Slope-Adjusted Surface Area Computations in Digital Terrain

Mehran Ghandehari and Barbara P. Buttenfield

May 22, 2018

Slope-Adjusted Surface Area Computations in Digital Terrain

Mehran Ghandehari
Department of Geography, University of Colorado Boulder
Boulder, USA
mehran.ghandehari@colorado.edu

Barbara P. Buttenfield
Department of Geography, University of Colorado Boulder
Boulder, USA
babs@colorado.edu

Abstract—**Area is an integral part of any spatial database and has a significant role in many geographic analyses and applications. Planar algorithms that are widely used to calculate area ignore the slope and curvature of the terrain and result in under-estimation, particularly as pixel size increases or in uneven terrain. Calculating surface area using a regular DEM can overcome this issue by considering localized variations on the terrain surface. This paper investigates the scale- and algorithm-dependence of surface area calculations. The expectation is that for any individual pixel, the improvement in measurements can be relatively small, however, the additive effects across the study area can become significant. The method of dividing each DEM pixel into eight 3D triangles is commonly used to calculate surface area. In this research, the elevation of triangle vertices are estimated using different interpolation methods to establish rates of under-estimation for progressively larger pixels. These methods are validated against vertex elevations on a 3 meter lidar data benchmark. Bi-Cubic interpolation outperforms other interpolation methods for calculating DEM surface areas, with Linear, Bi-Linear and Jenness methods performing nearly as well, especially at coarser resolution. Relative accuracies are shown to degrade somewhat in rougher terrain.**

I. INTRODUCTION

Estimating the area of features is very important in geographic analysis. Area of a land parcel, a forest patch or a wetland can be computed directly from land surveying or computed indirectly from a 2D map using grid coordinates. The key weakness of this approach is that the 2D method used for calculating area ignores the effects of topography and surface roughness, i.e., it does not account for the slope and curvature of the terrain. Planar projection and measurement of spatial features can distort information when areas are systematically underestimated. For example, Zhiming et al. [1] illustrate that surface area is significantly larger than planar area in two different mountainous areas using a parametric T-Test. Rogers et al. [2] show that the area of forest fire patches is underestimated by 20% when planar metrics are used. Jenness [3] demonstrates that surface area will always exceed planar area, adding that the ratio of the two can provide a useful measure of terrain roughness, as the discrepancy increases between the two measures. Differences between planar area and surface area can be expected to vary with size of geographic footprint, DEM resolution, terrain roughness, and landscape conditions. This error can be neglected for individual pixels, but it can propagate dramatically for measurements that encompass many pixels or where pixel sizes become quite large, corrupting terrain-based measurements and spatial modeling outcomes. This paper compares different interpolation methods for calculating surface area from DEMs with various resolutions.

Several available methods can determine surface area on terrain. Surface area can be calculated simply as: planar area / cos(slope). Dorner et al. [4] use this method to calculate surface area to account for non-uniform topography in landscape pattern analysis. A Triangulated Irregular Network (TIN) also can be used to calculate surface area, within each 3D triangle. This method takes the slope of the terrain into account. Xue et al. [5] have shown that TIN-based surface area calculations are preferable for vector data, while Jenness [3] works with raster data, calculating the surface area in a regular grid DEM using a focal window. Hoechstetter et al. [6] compare planar area with surface area to characterize patch area, patch perimeter, perimeter-area ratio, and terrain roughness for two different resolutions (2m and 20m). They conclude that terrain complexity significantly affects area and distance calculations. The surface area calculation methods also are compared in Zhang et al. [7] who conclude that when more than 30% of a region contains slopes greater than 18.2°, the difference between planar and surface area can exceed 5%.

A continuous surface also can be modeled to calculate surface area. Each pixel in a DEM represents a small area by a horizontal planar surface with an elevation assumed to be constant, which can be described by a zero-order polynomial function. Therefore, a DEM provides a discretized sample of the continuous elevation surface. The slope and curvature of the terrain is ignored under the horizontal pixel assumption and the slope-adjusted surface area of each pixel cannot be calculated directly. Instead, one can apply continuous interpolation methods to reconstruct the three-dimensional (3D) surface of each pixel using contextual information from adjacent pixels [8, 9]. Each DEM cell is modeled with a polynomial function z = f(x,y) whose surface area can be calculated with a double integral: $A = \iint_R \sqrt{1 + [f_x(x,y)]^2 + [f_y(x,y)]^2}\, dA$. In previous research by the authors of this paper, this double integral method was tested on regular mathematical surfaces but it proved computationally very slow. Even with the emergence of High Performance Computing (HPC), continuous polynomial estimation methods for surface area are not yet feasible in terrain modeling processes simply because the basic computation of surface area by such polynomials remains overly time-consuming.

On the other hand, Light Detection and Ranging (lidar) has provided finer resolution and more precise data but it introduces other challenges such as the need for initial filtering and additional data storage, and increased processing times. Furthermore, the current availability of lidar data is not exhaustive in developed nations, and is sparse or non-existent in rural and undeveloped regions. Lidar data is used as a benchmark in this research. It should be noted that the accurate measurement of surface area is similar to the well-known problem of coastline length. A terrain surface has a fractal nature and the surface area increases as resolution or level of detail improves [10]. The fractal dimension can be used to establish the relationship between surface area and DEM resolution [11].

This research reports on a pilot project that examines the sensitivity of slope-adjusted surface area estimation on a DEM incorporating slope and curvature across a progression of spatial resolutions. Surface area is estimated using a series of increasingly complex discrete interpolation methods, applied to digital terrain compiled independently at five different spatial resolutions. Results are validated using 3m lidar benchmark data to establish errors introduced as pixel sizes increase. The study areas reported here include an area of smooth terrain and an area of rougher terrain, both in a relatively humid landscape in the coastal plain of North Carolina. Results to be presented at the conference will incorporate additional smooth and rough terrain samples in dry landscapes, and at varying elevations.

II. Data Sets and Study Areas

The study areas are limited to areas for which 3m resolution lidar data is available for validation. DEM data is tested at 10m, 30m, 100m, and 1000m resolutions, and compared against a 3m lidar benchmark. The 3m, 10m, and 30m resolutions form part of the USGS National Elevation Dataset (NED); and the source for 100m and 1000m resolutions is the Shuttle Radar Topography Mission (SRTM) dataset (http://dds.cr.usgs.gov/srtm/version2_1/). The two terrain patches in North Carolina (Fig. 1) are drawn from a DEM centered on 35.798 degrees N and 81.473 degrees W. Its location at the southeast end of the Appalachian Mountains provides elevations ranging from 209m - 1602m. Located where the Blue Ridge Mountains drop towards the coastal plains is a humid, uneven landscape, with annual precipitation averaging 51 inches (129.5 centimeters). The study area provides a mix of uninhabited land with smaller rural settlements. Two different sub-regions of this study area are selected (A and B in Fig. 1) in the northwest as a rough terrain and in the east as a more flat terrain. For the 10m DEM, region A spans 1811 rows and 1471 columns. Region B spans 2085 rows and 1468 columns. The results of estimates are discussed for these two areas to investigate how the results vary with terrain roughness. In region A, the average elevation, standard deviation of elevation, and average slope are 765m, 239m, and 20 degrees (exceeding Zhang et al's [7] 18.2% slope threshold). In region B, by contrast, the respective values are 297m, 19m, and 5 degrees.

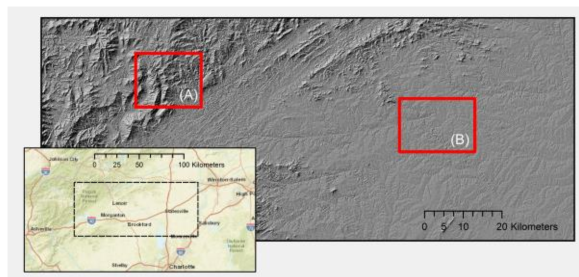


Figure 1. Study area located in North Carolina. Regions A and B are selected to examine how estimation errors vary with respect to terrain roughness, elevation, and local slope.

III. Methods

This research modifies the method proposed by Jenness [3] for calculating surface area from a DEM. In Jenness's original method, each pixel centroid is linked to the pixel centroid of the 8 surrounding pixels to generate 8 triangles (Fig. 2-a). The vertices of each triangle have differing elevations, taken from respective pixel centroids. The lengths for the sides of the 8 triangles can be calculated using the Pythagorean Theorem, incorporating slope because of each vertex's unique elevation value. Because the connecting vectors are bisected by pixel boundaries, all length values are divided by 2 to consider only the portion of triangles that fall within the central pixel's boundary. The area of each triangle given the lengths of sides a, b, and c is calculated as: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$. The surface area of each pixel is equal to the sum of the 8 triangle areas.

In the modified method, the 8 triangles laying within the pixel are created (Fig. 2-b). Then instead of simply dividing by 2 (i.e., bisecting the triangle edges), a new set of 8 vertices is created on the pixel boundary. The elevations of new vertices are estimated using five different interpolation methods. The accuracy of estimates is assessed using lidar data to capture actual elevations of the 8 estimated points, and lidar values are used to compute a validation surface area value. The surface area values computed using interpolation methods are compared with the lidar surface area values to find the interpolated results that are closest to the summed pixel areas of the lidar benchmark.

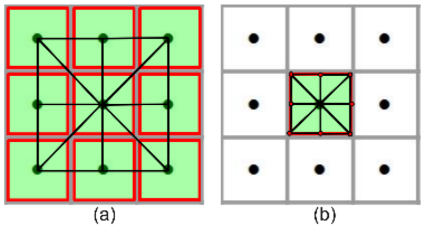
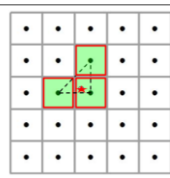


Figure 2. (a) Jenness's original method, and (b) modified method used in this research.

Given a regular elevation grid within the defined neighborhood, different interpolation techniques will generate differing elevation estimates. The methods compared in this research include Weighted Average and discrete polynomial surfaces (Linear, Bi-Linear, Bi-Quadratic, and Bi-Cubic), all of which are exact interpolators. Different contiguity configurations are used in the interpolation methods. The Linear, Bi-Linear, Weighted Average, Bi-Quadratic, and Bi-Cubic interpolators use 3, 4, 9, 9, and 16 neighboring pixels, respectively. Fig. 3 illustrates the mathematical function and the contiguity configurations used for each interpolation method.

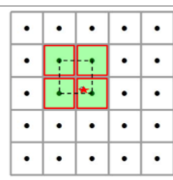
Linear 3

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y}$$



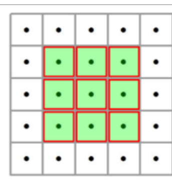
Bi-Linear 4

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y}$$



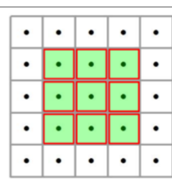
Weighted Average 9

$$z(x,y) = \frac{\sum_{i=1}^{n} w_i\, z_i}{\sum_{i=1}^{n} w_i}\ ; \quad w_i = \frac{1}{d_i^2}$$



Bi-Quadratic 9

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2 + a_6\,x^2y^2 + a_7\,x^2\,y + a_8\,xy^2$$



Bi-Cubic 16

$$z(x,y) = a_0 + a_1\,\mathrm{x} + a_2\,\mathrm{y} + a_3\,\mathrm{x\,y} + a_4\,x^2 + a_5\,y^2 + a_6\,x^2y^2 + a_7\,x^2\,y + a_8\,xy^2 + a_9\,x^3 + a_{10}\,y^3 + a_{11}\,x^3y^3 + a_{12}\,x^3y^2 + a_{13}x^2y^3 + a_{14}\,x^3\mathrm{y} + a_{15}\,xy^3$$

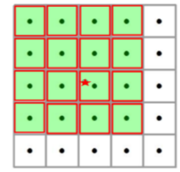


Figure 3. Different interpolation methods and their corresponding contiguity configuration

IV. Results

Tables I and II show the method comparison for region A and region B at various DEM resolutions. Surface areas are computed for a planar solution in each table, to provide a baseline "worst case" comparison. Also, the slope-adjusted surface area calculated based on the local slope at each DEM cell (Planar area / Cos(slope)) is reported. The original (unmodified) Jenness method is also reported to determine if the modification (that adds some processing time) is warranted. RMSEs are reported to evaluate the performance of various methods. Error magnitudes can be seen to vary with DEM resolution and with interpolation method within each table, with higher RMSEs overall for region A, in rough terrain, and lower RMSEs for region B, in flatter and smoother terrain. The general trend in either table is that RMSE values increase to varying degrees moving from finer to coarser DEM resolutions. In both regions, the Bi-Cubic interpolation shows lowest RMSEs at all resolutions, and the Planar and Weighted Average methods show the highest RMSEs. Linear, Bi-Linear and Jenness (original) interpolations show the next lowest RMSEs at all resolutions. Also evident from the tables for both regions, higher order polynomials do not appear to outperform lower order polynomials in every case. For example, Bi-Quadratic shows a larger RMSE than Linear and Bi-Linear methods.

When reported as percentages relative to the Bi-Cubic RMSEs, a slightly clearer pattern emerges, indicating that error percentages increase from 10m to 30m but then drop at 100m and 1000m resolutions. It is possible that the drop at coarsest resolutions is due to over-sampling relative to the 3m lidar validation; but the jump in error at 30m resolution bears further investigation, since it happens consistently across methods. In region A (the rougher terrain), percentages fall below 5% error for Linear interpolation at all resolutions, and for Bi-Linear and Jenness at coarser resolutions. Recall that Zhang et al [7] found that when slopes exceed 18.2 degrees, slope errors can exceed 5%. The finding that slope errors do not exceed 5% in region A indicates that any of these three methods could serve as an alternative to the Bi-Cubic interpolation, assuming the region in question is characterized by steep slopes.

TABLE I. RMSE VALUES (SQ. M.) FOR DIFFERENT METHODS AT DIFFERENT RESOLUTIONS FOR REGION A (ROUGH TERRAIN). BOLDFACE SHOWS THE LOWEST RMSE VALUES. UPPER TABLE SHOWS RMSE VALUES AND LOWER TABLE REPORTS THESE AS PERCENTAGES RELATIVE TO THE BI-CUBIC METHOD.

A	10m	30m	100m	1000m
Planar	14.649	111.923	810.425	30,417.716
Planar/ Cos (Slope)	3.983	40.361	745.191	25515.852
Jenness	3.504	30.691	423.386	20,064.473
Linear3	3.307	29.735	419.091	20,015.471
Bi-Linear4	3.440	30.974	427.636	20,551.611
Wtd Average9	7.802	61.262	579.130	22,816.463
Bi-Quadratic9	3.562	33.983	446.086	23,583.277
Bi-Cubic16	**3.123**	**27.358**	**411.628**	**19,971.391**
Planar	369.068	309.105	96.883	52.306
Planar/ Cos (Slope)	27.538	47.529	81.035	27.762
Jenness	12.200	12.183	2.856	0.466
Linear3	5.892	8.689	1.813	0.221
Bi-Linear4	10.150	13.217	3.889	2.905
Wtd Average9	149.824	123.927	40.693	14.246
Bi-Quadratic9	14.057	24.216	8.371	18.085
Bi-Cubic16	0.000	0.000	0.000	0.000

RMSE VALUES (SQ. M.) FOR DIFFERENT METHODS AT DIFFERENT RESOLUTIONS FOR REGION A (ROUGH TERRAIN). BOLDFACE SHOWS THE LOWEST RMSE VALUES. UPPER TABLE SHOWS RMSE VALUES AND LOWER TABLE REPORTS THESE AS PERCENTAGES RELATIVE TO THE BI-CUBIC METHOD.

B	10m	30m	100m	1000m
Planar	2.134	13.111	74.424	1016.487
Planar/ Cos (Slope)	1.100	7.467	61.105	998.146
Jenness	0.969	5.938	55.243	924.711
Linear3	0.931	5.691	55.041	924.844
Bi-Linear4	0.966	5.996	55.995	938.028
Wtd Average9	1.406	8.643	62.983	964.650
Bi-Quadratic9	1.006	6.455	57.958	955.460
Bi-Cubic16	**0.904**	**5.440**	**54.204**	**917.951**
Planar	136.062	141.011	37.304	10.734
Planar/ Cos (Slope)	21.681	37.261	12.732	8.736
Jenness	7.190	9.154	1.917	0.736
Linear3	2.987	4.614	1.544	0.751
Bi-Linear4	6.858	10.221	3.304	2.187
Wtd Average9	55.531	58.879	16.196	5.087
Bi-Quadratic9	11.283	18.658	6.926	4.086
Bi-Cubic16	0.000	0.000	0.000	0.000

V. SUMMARY

This research employs realistic terrain surface geometries using different interpolation methods and the information from adjacent pixels for purposes of incorporating terrain slope and curvature into surface area computations. Findings of this research indicate Bi-Cubic polynomial has the lowest RMSE. The Linear interpolations perform nearly as well and slightly better than Jenness's method. The Bi-Linear interpolation performs slightly worse except at 10m resolution. All three methods show nearly similar RMSEs for this pair of very small pilot study areas. One can expect that the improvement in RMSEs will be more pronounced for larger terrain footprints. Furthermore, the accuracy of slope-adjusted surface area estimation appears to bear some relation with terrain roughness, but this relationship will have to be tested to confirm statistical significance. In ongoing research, additional study areas and larger spatial footprints are tested to further explore these findings and understand error propagation in surface area calculations. Our research is investigating the extent to which slope- and curvature-adjusted surface area measurements can make a difference in higher order metrics utilized in spatial modeling, as for example in weighted flow accumulation, debris flow extents, and similar areal metrics. We are also investigating how much computational complexity the surface-adjusted area adds to the model processing.

REFERENCES

[1] Zhiming, Z., F. Van Coillie, R. De Wulf, E. M. De Clercq and O. Xiaokun, (2012). "Comparison of Surface and Planimetric Landscape Metrics for Mountainous Land Cover Pattern Quantification in Lancang Watershed, China." Mountain Research and Development 32(2): 213-25.

[2] Rogers, D., A. Cooper, P. McKenzie and T. McCann, (2012). "Assessing Regional Scale Habitat Area with a Three Dimensional Measure." Ecological informatics 7(1): 1-6.

[3] Jenness, J. S., (2004). "Calculating Landscape Surface Area from Digital Elevation Models." Wildlife Society Bulletin 32(3): 829-39.

[4] Dorner, B., K. Lertzman and J. Fall, (2002). "Landscape Pattern in Topographically Complex Landscapes: Issues and Techniques for Analysis." Landscape Ecology 17(8): 729-43.

[5] Xue, S., Y. Dang, J. Liu, J. Mi, C. Dong, Y. Cheng, X. Wang and J. Wan, (2016). "Bias Estimation and Correction for Triangle-Based Surface Area Calculations." International journal of geographical information science 30(11): 2155-70.

[6] Hoechstetter, S., U. Walz, L. Dang and N. Thinh, (2008). "Effects of Topography and Surface Roughness in Analyses of Landscape Structure—a Proposal to Modify the Existing Set of Landscape Metrics." Landscape Online 3(1): 1-14.

[7] Zhang, Y., L.-n. Zhang, W.-d. Bao and X.-x. Yuan (2011). "Surface Area Processing in Gis for Different Mountain Regions." Forestry Studies in China 13(4): 311-14.

[8] Ghandehari, M., B. P. Buttenfield and C. J. Q. Farmer, (2017). "Cross-Scale Analysis of Sub-pixel Variations in Digital Elevation Models.", Advances in Cartography and GIScience. ICACI 2017. Lecture Notes in Geoinformation and Cartography. Springer, Cham.

[9] Buttenfield, B. P., M. Ghandehari, S. Leyk, L. V. Stanislawski, M. Brantley and Y. Qiang, (2016). "Measuring Distance "As the Horse Runs": Cross-Scale Comparison of Terrain-Based Metrics." Paper presented at the Proceedings 9th International Conference on Geographic Information Science, GIScience 2016, Montreal, Canada.

[10] Feder, J., (1988). "Fractals." Plenum Press, New York.

[11] Goodchild, M. F., & Mark, D. M. (1987). "The fractal nature of geographic phenomena." Annals of the Association of American Geographers, 77(2), 265-278.