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Abstract

This article is based on the work of (Ferreira, Filipe and Coelho, 2016) presented at International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XVIII). There using results on the infinite servers queue systems with Poisson arrivals - M|G|∞ queues - busy period, it is presented an application of those queue systems in the unemployment periods time length parameters and distribution function study. These queue systems are adequate to the study of many population processes, and this quality is brought in here. The results presented are mainly on unemployment periods length and their number in a certain time interval. Also, some questions regarding the practical applications of the outlined formulas are briefly discussed.

1   THE MODEL RISING

In the queue systems used thoroughgoing this article the customers arrive according to a Poisson process at rate λ, receive a service which time length is a positive random variable with distribution function $G(.)$ and mean α, upon they arrive each one finds immediately an available server[1], each customer service is independent from the other customers' services and from the arrivals process and the traffic intensity is $\rho = \lambda\alpha$. That is: they are $M|G|\infty$ queues. It is easy to understand how these queues can be applied to the unemployment study. Then: λ is the rate at which occur the firings, supposed to happen according to a Poisson process and the service time, paradoxically, is the time between the worker firing and the time he/she finds a new job.

In any queue system, a busy period is a period that begins when a costumer arrives at the system finding it empty, ends when a costumer abandons the system letting it empty and there is always at least one customer present. So in a queuing system there is a sequence of idle and busy periods, during its operation time. In the $M|G|\infty$ queue system, as in any queue system with Poisson arrivals, the idle periods have an exponentially distributed length with mean λ^{-1}. But the busy period's distribution is much more complicated, being in general given by

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[1] Or there is no distinction between the customer and its server, as it happens in the application to be considered in this work.

infinite sums which parcels are convolutions (Ferreira and Andrade, 2009). In spite of it, it is possible to present some results as it will be seen. For what interests in this work: a busy period is a period of unemployment and an idle period is a period of full employment.

The results to be presented are on unemployment periods length and their number in a certain time interval. A study in following also this approach, over a public health situation, can be seen in Ferreira (2014).

2   UNEMPLOYMENT PERIOD TIME LENGTH DISTRIBUTION

Designate D the random variable unemployment period length. According to the results known for the $M|G|\infty$ queue busy period length distribution, see (Ferreira and Andrade, 2009),

$$E[D] = \frac{e^{\rho} - 1}{\lambda} \qquad (2.1),$$

whichever is the worker unemployment time length distribution, see Takács (1962).

As for $Var[D]$, it depends on the whole unemployment time length distribution probabilistic structure. But Sathe (1985) demonstrated that

$$\max \left[e^{2\rho} + e^{\rho}\rho^2\gamma_s^2 - 2\rho e^{\rho} - 1; 0 \right] \leq \lambda^2 Var[D] \leq \left[2e^{\rho}(\gamma_s^2 + 1)(e^{\rho} - 1 - \rho) - (e^{\rho} - 1)^2 \right],$$

$$(2.2)$$

where γ_s is the unemployment time length coefficient of variation.

For α and ρ great enough (very intense unemployment conditions) since $G(.)$ is such that for α great enough $G(t) \cong 0, t \geq 0$,

$$D(t) \cong 1 - e^{-\lambda e^{-\rho}t}, t \geq 0 \qquad (2.3),$$

see (Ramalhoto and Ferreira, 1994).

Calling N_D the mean number of unemployed people in the unemployment period, if $G(.)$ is an exponential distribution:

$$N_D = e^{\rho} \qquad (2.4).$$

For any other $G(.)$ probability distribution

$$N_D \cong \frac{e^{\rho(\gamma_s^2+1)}\left(\rho(\gamma_s^2+1)+1\right)+\rho(\gamma_s^2+1)-1}{2\rho(\gamma_s^2+1)}$$

(2.5),

see (Ferreira and Filipe, 2010). Of course, multiplying (2.4) or (2.5), as appropriate, by the mean cost of each unemployment subsidy it is possible to estimate the assistance costs caused by the unemployment period.

3 UNEMPLOYMENT PERIODS IN A TIME INTERVAL MEAN NUMBER

After the renewal processes theory, see Çinlar (1975), calling $R(t)$ the mean number of unemployment periods that begin in $[0,t]$, being $t=0$ the beginning instant of an unemployment period, it is possible to obtain, see Ferreira (1995),

$$R(t) = e^{-\lambda\int_0^t[1-G(v)]dv} + \lambda\int_0^t e^{-\lambda\int_0^u[1-G(v)]dv}\,du \qquad (3.1)$$

and, consequently,

$$e^{-\rho}\left(1+\lambda t\right) \le R(t) \le 1+\lambda t \qquad (3.2),$$

see Ferreira (2004).

4 CONCLUSIONS

So that this model can be applied it is necessary that the firings occur according to a Poisson process at constant rate. It is a hypothesis that must be tested. Thus remain outside of this study, periods of unemployment caused by mass firings.

Among the results presented, (2.1), (2.2), (2.3), (2.5) and (3.2) are remarkable for its simplicity and also for requiring only the knowledge of the firings rate λ, the mean unemployment time α, and the unemployment time variance.

References

[1] Çinlar, E.: *Introduction to Stochastic Processes*. Prentice Hall, Englewood Cliffs, NJ.1975.
[2] Ferreira, M. A. M.: Um sistema M|G|∞ com período de ocupação exponencial. In: *Actas das XV Jornadas Luso-Espanholas de Matemática* (C. Braumann, ed.), Vol. IV, Universidade de Évora, Évora, 1991, D63-D66.
[3] Ferreira, M. A. M.: *Comportamento transeunte e período de ocupação de sistemas de filas de espera sem espera*. PhD thesis. ISCTE-IUL, Lisboa,1995.
[4] Ferreira, M. A. M.: $M\,|\,G\,|\,\infty$ queue busy cycle renewal function for some particular service time distributions. In: *Proceedings of the International Conference Quantitative*

Methods in Economics (Multiple Criteria Decision Making XII) (M. Lukáèik, ed.), Virt, Slovakia, 2004, 42-47.

[5] Ferreira, M. A. M. and Andrade, M.: The Ties between the $M|G|\infty$ Queue System Transient Behaviour and the Busy Period. *International Journal of Academic Research*, **1**(2009), 84-92.

[6] Ferreira, M. A. M. and Filipe, J. A.: Economic Crisis: Using $M|G|\infty$ Queue Systems Busy Period to solve Logistics Problems in an Organization. *China-USA Business Review*, **9** (2010), 59-63.

[7] Ferreira, M. A. M., Filipe, J. A. and Coelho, M.: Unemployment Modelled Through M|G|∞ Systems. *International Journal of Latest Trends in Finance and Economic Sciences*, **4**(2014), 677-679.

[8] Ferreira, M. A. M.: The pandemic period length modelled through queue systems. In: *Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XVII)* (R. Marian, P. Gezik, and L. Martin, eds.), Virt, Slovakia, 2014, 43-47.

[9] Ferreira, M. A. M., Filipe, J. A. and Coelho, M.: Unemployment period quantitative approach through infinite servers queue systems. In*: Proceedings of the International Conference Quantitative Methods in Economics (Multiple Criteria Decision Making XVIII)* (R. Marian and P. Gezik, eds.), Vratna, Slovakia, 2016, 89-94.

[10] Ramalhoto, M. F. and Ferreira, M. A. M.: Some further properties of the busy period of a $M|G|\infty$ queue. *Central European Journal for Operations Research and Economics*, **4**(1996), 251-278.

[11] Sathe, Y. S.: Improved bounds for the variance of the busy period of the $M|G|\infty$ queue. *Advances in Applied Probability*, **17**(1985), 913-914.

[12] Takács L.: *An introduction to queueing theory*. Oxford University Press. 1962.