system may be incomplete information. The fuzzy logic [10] deals incomplete information with belief rather than probable [5]. The fuzzy modulations[8] and fuzzy predicate logic[9] are discussed for incomplete information.

The fuzzy propositions may contain time constrains. For instance " The train "x" will come shortly". This situation is falls under fuzzy temporal. There is a need for KR for fuzzy temporal logic.

The Incomplete information time constraints are represented with fuzzy temporal modulations and later represented by Fuzzy Temporal Predicate Logic (FTPL). The Fuzzy Temporal Agent is discussed for Automation with Prolog.

II. FUZZY LOGIC

Zadeh [11] introduced fuzzy set as model to deal with imprecise, inconsistent and inexact information. The fuzzy set A of X is defined by its membership function $\mu_A(x)$ and take the values in the unit interval [0, 1]

$\mu_A(x) \rightarrow [0, 1]$, where $x \in X$ is in some Universe of discourse.

For example,

Consider the fuzzy proposition "x is tall" and the fuzzy set 'tall" is defined as

$\mu_{tall}(x) \rightarrow [0, 1]$, $x \in X$

tall= 0.5/x1 +0.6/x2 + 0.7/x3 +0.8/x4 +0.9/x5

. Let A, B and C be the fuzzy sets, and the operations on fuzzy sets are given below

$AVB=\max(\mu_A(x) , \mu_B(x)\}$ Disjunction

$A\Lambda B=\min(\mu_A(x) , \mu_B(x)\}$ Conjunction

$A'=1- \mu_A(x)$ Negation

$A\rightarrow B=\min \{1, (1- \mu_A(x) +\mu_B(x)\}$ Implication

$AXB=\min \{ \mu_A(x) , \mu_B(y)\}/(x,y)$ Relation

$AoR=\min_x\{ \mu_A(x) , \mu_R(x,y)\}/y$ Composition

The fuzzy propositions may contain quantifiers like "very ", "More or Less" etc. These fuzzy quantifiers may be eliminated as[10]

$\mu_{very}(x) = \mu_A(x)^2$ Concentration

$\mu_{more\ or\ less}(x) = \mu_A(x)_{0.5}$ Diffusion

III. FUZZY TEMORAL LOGIC

. the temporal logic is a logic with time constraints and Time variables "t1-t0" like "before", "meet", "after", where starting time t0 and ending time t1.

Fuzzy temporal logic has to deal with incomplete information of time constraints[1].

A temporal variable is "t1-t0", where t0 is starting time and t1 ending time.

For instance "past"=t 1-t0, t0<t1

"Present"= t1 approximately =t0

"feature"=t 0-t1, t0>t1

A temporal set is set of temporal variables with interval "t1-t0

.The Temporal logic is interpreted in simple methodl.

A Temporal logic is logic combinations of temporal sets.

Let (I, t) and (J.t) are temporal sets.

For instance " x was rich"
Was rich=richXpast.
(Rich, past)
not(I), t)=not (I,t)
(not(rich), past)=not (rich,past)
(I,t) and (J,t) = (I,t) Λ(J,t) conjunction
"x was rich and poor"
(rich,past) and (poor,past) = (rich, past) Λ(poor, past)
(I,t) or (J,t) =(I,t) V (J,t) disjunction
"x was rich or poor"
(rich,past) or (poor,past) = (rich, past) V (poor, past)
If (I,t) then (J,t)= (I,t)→ (J,t) implication
"if x was rich thenx was poor"
If (rich,past) then (poor,past) = (rich, past)→ (poor, past)

Definition: Let p be the fuzzy temporal proposition of the form like ' x was A". The fuzzy temporal set Ã may be defined in terms of possibility Π as

$p \rightarrow \Pi_{R(x)}=A$, where "R" is relation and xЄX is universe of discourse.

For instance ,
The fuzzy proposition may contain time variables like .
" x was rich"
was rich=$\Pi_{wealth}(x)$= rich X past

Definition: The fuzzy temporal set Ã is characterized by membership function $\mu_{\tilde{A}}$:XxT→ [0,1], xЄX and TЄA

Suppose X is a finite set. The fuzzy temporal set Ã of X may be represented by

$\tilde{A}=\int\int \mu_{\tilde{A}}(x,t)/x/t=\sum\sum \mu_{\tilde{A}}(x,t) = (\mu_{\tilde{A}}(x_1,t_1)/x_1 + \mu_{\tilde{A}}(x_2,t_1)/x_2 {}_{+\ldots+} \mu_{\tilde{A}}(x_n,t_1)/x_n)/t_1$
$+ (\mu_{\tilde{A}}(x_1,t_2)/x_1 + \mu_{\tilde{A}}(x_2,t_2)/x_2 {}_{+\ldots+} \mu_{\tilde{A}}(x_n,t_2)/x_n)/t_2 +\ldots+ (\mu_{\tilde{A}}(x_1,t_m)/x_1 + \mu_{\tilde{A}}(x_2,t_m)/x_2 {}_{+\ldots+} \mu_{\tilde{A}}(x_n,t_1)/x_n)/t_m$
$\tilde{A}'=1-\mu_{\tilde{A}}(x,t)$
$\tilde{A}= \{ (0.1/x_1+ 0.2/x_2+0.3/x_3+0.35/x_4+0.4/x_5)/t_1 +(0.4/x_1+0.45/x_2+0.5/x_3+0.55/x_4+0.6/x_5)/t_2 +(0.7/x_1+0.75/x_2+0.8/x_3+0.85/x_4+0.9/x_5)/t_3 \}$
Train arrival= $\{ (0.2x_1+ 0.3/x_2+0.5/x_3+0.7/x_4+0.9/x_5)/before +(0.6/x_1+0.65/x_2+0.7/x_3+0.8/x_4+0.9/x_5)/normal +(0.7/x_1+0.75/x_2+0.8/x_3+0.85/x_4+0.9/x_5)/after \}$

For instance "Train came in normal time"
"Train will come after 10 minutes"
"Train left 10 munities before"
"usually the train x is late"→ $\mu_{\tilde{A}}(x)^2$
"the train x comes more or less in time"→ $\mu_{\tilde{A}}(x)^{0.5}$
"x was rich"
$\Pi_{was\ rich}(x)$=rich X past
where rich X past = min { rich, past}
rich= $0.5/x_1 +0.5.5/x_2 + 0.7/x_3 +0.7.5/x_4 +0.8/x_5$
past= $0.4/t_1 +0.6/t_2 + 0.7/t_3 +0.8/t_4 +0.85/t_5$
was rich =rich X past =min { $0.5/x_1 +0.5.5/x_2 + 0.7/x_3 +0.7.5/x_4 +0.8/x_5$, $0.4/t_1 +0.6/t_2 + 0.7/t_3 +0.8/t_4 +0.85/t_5$ }
= $0.4/t_1 +0.55/t_2 + 0.7/t_3 +0.7.5/t_4 +0.8/t_5$

The fuzzy temporal propositions like "x was A" may contain quantifiers like "very ", "More or Less" etc. These fuzzy quantifiers may be eliminated as

$\mu_{very\ richXpast}(x) = \mu_{richXpast}(x)^2$ Concentration
$\mu_{more\ or\ less\ richXpast}(x) = \mu_{richXpast}(x)^{0.5}$ Diffusion

The operations on fuzzy temporal are similar to fuzzy sets type-2 are given as

ĈVĎ=max{$\mu_{\hat{C}}(x,t)$, $\mu_{\check{D}}(x,t)$} Disjunction(Ĉ overlap Ď)
ĈΛĎ=min{$\mu_{\hat{C}}(x,t)$, $\mu_{\check{D}}(x,t)$} Conjunction(Ĉ before Ď)
Ĉ→Ď=min{1, 1-$\mu_{\hat{C}}(x,t)$ +$\mu_{\check{D}}(x,t)$}Implication(Ĉ proceeds Ď)
ĈxĎ=min{$\mu_{\hat{C}}(x,t)$,$\mu_{\check{D}}(x,t)$} Relation

IV. FUZZY MODULATIONS

The Automated Fuzzy Reasoning System is problem solving system using fuzzy reasoning with fuzzy facts and rules. These fuzzy facts and rules are modulated to represent the Knowledge available to the system. The fuzzy agent is independent component which performs fuzzy reasoning.

The fuzzy modulations for Knowledge representation are type of modules for fuzzy propositions "x is A". "x is A" is defined as

[A]R(x),

where A is fuzzy set, R is relation and x is individual in the Unversed of discourse X.

For instance
"Rama is tall "represented as
[tall]Hight(Rama), where "tall" is fuzzy set, "Hight" is relation and "Rama" is individual.

The fuzzy modules[13] are knowledge representation technique of the fuzzy propositions. The Fuzzy Modulations are combined with logical operators.

Let A and B be fuzzy sets.
x is ¬A
[¬A]R(x)
x is A or x is B
[A V B]R(x)
x is A and x is B
[A Λ B]R(x)
if x is A then x is B
[A → B]R(x)

Some of the Fuzzy Reasoning rules are given as

R1: [A]R(x)
[B](R(x) or R(y))
____
[AΛB]R(y)

R2: [A]R(x)
[B](R(x) or R(y)
____
[AVB]R(y)

R3: [A](R(x,y)
[B](R(y,z)
____
[AΛ B](R(x,z)

R4: [A](R(x) or R(y))
[B](R(y) or R(z))
____
[AV B](R(x) or R(z))

R5: : [A]R(x)
if [A]R(x) then [B]R(y)
____

[[Ao (A→B)]R(y)

Patient has cold

If Patient has cold then Patient has headache

The inference is given as using the above fuzzy fact and fuzzy rule

[cold] symptom(Patient)

if [cold] symptom(Patient) than [headache]symptom(Patient)

The fuzzy reasoning is given as using Fuzzy Knowledge Base

[cold] ^[cold → headache] symptom (Patient, headache

V. FUZZY PREDICATE LOGIC

The Predicate is Relation with arguments. The arguments may be variables or constants.

The Predicate Logic is a combination of predicates with logical operators 'not', 'and', 'or', implication', for every ' 'there exists' etc.

Marcus was a man

Man(Marcus)

All man are people

$\forall$ Man(x)→person(x)

Every one loyal to every one

$\forall$ x $\exists$ y loyal(x,y)

Basic resolution

p

p→q= (¬p v q)

---

q

For instance

winter

winter → cold

---

cold

p v q

¬p v r

---

q v r

for instance

winter v summer

¬summer v cold

Winter v cold

The fuzzy predicate may be defined as relation with arguments and the arguments may be constants or variables

R(x,A)

For instance,

x is tall

Height(x,tall)

Rama is tall with fuzziness 0.7

Height(Rama,0.7)

where A is fuzzy set, R is relation and x is individual in the Unversed of discourse X.

For instance

"Rama is tall " may be modulated as

[tall]Hight(Rama)

The fuzzy predicate is given by

Hight(Rama, tall)

where "tall" is fuzzy set, "Hight" is relation and "Rama" is individual.

The fuzzy propositions may contain quantifiers like "for every", "there exits" "very ", "More or Less" etc. These fuzzy quantifiers may be eliminated as

$\mu_{very}(x) = \mu_A(x)^2$

for instance,

young= $0.8/x_1 + 0.7/x_2+0.7/x_3+0.6/x_4+0.6/x5$

very young= $0.64/x_1 +0.49/x_2+0.49/x_3+0.36/x_4+0.36/x_5$

$\mu_{more\ or\ less}(x) = \mu_A(x)^{0.5}$

more or less young

$=0.89/x_1 +0.83/x_2+0.83/x_3+0.77/x_4+0.77/x_5$

$\forall$ x (if x is A then x is B)

$\min_x$ (R(x,A)→R(y,B))

Consider the fuzzy proposition

"for every young person runs fast" may be given as

$\forall$ x (age(x, young)→run(x,fast)

$\min_x$ (age(x, young)→run(x,fast)

For instance,

young= $0.8/x_1 + 0.7/x_2+0.7/x_3+0.6/x_4+0.6/x_5$

fast= $0.2/x_1 + 0.3/x_2+0.4/x_3+0.5/x_4+0.4/x_5$

young→fast = $0.4/x_1 + 0.7/x_2+0.8/x_3+0.9/x_4+.8/x_5$

$\min_x$ {young→fast} =

$\min_x \{0.4/x_1 + 0.7/x_2+0.8/x_3+0.9/x_4+0.8/x_5 \}=0.4/x_1$

$\exists$ x (if x is A then x is B)

$\max_x$ (R(x,A)→R(y,B))

Consider the fuzzy proposition

"there exists young person who runs fast" may be given as

$\exists$ x (age(x, young)→run(x,fast)

$\max_x$ (age(x, young)→run(x,fast)

For instance,

young= $0.8/x_1 + 0.7/x_2+0.7/x_3+0.6/x_4+0.6/x_5$

fast= $0.2/x_1 + 0.3/x_2+0.4/x_3+0.5/x_4+0.4/x_5$

young→fast = $0.4/x_1 + 0.7/x_2+0.8/x_3+0.9/x_4+.8/x_5$

$\max_x$ {young→fast} =

$\max_x \{0.4/x_1 + 0.7/x_2+0.8/x_3+0.9/x_4+0.8/x_5 \}=0.9/x_4$

The fuzzy reasoning is drawing conclusions from the fuzzy propositions. Some of the Fuzzy Reasoning rules are given as

R1: [A]R(x,A)

R(x, B) and R(y,B)

R(y, A∧B)

R2: R(x, A)

R(x,B) or R(y,B)

R(y, AV B)

R3: R(x,y,A)

R(y,z,B)

R(x,z, A∧ B)

R4: R(x,A) or R(y,A)

R(y,B) or R(z,B)

R(x, AV B) VR(z, AV B))

R5: :R(x,A)

if R(x, A) then R(y,B)

R(y,Ao A→B)

Patient has cold

If Patient has cold then Patient has headache

The inference is given as Using the above fuzzy fact and fuzzy rule

symptom(Patient, cold)

if symptom(Patient, cold) than symptom(Patient, headache)

The fuzzy reasoning is given as using Fuzzy Knowledge Base

[cold] ^[cold → headache] symptom (Patient, headache)

The fuzzy logic and fuzzy reasoning are discussed in the following for the FPL.

VI. Fuzzy Temporal AGENT

The Fuzzy Temporal Agent is an Automated Fuzzy Reasoning System with fuzzy temporal facts and fuzzy temporal rules. These fuzzy temporal facts and fuzzy temporal rule are modulated to represent Knowledge to the system. The Fuzzy Temporal Agent is independent component which performs fuzzy reasoning is shown in Fig.1

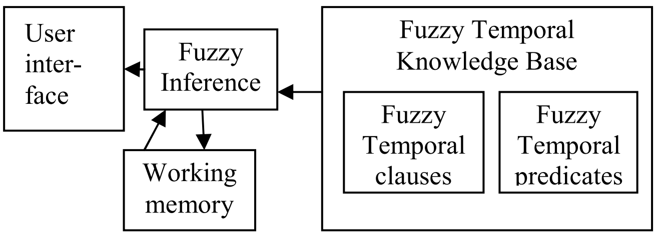

Fig.1 Fuzzy Temporal Agent

Definition: The fuzzy temporal modulations are the possibility distribution.

[A]R(x),

where A is fuzzy set, R is relation and x is individual in the Unversed of discourse X.

For instance

"x was rich" may be modulated as represented as

[rich]wealth(x), where "young" is fuzzy set, "wealth" is relation and "x" is individual.

The fuzzy temporal modules are type of knowledge representation technique for the fuzzy temporal propositions. The fuzzy temporal modulations are combined with logical operators.

Let A and B be fuzzy temporal sets. The fuzzy temporal operations are given as

x is A

[A]R(x)

x is ¬A

[¬A]R(x) Negation

x is A or y is B

[A]R(x)V [B]R(y) Disjunction

x is A and x is B

[A Λ B]R(x)

if x is A then y is B

[A]R(x)→ [B]R(y) Implication

[A']R(x) o [A]R(x)→ [B]R(y) Comosition

Consider the two propositions

x is more or less late

If x comes late then y comes very late

The inference is given as using the above fuzzy fact and fuzzy rule

[very late] arrival(x)

[more or less late] arrival(x) → [livery late] arrival(y)

The fuzzy reasoning is given as

[very late]arrival(x) o [more or less late] arrival(x) → [livery late] arrival(y)

VII. Fuzzy Temporal Predicate Logic

First Order Fuzzy Temporal Predicate Logic is transformation of fuzzy temporal modulations.

Consider the fuzzy temporal proposition of type "x was A" may be modulated as

[Ã]R(x)

The fuzzy temporal predicate may be defined as relation with arguments and the arguments may be constants or variables

R(x,Ã)

For instance,

x was rich

wealth t(x,richXpast)

The fuzzy temporal predicate logic is the combination of fuzzy temporal predicate with logical operators.

Let A and B be fuzzy sets.

x is A

R(x, A)

arrival(x,late)

x is not A

R(x, not A)

arrival(x,not late)

x is A or y is B

R(x, A) V R(y,B)

arrival(x, late) V arrival(y, very late)

x is A and y is B

R(x, A) Λ R(y,B)

arrival(x, late) Λ arrival(y, very late)

if x is A then y is B

R(x,A)→R(y,B)

arrival(x,late) → arrival(y very late)

Reasonig is drawing conclusion. The reasoning with fuzzy temporal predicate logic is given as

x is late

if x comes late then y comes very late

The fuzzy temporal fact and rule may be modulated as

arrival(x, late)

arrival(x, more or less late) → arrival(y, very late)

arrival(x, late)o arrival(x, more or less late) → arrival(y, very late)

arrival(y, lateo(more or less late → very late))
let fuzziness late =0.8
arrival(y, 0.8o(0.89 → 0.64))
arrival(y, min{0.8, min(1, 1-(0.89 + 0.64))
arrival(y, min{0.8,0.53)
arrival(y, 0.53)
i.e., y is late with fuzziness 0.53

VIII. PROLOG FOR FUZZY TEMPORAL PREDICATE LOGIC

The Prolog is mainly used for Predicate Logic. The Prolog is a Logic Programming language [2]. It contains mainly predicates and Clauses.

A predicate is a relation with name of the relation and arguments. The arguments may be containing variables or constants.

For instance,

arrival(x, late)

A clause is combination of and/ or more predicates for the rules

arrival(y, very late) :-arrival(x, more or less late)

Consider the logic programming for fuzzy temporal logic

x is late

if x comes late then y comes very late

arrival(x, late)

arrival(x, more or less late) → arrival(y, very late)

arrival(x, late)o arrival(x, more or less late) → arrival(y, very late)

arrival(y, lateo(more or less late → very late))

The fuzzy reasoning may be given as

arrival(x, late)

arrival(y, very late) :-arrival(x, more or less late)

---

arrival(y, lateo(more or less late → very late))

The Logic Programming may be written in SWI-Prolog as

```
fuzzy(A,B,M) :- A >  B, M is 1-A+B.
fuzzy(A,B,M) :- M=1.
fuzzy1(C,M,F):-C<M,F is C.
C:-0.8.
A:-089
B:-0.64.
fuzzy1(C,M,F):-F=M.
arrival(x,late, A).
arrival(y, late,  B).
arrival(x,late, C).
arrival(y, late,  M):- arrival(x,late, A).
arrival(y, late ,F) :- arrival(y,late,M), arrival(x,late,C), fuzzy1(C,M,F).
```

The output may be given for input more or less late =A=0.8, very late=B=0.64, and late =C=0.8

?-arrival(y,late,F).

F=0.53

Yes

The prolog will give a fuzziness is 0.64 for "y is late".

IX. CONCLUSION

The AI Problems may contain temporal constraints with incomplete information. The Knowledge Representation is the key factor for problem representation In AI. The traditional predicate logic is difficult to study with the fuzzy temporal constraints. The fuzzy temporal logic is studied for reasoning in AI problems with incomplete information time constraints. The fuzzy temporal agent is independent intelligent problem solving system with time constraints and incomplete information for Automation. The fuzzy temporal modulations are discussed for fuzzy temporal predicate logic to represent incomplete information of time constraints . The Prolog is discussed for fuzzy temporal agent for automated reasoning.

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