High-g Barrel Roll Maneuvers Against Proportional Navigation

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Abstract*— This research explores the effectiveness of high-g barrel roll maneuvers as a defensive tactic against proportional navigation (PN) guidance in missile-target engagements. Proportional navigation, widely used in missile guidance systems, aims to steer the missile along a collision course with a target by adjusting its path based on the line-of-sight rate. However, evasive barrel roll maneuvers, which involve high lateral accelerations and rapid changes in direction, can significantly challenge the missile's ability to maintain an effective trajectory toward the target. This research analyzes the dynamics of high-g barrel rolls and evaluates how these maneuvers affect the missile's interception performance under different navigation constants. Through simulation, we examine how variations in the missile's response to the target's maneuvers influence interception accuracy and terminal phase stability. The findings provide valuable insights into missile-target engagements, highlighting the potential of high-g maneuvers as a countermeasure to PN guidance and offering implications for defensive strategy development and missile guidance optimization.

I. Introduction

The effectiveness of air combat tactics hinges on a pilot's ability to evade incoming threats, particularly guided missiles. One of the most sophisticated guidance systems employed by modern missiles is proportional navigation, which allows the missile to adjust its trajectory based on the relative motion between itself and its target. This approach aims to minimize the time to intercept by constantly refining its flight path to maintain a consistent line-of-sight angle to the target [1]. In this context, high-g barrel roll maneuvers emerge as a critical defensive tactic. By executing rapid and dynamic changes in flight path, these maneuvers exploit the limitations of proportional navigation systems. The objective is to create an unpredictable flight trajectory that can confuse the missile's guidance calculations, making it difficult for the missile to accurately predict and intercept the target [2].

II. Methodology

This study employs a simulation-based approach to investigate the effectiveness of high-g barrel roll maneuvers as a countermeasure against proportional navigation (PN) guidance in missile-target engagements. First, we model the missile's PN guidance system, where the missile's acceleration command is proportional to the rate of change in the line-of-sight angle to the target. Different navigation constants (gains) are tested to observe how variations in PN parameters impact the missile's performance against evasive maneuvers. The target's high-g barrel roll maneuver is designed to create rapid lateral accelerations and unpredictable changes in direction, characterized by a spiral motion with high lateral g-forces. This maneuver is specifically structured to exploit the limitations of PN guidance, and parameters such as roll frequency, radius, and peak acceleration are adjusted to simulate realistic high-g conditions [3]. A three-dimensional missile-target engagement scenario is set up, with predefined initial conditions for both missile and target, including velocity, position, and heading. The simulation tracks the missile's trajectory in response to the target's maneuver, with key metrics like miss distance, terminal phase error, and missile acceleration responses recorded. To understand the impact of the PN guidance constant (τ) on interception performance, simulations are conducted with varying values of τ, providing insight into the sensitivity of the missile's interception capability when facing a high-g barrel roll [4].

The simulation data are then analyzed to evaluate the effectiveness of high-g maneuvers in increasing miss distance and preventing interception. Metrics such as miss distance, required missile acceleration, and time-to-hit are assessed, along with trajectory plots and command accelerations (n_c) in both vertical and horizontal planes to interpret the missile's response to the evasive maneuver. Finally, sensitivity analysis is conducted to validate the model, varying key parameters of both the missile's guidance system and the target's barrel roll characteristics. This approach ensures robustness of the findings and identifies optimal evasive parameters for the target, as well as critical weaknesses in PN guidance. Through this methodology, the study provides insights into how high-g barrel rolls can exploit PN guidance limitations, highlighting potential defensive strategies and opportunities for improving missile guidance algorithms.

III. Three Dimensional Simulation of HGP Manuever

Consider an inertial coordinate system fixed to the surface of a flat -earth model (i.e. the 1-axis is down range, 2-axis is cross range, and 3-axis is altitude range). The 2- 3 plane will be called the roll plane. And 1-2 plane will be called longitudinal plane. For simplicity, it is assumed that both the missile and target travel at constant velocity. gravitational and drag force are neglected The missile demanded normal accelerations are given by [5]:

$$N_{C1} = N^{`} V_C \dot{\lambda_1} \qquad (1)$$

$$N_{C2} = N`V_C\dot{\lambda}_2 \quad (2)$$

Where N_C are the acceleration commands, N' is a unit less designer chosen gain (usually in the range of 3-5), V_c is the missile target closing velocity, and λ is the line of sight angle.

In the engagement model of the target produce barrel roll maneuver with lateral acceleration N_C and barrel roll rate perpendicular to the target velocity vector, the angular velocities of the target can be expressed as.

$$\dot{\beta}_1 = \frac{N_1}{V_{T1}} \quad (3)$$

$$\dot{\beta}_2 = \frac{N_1}{V_{T3}} \quad (4)$$

The components of the target velocity vector in the Earth coordinate system are

$$V_{T1} = -V_T \quad (5)$$

$$V_{T2} = \frac{N_T}{\omega}\sin\omega t \quad (6)$$

$$V_{T3} = -\frac{N_T}{\omega}\cos\omega t \quad (7)$$

The differential equations of the target position in the earth coordinate system given by

$$\dot{R}_{T1} = V_{T1} \quad (8)$$

$$\dot{R}_{T2} = V_{T2} \quad (9)$$

$$\dot{R}_{T2} = V_{T2} \quad (10)$$

Similarly, the missile velocity and position differential equations are given by

$$\dot{V}_{M1} = A_{M1} \quad (11)$$

$$\dot{V}_{M2} = A_{M2} \quad (12)$$

$$\dot{V}_{M3} = A_{M3} \quad (13)$$

$$\dot{R}_{M1} = V_{M1} \quad (14)$$

$$\dot{R}_{M2} = V_{M2} \quad (15)$$

$$\dot{R}_{M3} = V_{M3} \quad (16)$$

Where A_{M1}, A_{M2}, A_{M3}, $\dot{V}_{M1}$, $\dot{V}_{M2}$, $\dot{V}_{M3}$ are the acceleration and velocity components in the earth coordinate system. The components of the relative missile-target separation are

$$R_{TM1} = R_{T1} - R_{M1} \quad (17)$$

$$R_{TM2} = R_{T2} - R_{M2} \quad (18)$$

$$R_{TM3} = R_{T3} - R_{M3} \quad (19)$$

The LOS angles can be expressed as

$$\lambda_1 = \tan^{-1}\left(\frac{R_{TM3}}{\sqrt{(R_{TM1})^2 + (R_{TM3})^2}}\right) \quad (20)$$

$$\lambda_2 = \tan^{-1}\frac{\sqrt{(R_{TM1})^2 + (R_{TM3})^2}}{R_{TM2}} \quad (21)$$

The relative velocity components in Earth coordinate system

$$V_{TM1} = V_{T1} - V_{M1} \quad (22)$$

$$V_{TM1} = V_{T1} - V_{M1} \quad (23)$$

$$V_{TM3} = V_{T3} - V_{M3} \quad (24)$$

Thus, the closing velocity can be expressed as

$$V_C = -R_{TM}\frac{R_{TM1}V_{TM2} + R_{TM2}V_{TM2} + R_{TM3}V_{TM3}}{R_{TM}} \quad (25)$$

and the components of the missile acceleration are

$$A_{M1} = -N_{C1}\sin\lambda_1 + N_{C2}\cos\lambda_2\cos\lambda_1 \quad (26)$$

$$A_{M2} = -N_{C2}\sin\lambda_2 \quad (27)$$

$$A_{M3} = N_{C1}\cos\lambda_1 + N_{C2}\cos\lambda_2\sin\lambda_1 \quad (28)$$

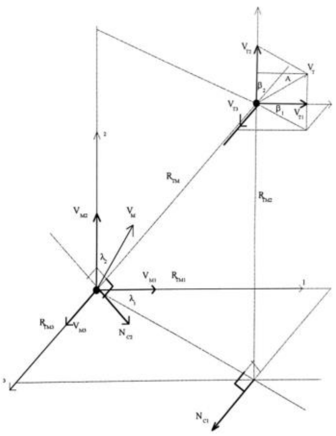

Fig. 1. Three-dimensional missile-target engagement geometry

IV. Simulation Results

A. Non-maneuvering target scenario

Table I shows the states of the missile and target at the beginning of the simulation

TABLE I. NON-MANEUVERING TARGET SCENARIO STATES

Missile		Target	
Initial Position	(0,490,0) m	Initial position	(1600,150,1000) m
Velocity	2000 m/s	Velocity	1000 m/s
N	5	n_t	0
ω	0.4	β	0

The fig.2 shows the result of the interception

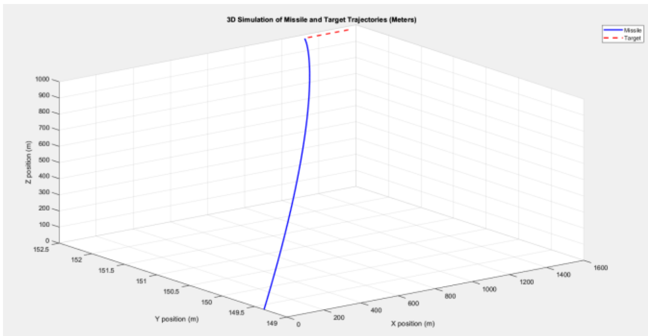

Fig. 2. Trajectory of the missile and target in case of non-maneuver.

A key observation from the 3D simulation of the missile and target trajectories is that the missile follows a curved path in its attempt to intercept the target, which moves in a straight line along the x-axis. The missile's trajectory curves upward as it adjusts its course using proportional navigation to reduce the relative distance to the target. However, the separation between the two indicates that the missile's lateral acceleration may not be sufficient to close the distance quickly. This demonstrates that even against a non-maneuvering target, interception accuracy is highly dependent on factors like the missile's guidance law, navigation constants, and the relative velocities of the missile and target.

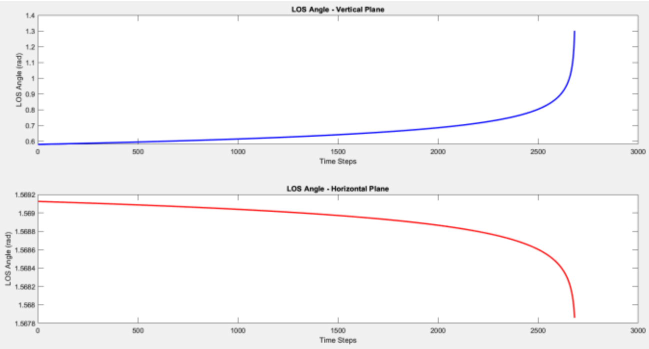

Fig. 3. Line of sight in case of non-maneuvering target

The fig.3 shows the Line-of-Sight (LOS) angles for both the vertical and horizontal planes during a missile's pursuit of a non-maneuvering target. The bottom graph, which depicts the LOS angle in the horizontal plane, shows a gradual decrease in the LOS angle over time, with a sharp drop near the end of the trajectory. This gradual change indicates that the missile is adjusting its trajectory in the horizontal plane to maintain pursuit of the target. The sharp decrease at the end suggests that the missile is nearing interception or the final phase of its trajectory, where the guidance system is actively steering to minimize the remaining distance to the target.

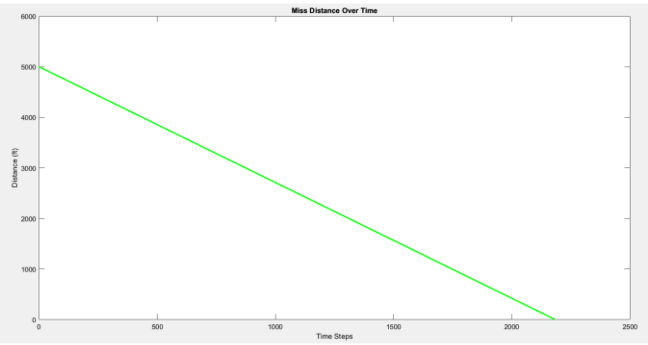

Fig. 4. Miss distance over time during simulation.

The Fig.4 displays the "Miss Distance Over Time" for the missile-target interception scenario. The initial distance between the missile and the target starts at approximately 5000 m. As time progresses, the distance decreases linearly until it reaches zero, indicating successful interception.

B. HGB maneuver scenario

Table II shows the states of the missile and target at the beginning of the simulation

TABLE II. HGB MANEUVERING TARGET SCENARIO STATES

Missile		Target	
Initial Position	(0,0,0) m	Initial position	(50000,10000,0) m
Velocity	2000 m/s	Velocity	1000 m/s
N	5	n_t	9
ω	0.4	β	0

Fig.5 shows the simulated trajectories of the missile and target in a scenario involving the HGB maneuver.

The missile's trajectory, shown by the blue line, responds to the HGB's maneuvers by attempting to follow and match its path. The missile starts with a more gradual approach but eventually begins to mirror some of the target's sharp turns as it closes in on the target. The complex and looping shape of the missile's trajectory suggests that it has to continuously adjust its path to maintain pursuit, showing the effectiveness of the missile's guidance system in tracking a highly maneuverable target like the HGB.

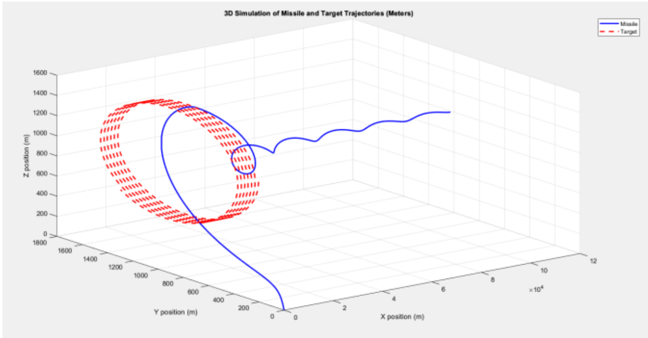

Fig. 5. Trajectory of the missile and target in case of HGB.

Fig.6 depict the Line of Sight (LOS) angle rates for the vertical and horizontal planes during an HGB maneuver. These angle rates are crucial for understanding how the missile adjusts its trajectory in relation to the moving target. In the vertical plane (upper plot), the LOS angle begins with a sharp drop and then oscillates with diminishing amplitude as time progresses. The initial rapid variation signifies a strong correction in the missile’s trajectory in response to the HGB’s initial movement. As the time steps increase, the oscillations smooth out, indicating that the missile is gradually stabilizing its course, but still adjusting to minor fluctuations in the target's motion. The horizontal plane (lower plot) shows a similar trend, with pronounced oscillations in the LOS angle, although it starts from a slightly different initial angle compared to the vertical plane. The magnitude of oscillations is more significant here, likely reflecting the higher degree of lateral movement in the target's trajectory. The consistent oscillations suggest that the missile is continuously adjusting its heading to align with the evasive maneuvers performed by the HGB.

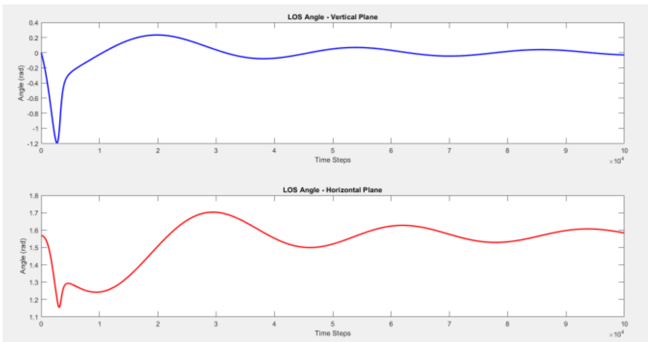

Fig. 6. Line of sight in case of HGB maneuvering.

Fig.7 shows the "Miss Distance Over Time," illustrating the distance between the missile and the target during the engagement. The miss distance starts at 5000 ft and decreases gradually over time, eventually reaching 1000 ft. However, the fact that the missile doesn't reduce the miss distance to zero indicates that the missile fails to hit the target. Instead, the engagement ends with a substantial miss distance of 1000 m, suggesting that the target successfully evaded interception.

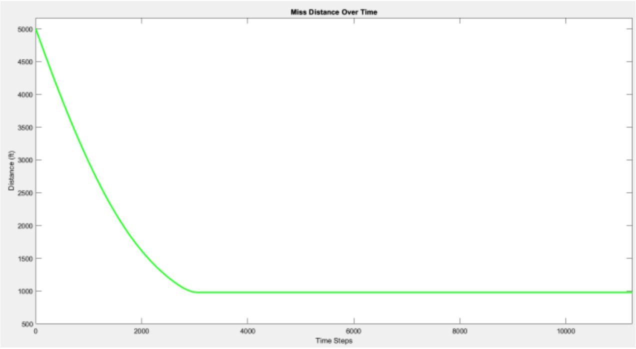

Fig. 7. Miss distance over time during simulation.

C. *Effect of Target Maneuver Frequency w on Missile Interception*

TABLE III. HGB MANEUVERING TARGET SCENARIO STATES

Missile		Target	
Initial Position	(0,0,0) m	Initial position	(5000,500,1000) m
velocity	2000 m/s	Velocity	1000 m/s
N	5	n_t	9
ω	1, 0.2	β	0

Fig.8 illustrates a missile's Proportional Navigation (PN) trajectory in the longitudinal plane (X-Z plane), where the missile attempts to intercept a maneuvering target. The missile's path (blue line) shows a gradual adjustment towards the target's trajectory (red dashed line), which ascends smoothly. With a target movement frequency of w=1, the target executes a moderate sinusoidal maneuver, allowing the missile to follow a relatively smooth intercept course. The navigation scaling factor, nt=9, causes the missile to respond aggressively to changes in the target's movement, resulting in a tighter and more accurate intercept path towards the end.

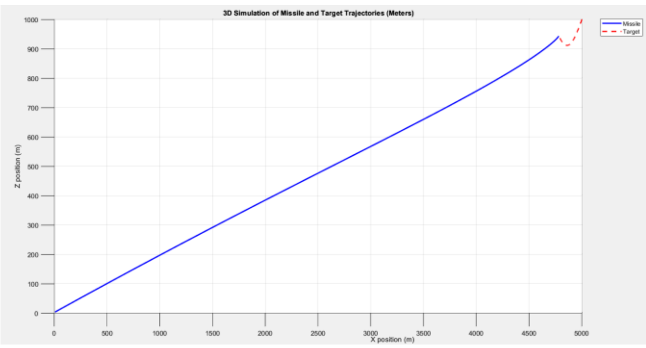

Fig. 8. PN trajectory in longitudinal plane for w=1 and nt=9.

Fig.9 represents the missile-target Proportional Navigation (PN) trajectory in the lateral (Y-Z) plane. The trajectory demonstrates how PN ensures that the missile responds to both vertical and lateral target motion, dynamically adjusting its path for a successful intercept.

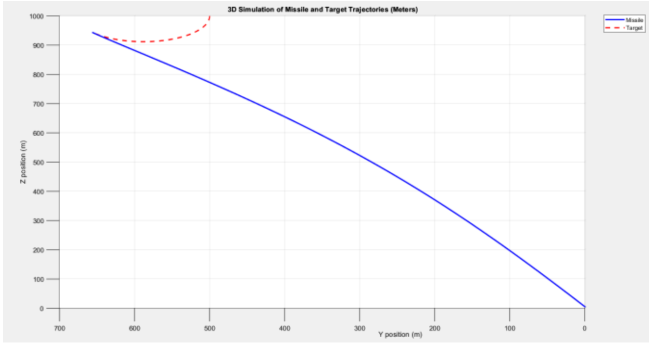

Fig. 9. PN trajectory in lateral plane for w=1 and nt=9.

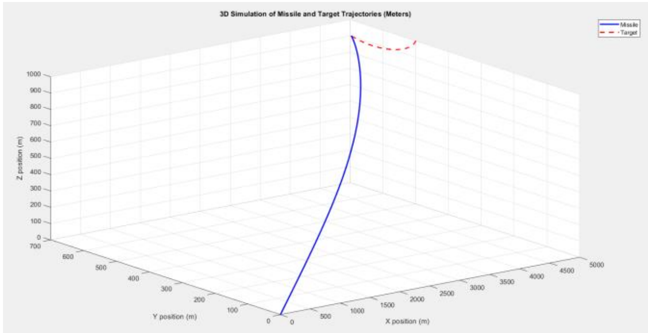

Fig. 10. PN trajectory in 3D view for w=1 and nt=9.

Fig.11 shows LOS angle in the vertical plane (top plot) starts at approximately 0.2 radians and remains relatively constant for a significant duration. However, it rapidly increases toward the end, indicating that the missile is nearing the target and adjusting its trajectory steeply. The horizontal LOS angle (bottom plot) follows a similar trend, starting at around 1.45 radians and decreasing slightly over time, showing minor adjustments in the missile's lateral orientation, before spiking near the end. This rapid change in LOS angles towards the final time steps suggests the imminent interception as the missile homes in on the target, requiring substantial directional corrections in both vertical and horizontal planes to achieve a successful hit.

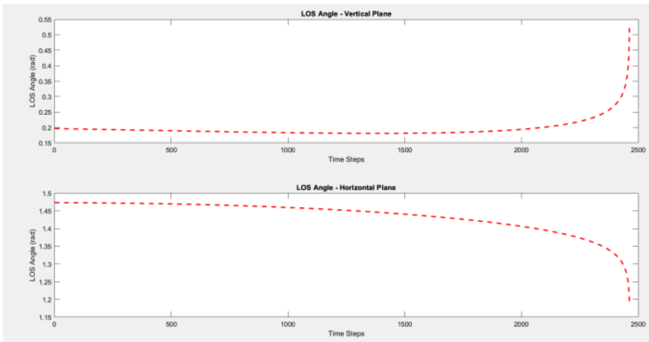

Fig. 11. LOS angle for w=1 and nt=9.

In Fig.12, the navigation command decreases rapidly during the initial time steps. For the vertical plane (top plot), N_c starts at a value near 12×10 m/s2 and quickly drops to zero after about 2 time steps, where it remains constant for the rest of the simulation. Similarly, in the horizontal plane (bottom plot), N_c begins at 7×103 m/s2 and follows a sharp decline to zero within 2 time steps, also stabilizing afterward. This sharp decline in the navigation command at the early stages suggests that most of the missile's corrective maneuvers take place shortly after launch, and it maintains a constant trajectory as it approaches the target, implying that minimal corrections are needed as it gets closer.

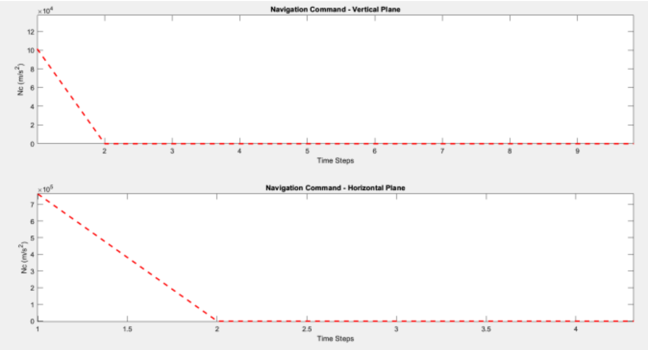

Fig. 12. : Nc command for w=1 and nt=9.

Fig.13 shows the 3D simulation of the missile and target trajectories in the longitudinal plane for w=0.2 and n_t=9. The missile's path features large oscillations early on, indicating significant corrective maneuvers. Despite these adjustments, the missile fails to converge with the target's more direct trajectory and does not hit the target. The oscillations highlight the effect of the low w, making the missile's guidance overly sensitive and unstable.

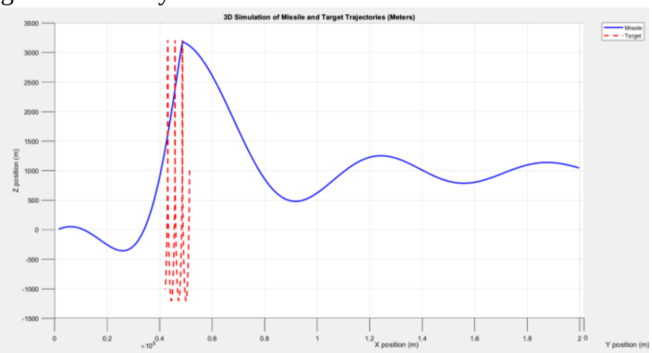

Fig. 13. PN trajectory in longitudinal plane for w=0.2 and nt=9.

In this simulation of the missile and target trajectories in the lateral plane for ω=0.2 and n_t=9, the missile's path shows a spiraling pattern, which indicates an unstable guidance system. The missile continuously spirals inward but fails to intercept the target's more consistent, direct trajectory. This spiraling suggests that the low navigation constant results in poor guidance, preventing the missile from successfully reaching the target.

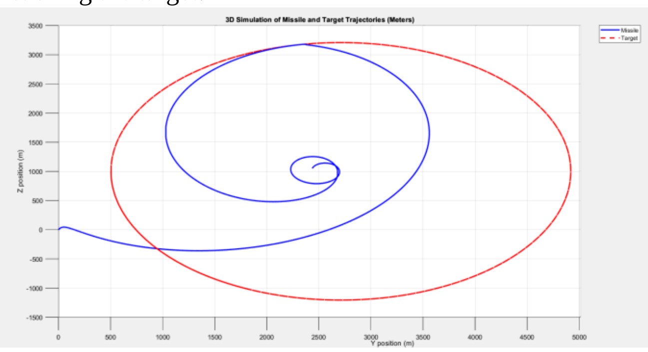

Fig. 14. PN trajectory in leteral plane for w=0.2 and nt=9.

Fig.15 shows a 3D view of the interception process

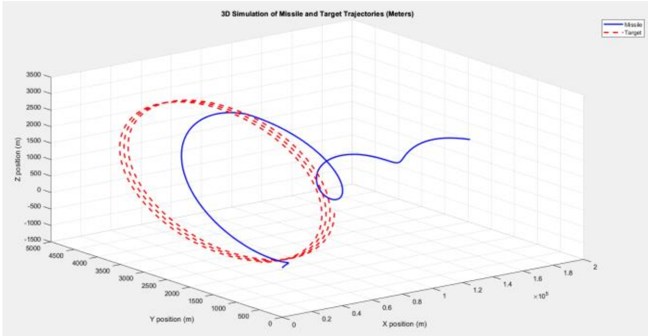

Fig. 15. PN trajectory in 3D view for w=0.2 and nt=9.

In Fig.16 the Line-of-Sight (LOS) angles for w=0.2and nt=9, both the vertical and horizontal planes exhibit significant oscillations over time. The fluctuations in the LOS angle indicate that the missile struggles to maintain a stable lock on the target, likely due to the low maneuver frequency (w=0.2), leading to an unstable guidance process. This instability correlates with the missile's spiraling trajectory, as seen in the previous plots, and its failure to intercept the target.

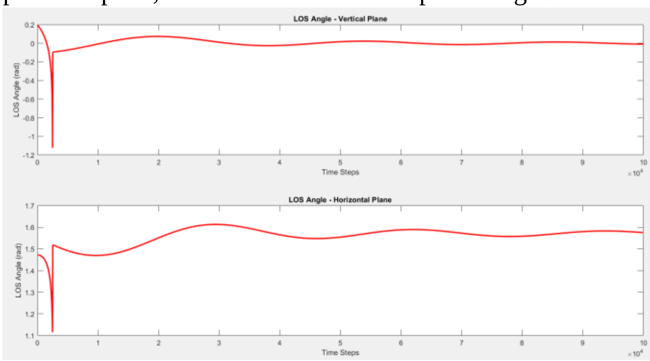

Fig. 16. LOS angle for w=0.2 and nt=9.

The navigation command in fig.17 for w=0.2 and nt=9 in both the vertical and horizontal planes reveal a significant spike early in the simulation, followed by a gradual stabilization. These sudden changes in the navigation command suggest that the missile is attempting aggressive course corrections, likely contributing to the oscillatory behavior observed in the trajectory and LOS angle plots.

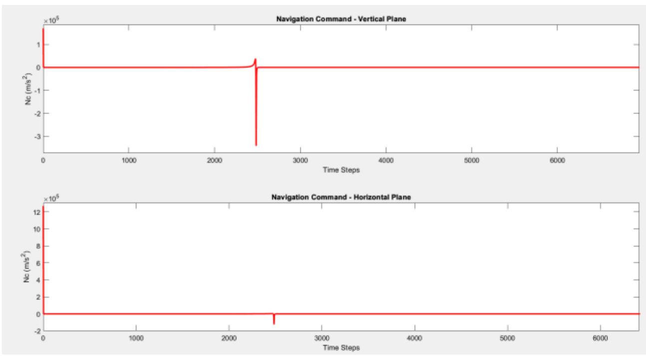

Fig. 17. nc command for w=0.2 and nt=9.

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D. Impact of radius of the HGB maneuver on Missile-Target interception

In fig.18, we observe the 3D simulation in longitudinal plane of the missile's trajectory as it successfully intercepts a target maneuvering along a high-g barrel maneuver (HGB) path with a radius of 490 meters in the longitudinal plane.

The red dashed region highlights the target's maneuvering zone. Despite the increased maneuver radius, the missile maintains alignment and achieves interception. The missile's path shows the necessary adjustments made by the guidance system to follow the target's maneuvers closely, ultimately leading to a successful hit.

This successful interception, even with a larger maneuver radius, suggests that the missile's guidance algorithm and navigation constants are effectively calibrated for engaging highly maneuverable targets.

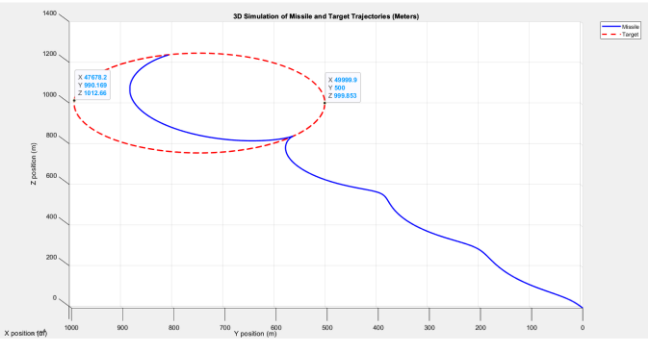

Fig. 18. HGB in case of 490 m radios.

Fig.19 show the successful of the intersection process in 3D.

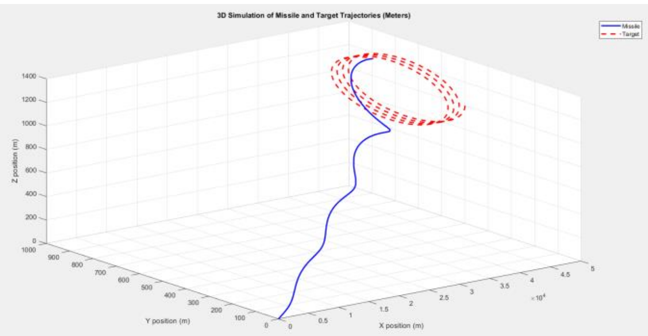

Fig. 19. 3D view in case of 490 m radios.

In fig.20 the Line of Sight (LOS) angle for a 490-meter radius engagement, the LOS angle in both the vertical and horizontal planes remains relatively steady for most of the simulation but increases sharply towards the end.

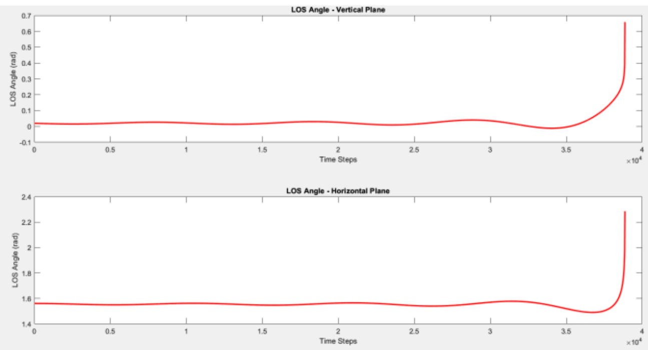

Fig. 20. LOS angle in case of 490 m radios

Fig.21 shows the navigation command (nc) for a 490-meter radius engagement, the values in both the vertical and horizontal planes remain nearly constant throughout the simulation. This stability in the command inputs suggests that the missile system is receiving a steady, continuous input for its course corrections.

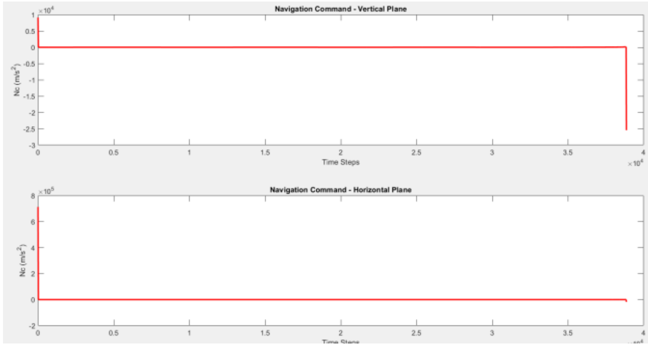

Fig. 21. nc command in case of 490 m radios.

Fig.22 shows trajectory simulation with a 1100-meter radius for the HGB, the missile path demonstrates a spiral approach as it attempts to align with the target's orbit. The missile's trajectory does not intersect with the target path, indicating a failure to intercept.

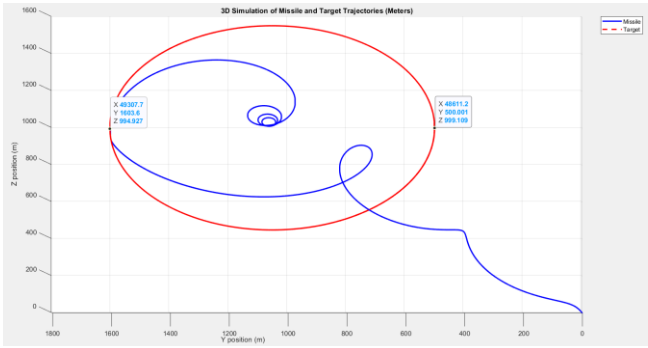

Fig. 22. HGB in case of 1100 m radios.

Fig.23 shows a 3D simulation of the interception process.

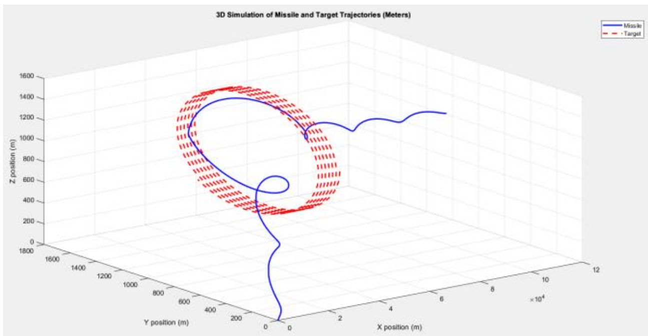

Fig. 23. 3D view in case of 1100 m radios.

Fig.24 shows Line of Sight (LOS) angle plot for a 1100-meter radius scenario, there is a noticeable spike around the fourth time step in both the vertical and horizontal planes. This sharp change suggests a sudden adjustment in the missile's trajectory in an attempt to realign with the target.

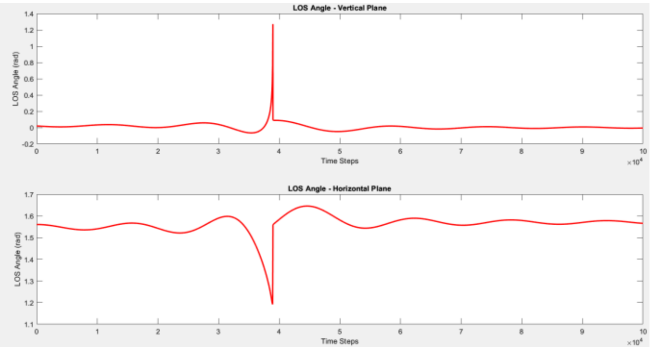

Fig. 24. LOS angle in case of 1100 m radios.

In the navigation command in fig.25 for the 1100-meter radius scenario, there is a significant spike in the vertical plane around the 3.88-time step, corresponding to the sudden change in the Line of Sight (LOS) angle observed earlier. This indicates a high demand for acceleration to adjust the missile's trajectory.

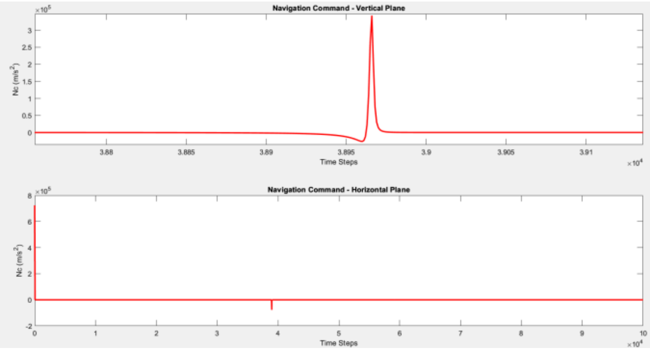

Fig. 25. nc command in case of 1100 m radios.

V. Conclusion

The overall conclusion of this research highlights both the strengths and limitations of the proportional navigation (PN) guidance strategy. PN guidance proved effective in scenarios with constant, predictable target trajectories, but its performance diminished against complex and highly dynamic maneuvers, suggesting the need for more adaptive strategies for such targets. The study found that the navigation constant, a crucial parameter in PN guidance, had a significant impact on missile trajectory and interception success: higher values resulted in aggressive maneuvers, which often led to overshooting or spiraling trajectories, while lower values enhanced stability but reduced responsiveness. Analyzing the line-of-sight (LOS) angle revealed challenges in maintaining alignment with maneuvering targets, especially when large maneuver radii were involved, which led to fluctuations in LOS angles and increased the likelihood of interception failure. Additionally, simulations with varying maneuver radii for both the missile and target offered insights into the effects of lateral and longitudinal adjustments on interception accuracy. Smaller radii generally improved tracking precision, while larger radii introduced greater trajectory deviations, reducing interception success. These findings underscore the need for enhancements in PN guidance to improve its effectiveness against unpredictable and evasive targets.

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