A Performance Comparison of Several Lie Group Integration Methods for Solving the Equations of Constrained Multibody Dynamics

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Abstract

When expressed in absolute coordinates, the constrained multibody dynamics problem results in a system of index-3 differential algebraic equations (DAEs). In [1, 2, 3, 4, 5], we explored three Lie group integration methods for solving these DAEs: fully-implicit, half-implicit, and velocity coordinate partitioning. It was found that these methods are computationally more efficient in approximating the solution to the equations of motion than methods which rely on Euler angles or Euler parameters as generalized coordinates. In this work, we directly compare the accuracy and performance of the three Lie group approaches to solving the index-3 DAEs. The Python code developed to generate the reported results is open-source and available in a public repository for reproducibility studies [6].

Methods comparison

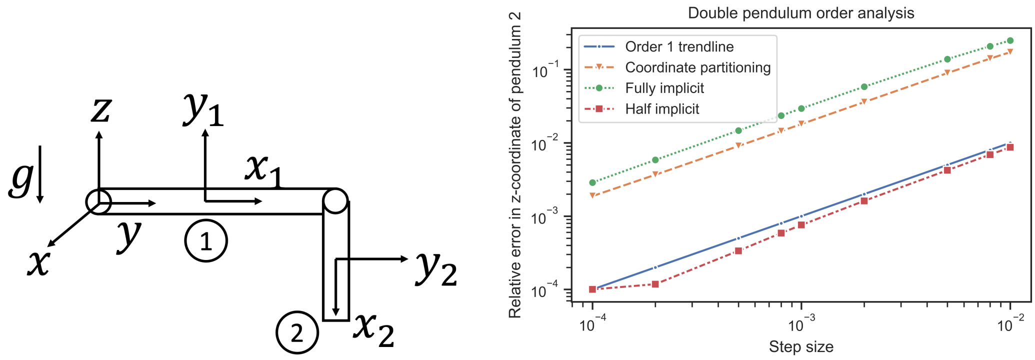
Implementation details for the three methods can be found in [1, 2, 3, 4, 5]. In the results reported below, a tolerance of $\varepsilon_{tol} = 10^{-5}$ was used for all numerical experiments. In [4], a similar scaling analysis was done to compare the fully- and half-implicit integrators. Therein, the tolerances were chosen to ensure that the methods achieved a similar quality of solution.

Convergence analysis: The fully-implicit, half-implicit, and coordinate partitioning methods employ first-order Euler explicit and/or implicit integration schemes. We confirmed their first-order accuracy by conducting an order analysis on the double pendulum in Fig. 1a with physical properties that match those used in [1, 2, 3, 4, 5]. A reference solution was obtained by integrating a set of ordinary differential equations (ODEs) using a highly accurate, 8th order Runge-Kutta solver DoPri853 with a step size of $h = 1 \times 10^{-6}$s. Relative errors were calculated as $||y_i - y_i^{\text{ref}}||_2/||y_i^{\text{ref}}||_2$, where y_i and y_i^{ref} are the values of the solution at time t_i produced by the Lie group integrator and the reference solution, respectively. The relative errors in position were plotted as a function of the step size h on a log-log scale in Fig. 1b. The half-implicit integrator is shown to be significantly more accurate, very likely due to its symplectic property (detailed in [4]).

Scaling Analysis: A scaling analysis was performed using a N-body pendulum to compare the computational performance of the three methods. As shown in Fig. 2a, N rigid links were connected by spherical joints and subjected to a randomly directed gravitational force $\vec{g}$. At time $t = 0$, the links are assigned an angular velocity of random magnitude $\vec{\omega}_i$ around their longitudinal axis. Tests were conducted for $N = 1, 2, 4, 8, 16, 32$. The timing results reported in Fig. 2 indicate that the half-implicit method is the most time-efficient. This computational advantage is likely due to its Newton iteration matrix, which is both more accurate and less costly to compute, as discussed in [4]. Conversely, the coordinate partitioning approach demonstrated poor scalability compared to the fully- and half-implicit methods. This result is mainly linked to the increasing cost of the partitioning step as the number of system degrees of freedom grows [5].

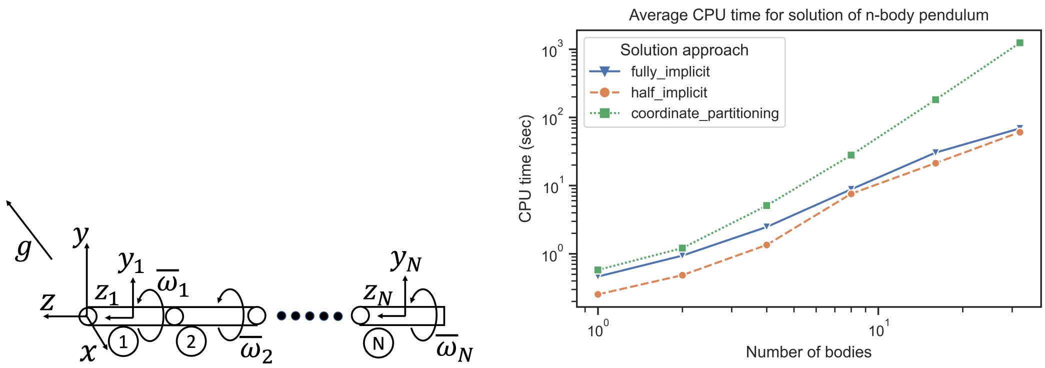
Discussion. Future work

When comparing the three Lie group methods, the half-implicit solver stands out in both speed and accuracy. While it was only marginally faster than the fully-implicit solver in the scaling analysis, it is assumed that the higher accuracy reported above translates to the N-body pendulum. Thus, a looser tolerance and/or larger step size could achieve faster simulation times depending on solution accuracy requirements. On the other hand, the coordinate partitioning method, while slower than the direct integration methods, offers advantages in applications which favor ODEs [5]. A similar comparative analysis will be presented for a seven-body mechanism to provide insights for a complex closed-loop system, but it has been omitted here due to space limitations.



(a) Double pendulum schematic. (b) Order analysis; position coordinate.

Figure 1: Integrator order analysis using a double pendulum.



(a) N-body pendulum schematic. (b) N-body pendulum scaling results; log-log.

Figure 2: Scaling analysis using a N-body pendulum.

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