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***Abstract* — The aim of calibration of measuring instruments (MI) is providing metrological traceability of measurement results obtained using calibrated measuring instruments. Quite often the deviation of indications of a measuring instrument from reference values given by a measurement standard is estimated in calibration. Calibration can also include conformation of compliance of this deviation with a maximum permissible error, when it's necessary. Design of calibration procedure is caused by further application of calibration results and requirements of measuring instruments owners. Some issues of validation of calibration procedures according to intended usage of their results are analysed in the given paper. Different approaches for using calibration results for obtaining measurements results and associated measurement uncertainties, to be more precisely calculating of instrumental component of measurement uncertainty, are considered.**

Keywords — validation, calibration procedure, metrological traceability, measurement uncertainty, measurement error, instrumental measurement uncertainty, target measurement uncertainty

I. Introduction

The aim of MI calibration is establishment of metrological traceability of measurement results to SI units through a calibration hierarchy. In the International vocabulary of metrology VIM [1], calibration is defined as a two-step operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication. The second step is largely related to the use of calibration results and relates to the specific application of MI. Accordingly, users of MI should have clear interpretation of calibration results reported in calibration certificates. As well as specialists from calibration laboratories should design calibration procedures and present calibration results in a way to meet the requirements of MI users. In this paper application of calibration results is considered in the context of evaluation of measurement uncertainty. We consider both evaluation of measurement uncertainty associated with deviation of MI indication from a value given by measurement standard as well as in calculating instrumental component of measurement uncertainty associated with measurement result.

II. Evaluation of measurement uncertainty during calibration

In the case considered measurement model is reported in the form of the following equation:

$$\Delta = x_{cal} - x_{ref}, \qquad (1)$$

where

x_{cal} – measured value obtained by MI/indication;

x_{ref} – value obtained by measurement standard;

Δ – systematic errors of a measuring instrument, which is determined at specified measured values (calibration points) in the measurement range.

An estimate of the systematic error Δ (measurement bias) is establishing with the associated measurement uncertainty [2]:

$$U_{0.95}(\Delta) = 2u(\Delta) = 2\sqrt{u^2(x_{cal}) + u^2(x_{ref})}, \qquad (2)$$

where $u(x_{cal})$ – standard uncertainty associated with indications of MI, which is often calculated by type A evaluation on the bases of repeated measurement;

$u(x_{ref})$ – standard uncertainty caused by measurement standard used which is often calculated by type B evaluation on the bases of available a priory information;

$\sqrt{u^2(x_{cal}) + u^2(x_{ref})}$ – combined standard measurement uncertainty;

$U_{0.95}(\Delta)$ – expanded measurement uncertainty for coverage probability 0,95; coverage factor equals to 2 is often used assuming normal distriburion.

Typical sources of uncertainty in calibration are the following:

- applicable measurement standards, including uncertainty of metrological characteristics (established in calibration of a given measurement by measurement standard of higher level in calibration hierarchy), instability or drift of metrological characteristics, non-linearity of calibration functions, etc.;
- random errors of measurement standards, calibrating measuring instruments and calibration procedures;
- a measurement method used in calibration (direct or indirect measurements),
- an algorithm used for measurement data evaluation, for an example, for estimating the parameters of the calibration function (for example, calculating the uncertainty for estimating the parameters of a linear calibration function using the least squares method);
- corrections to systematic errors, caused by deviations from normal conditions.

The methods for evaluation the components of uncertainty during calibration and forming of uncertainty budget are described in detail in a international and national documents, in particular [2-4].

Thus, in this paper the calibration result is considered as an estimate of the MI systematic error, Δ_i, and the associated expanded uncertainty, $U_{0.95}(\Delta_i)$, which are calicated at different measured values over the measurement range (calibration points). In general case the covarience matrix is also given $cov(\Delta_i, \Delta_j)$. Covariences are caused by aplication one and the same measurement standard for calculating bises at the calibration points.

III. USAGE OF CALIBRATION RESULTS

The calibration results are used for calculating measured values for MI indication and the following questions can arise in practice:

1. Is it justified to introduce a correction to the indication of a calibrated MI?
2. How to confirm compliance of a calibrated MI error with established requirements (MPE)?
3. How to calculate the instrumental component of measurement uncertainty when using calibrated MI?

The answer to the first question depends on the relationship between the magnitude of the bias and the expanded uncertainty of establishing this bias, as well as significance of the input of the bias to the total measurement uncertainty associated with measurement result. The second point that should be taken into account is behaviour of the bias upon the measurement range and its stability over time. If the measurement bias is insignificant compared with the corresponding uncertainty, therefore, correction is not justified. If the measurement bias exceeds the corresponding expanded uncertainty at some (or all) calibration points, then correction can improve the accuracy of measurement results. But behaviour of bias should be examined upon the measurement range and over time. If the sign of the bias does not change upon the measurement range and the magnitude of the bias exceeds the corresponding uncertainty, therefore, the introduction of the correction according is justified. If a correction to the MI indications is made, then the corresponding uncertainty of the correction should be taken into account in calculating the instrumental component of the measurement uncertainty. Correction can improve the measurement accuracy, but the question arises about the stability of the established biases. Since, as a rule, calibration of a measuring instrument is carried out in the same calibration laboratory, so experimental data are accumulated that make it possible to estimate the drift of the biases of a calibrated measuring instrument and establish the inter-calibration interval at the request of the customer.

Quite often, calibration ends with confirmation of the accuracy of the measuring instrument with the established requirements. For measuring instruments, the limits of error (MPE), $\Delta_{\lim}$, is usually spicified. When compliance with the limits of permissible error is checked the measurement uncertainty associated with the bias should be taken inti account. One of the possible criteria used for confirming compliance of the bias of a calibrated measuring instrument with the established requirements is the application of following condition [6]:

$$|\Delta| + 2\sqrt{u^2(x_{cal}) + u^2(x_{ref})} < \Delta_{\lim}, \tag{3}$$

Condition (3) is a very stringent condition comparining with a simple condition, $|\Delta| \leq \Delta_{\lim}$, which is checked at verification of measuring instrumrnt in legal metrology. Therefore, other criteria can be also considered and applied. For an example, the following criterion is also applied in practice:

$$\begin{cases} |\Delta| \leq \Delta_{lim} \\ U_{0,95}(\Delta) \leq \frac{\Delta_{lim}}{q} \end{cases} \tag{4}$$

Parameter , q, is often chosen eqial to 3 in order to optimise the influence of measurement uncertainty on decision made regarding checking compliance of the bias with specified requirements. In general the approach used for accounting measurement uncertainty when deciding on compliance with the requirements should be cliarly discribed.

IV. CALCULATION OF INSTRUMENTAL MEASUREMENT UNCERTAINTY

The calibration results are the initial information for calculating the component of the measurement uncertainty associated with the applied MI, i.e. instrumental uncertainty. In the general case, the contribution of this component of uncertainty to the total uncertainty is determined by the measurement model/ measurement equation. However, even in the simplest case of direct measurements, it is necessary to take into account that the measurement conditions may be different from the calibration conditions, which leads to the need to introduce corrections for influence quantities and/or to perform repeated measurement for estimating a random measurement error. It must be emphasized that, as a rule, during calibration, repeated measurements are also performed when the systematic error of the measuring instruments is established. Moreover in some cases the precision of MI is also reported by standard deviation (repeatability standard deviation S_r). However, when using a calibrated measuring instrument the component of measurement uncertainty caused by random errors should be estimated. Quite often it's realised in frames of assessment of a measurement procedure when measurement precision is reported by standard deviation. And this standard deviation is used for control of random measurement error behaviour and for calculating total measurement uncertainty associated with measurement result.

In practice, the most questionable issue is a way for incorporating a bias into total measurement uncertainty in the case when no correction is applied. In some cases, one of the requirements for calibration (calibration procedure) from a customer is directly the assessment of the instrumental component of measurement uncertainty caused by using calibrated measuring instrument. Application of two formulas below is a common practice for incorporating measurement bias into uncertainty calculation:

$$U_{0.95}^{(1)} = 2\sqrt{\Delta^2 + u(\Delta)^2}\ , \tag{5}$$

$$U_{0.95}^{(2)} = |\Delta| + 2u(\Delta), \tag{6}$$

An analysis of the above two expressions shows that the corresponding values of the expanded uncertainty differ significantly depending on magnitude of a bias. In particular, for $\Delta/u(\Delta)=3$, the difference is about 20%. This relation between Δ and u is the greatest reasonable one because for lager rations the corrections seems to be reasonable.

The above analysis does not take into account the fact that for a number of measuring instruments, the random component of the error is predominant. In this case, it is necessary to separately evaluate and report the standard deviation of repeatability, S_r in the calibration certificate. Typically, when performing a calibration, the random measurement error associated with bias evaluation is significantly reduced by performing repeated measurements. However, when using measuring instruments and calculating the standard instrumental measurement uncertainty, it is necessary to take into account the standard deviation of the repeatability of the indications of the measuring instruments. The repeatability standard deviation (taking into account the number of repeated measurements m) is added up with the corresponding uncertainty of bias determination. Accordingly, we obtain analogues of formulas (5) and (6) relating to total measurement uncertainty:

$$\tilde{U}^{(1)}_{0.95} = 2\sqrt{\Delta^2 + u^2(\Delta) + \frac{S_r^2}{m}} = 2\sqrt{\Delta^2 + u^2(x_{ref}) + S_r^2\left(\frac{1}{n} + \frac{1}{m}\right)}, \quad (7)$$

$$\tilde{U}^{(2)}_{0.95} = |\Delta| + 2\sqrt{u^2(\Delta) + \frac{S_r^2}{m}} = |\Delta| + 2\sqrt{u^2(x_{ref}) + S_r^2\left(\frac{1}{m} + \frac{1}{n}\right)}, \quad (8)$$

In Table I the results obtained by (7) and (8) are compared assuming that m=1 and n is large enough to ignore a random error in calibration. In Table I the values of ratio $\tilde{U}^{(1)}_{0,95}/\tilde{U}^{(2)}_{0,95}$ are presented for different parameters values of $\gamma_1=\Delta/u(x_{ref})$, $\gamma_2=S_r/u(x_{ref})$. The differences can be up to about 50%, for most cases $\tilde{U}^{(1)}_{0.95}$ is greater then $\tilde{U}^{(2)}_{0.95}$. The opposite relation don not exceed 10% so in general $\tilde{U}^{(1)}_{0.95}$ can be recommended as a safe estimator for expanded measurement uncertainty.

TABLE I. COMPARISON OF VARIOUS ESTIMATES OF EXPANDED INSTRUMENTAL UNCERTAINTIES OBTAINED BY (7) AND (8)

γ_2 \ γ_1	1	2	3	4	5
0,1	0,94	1,12	1,26	1,37	1,46
0,5	0,93	1,08	1,22	1,33	1,42
1	0,91	1,02	1,14	1,24	1,33
2	0,89	0,93	1,00	1,08	1,16
3	0,91	0,89	0,94	0,99	1,05
4	0,92	0,89	0,91	0,94	0,98
5	0,93	0,89	0,89	0,91	0,94

There are cases when in calibration standard deviation from reference value is directly calculated by formula (9). In this case deviations of measuring instruments indications are considered not from the average value, but from the value obtained by the measurement standard (n is the number of repeated measurements at calibration, which should not be confused with a number of repeated indication at measurement, m):

$$S^{*2} = \frac{\sum\left(x_i - x_{ref}\right)^2}{n}, \quad (9)$$

The number of repeated measurements should be large enough to get a valid estimate of standard deviation of precision. Note that the standard deviation calculated according (9) already includes the measurement bias of the indications of the calibrated measuring instrument. Therefore, in order to calculate the instrumental component of measurement the uncertainty in the case of single measurements, only the uncertainty due to the measurement standard should be added.

$$\tilde{U}^{(3)}_{0,95} = 2\sqrt{S^{*2} + u^2(x_{ref})}, \quad (10)$$

Formulas (7) and (10) are comparable for the case of a single measurement, formulae (10) is not applicable for multiple measurements.

Below formula (7) is investigated for different number of repeated indications, m. In Table II the ratios of $\tilde{U}^{(1)}$(m=1)/ $\tilde{U}^{(1)}$ (m=2,3,5,10) are given for different values of $S_r/\sqrt{\Delta^2 + u(\Delta)^2}$. Even for $S_r/\sqrt{\Delta^2 + u(\Delta)^2} = 1$ repeated measurement can reduce the total uncertainty up to 35%. In should be stressed that the analyses is based on standard deviation of precision of MI indications calculated and reported at calibration. But at applying MI the standard deviation of repeated indication can be greater because of additional sources of dispersion. So data in Table II can be used as preliminary information for rational choice of a number of indication. But this number should be refined in measurement.

TABLE II. COMPARISON OF ESTIMATE OF EXPANDED INSTRUMENTAL UNCERTAINTY $\tilde{U}^{(1)}_{0.95}$ FOR CASES WHEN M=1 AND M =2,3,5,10

Number of measurements	$S_r/\sqrt{\Delta^2 + u(\Delta)^2}$				
	0.5	***1***	***2***	***3***	***5***
m=2	1,05	1,15	1,29	1,35	1,37
m=3	1,07	1,22	1,46	1,58	1,64
m=5	1,09	1,29	1,67	1,89	2,01
m=10	1,10	1,35	1,89	2,29	2,56

To calculate the expanded measurement uncertainty, the standard uncertainty u_Σ, is multiplied by the coverage factor $k_{0.95}$ for a confidence factor of 0.95. Typically, the coverage factor is assumed to be 2, but in general, the coverage factor depends on the type of distribution law. In practice, they are usually guided by the following rule. If random factors dominate when performing measurements, then they assume the normal distribution law and the coverage factor, $k_{0.95}$, are taken to be 2. Otherwise, they assume a uniform distribution law and the coverage factor, $k_{0.95}$, is taken to be 1.65.

V. CONCLUSION

The article considers issues of using the calibration results for calculating measurement uncertainty associated with a measurement result obtained by calibrated measuring instrument. Various approaches to calculating the instrumental expanded measurement uncertainty and incorporating measurement bias into measurement uncertainty are analyzed and compared. The previously proposed approaches are generalized for accounting a significant random measurement error of MI. Some recommendations for rational choice of a number of repeated measurements are given.

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