Fuzzy Temporal Non-Monotonic Reasoning

Poli Venkatasubba Reddy

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***Abstract*— Non-monotonic reasoning is reasoning is kind of knowledge in which many conclusion will be drawn. Temporal non-monotonic reasoning in which inference will be changed when time goes on. For instance, the bird can't fly at certain age. The non-monotonic problem is undividable. An undecided problem has no solution. Fuzzy logic will made undecided problem into decidable problem. Some time, fuzzy non-monotonic reasoning deal with fuzzy temporal constraint like past, present feature etc. A young bird can fly at the age t0 but young bird may not fly at the age t1. In this paper, Fuzzy temporal non-monotonic reasoning is studied with unknown and known twofold fuzzy set to made undecided problem in to decidable. Fuzzy temporal truth maintenance system (FTTMS) is studied for computation of fuzzy non-monotonic reasoning. Some examples are given.**

Keywords—non-monotonic reasoning, fuzzy Sets, twofold fuzzy sets, fuzzy non-monotonic reasoning, FTTMS, incomplete knowledge

I. INTRODUCTION

Non-monotonic reasoning will draw different conclusions and it is undecided reasoning. Sometimes Artificial Intelligence (AI) has to deal with undecided problems. Non-monotonic problem is undecided. In non.-monotonic reasoning, if some knowledge is added to the system then the conclusion will be changed, if knowledge base is incomplete then the reasoning is also incomplete. Knowledge bases are collecting the facts and give the conclusion based on facts. If the knowledge is temporal; then the reasoning also changes. For instance, a young bird is kept in room. After sometime it is escaped. Whether the bird can fly or not? Here age of the bird is involved.

John McCarthy [5] formalized non-monotonic reasoning with predicates P(x1,x2,…,xn) for the propositions of type "x is A".

The non-monotonic logic may be defined as

∀x (P(x) Λ Q(x) →R(x))

ꓱP(x) Λ Q(x) →┐R(x)

For instance ,

∀x (bird(x) Λ wings(x) →fly(x))

bird(peacock) Λ wings(peacock) →fly(peacock)

ꓱx (bird(x) Λ wings(x) →f) →┐fly(x))

ꓱx (bird(penguin) Λ wings(penguin) →┐fly(penguin))

The temporal logic is logic with time constraints and Time variables "t1-t0 "like "before", "meet", "after", where starting time t0 and ending time t1. The time constraints are necessary to deal with data [1, 4].

Sometimes temporal logic may con FL2n incomplete information of time constraints. Fuzzy will deal with incomplete information.

Fuzzy temporal proposition is of the form " x" is Ã", where Ã is temporal fuzzy set.

Definition: A temporal set Ã is characterized by its membership function $\mu_{\tilde{A}}(t)$, where $t=t_e-t_s$, t_s is starting time and t_e ending time and $t1>t_0$

For instance

past=$ts>t_e$

Present=$t_e=t_s$

feature=$ts<t_e$

latte=$ts>t_e$

in time=$t_e=t_s$

early=$ts<t_e$

For instance,

$$late= \mu_{late}(t)/t= \mu_{late}(t1)/t1 +\ldots+ \mu_{late}(tn)/tn$$

For instance,

The fuzzy proposition may con FL2n time variables like.

"x is early"

"x is late"

fly-age=0.0/5+0.3/20+0.6/30+0.8/40+0.9/50 +1/60

at the age of 5 days, the bird can't fly but at age of 20 days, the bird can fly.

I. FUZZY LOGIC

Zadeh [15] fuzzy logic is based on belief rather than probable (likelihood). The fuzzy logic made imprecise information in to precise.

Zadeh fuzzy logic is characterized by single membership function.

Zadeh[16] has introduced fuzzy set as a model to deal with incomplete information. Fuzzy set is a class of objects with a continuum of grades.

The set A of X is characterized by its membership function $\mu_A(x)$and ranging values in the unit interval [0, 1]

$\mu_A(x)$: X →[0, 1], x Є X, where X is Universe of discourse.

$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(a_n)/x_n$, where "+" is union

$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)$= 0.1/Penguin + 0.3/Hen+0.5/Cock + 0.6/Parrot + 0.8/eagle + 1.0/flamingos

$\mu_{┐A}(x)= 1-\mu_A(x)$ Negation

$\mu_A(x) \Lambda \mu_A(x) =\min\{ \mu_A(x) \Lambda,\mu_A(x)\}$ Conjunction

$\mu_A(x) V \mu_A(x) =\max\{\mu_A(x),\mu_A(x)\}$ Disjunction

$\mu_{QA}(x) =\mu_A(x)^q$ Quantifier

The fuzzy rules are of the form “if <Precedent Part> then <Consequent Part>”
if x is P the n x is Q.
. if x is P_1 and x is P_2 …. x is P_n then x is Q

The Zadeh [14] fuzzy conditional inference s given by
if x is P_1 and x is P_2 …. x is P_n then x is Q =
min 1, (1-min($\mu_{P1}(x)$, $\mu_{P2}(x)$, …, $\mu_{Pn}(x)$) +$\mu_Q(x)$} (2.1)

The Mamdani [8] fuzzy conditional inference s given by
if x is P_1 and x is P_2 …. x is P_n then x is Q =
min {$\mu_{P1}(x)$, $\mu_{P2}(x)$, …, $\mu_{Pn}(x)$, $\mu_Q(x)$} (2.2)

The fuzzy conditional inference may be derived “Consequent Part” from “Precedent Part” . [15].
Fuzzy conditional inference is given by fuzzy conditional inference

If x is then x is Q= {$\mu_P(x)$}

if x is P_1 and x is P_2 ... x is P_n then x is Q
={ min($\mu_{P1}(x)$, $\mu_{P2}(x)$, …, $\mu_{Pn}(x)$,)} (2.3)
.
P(x) Λ Q(x) → R(x) ={P(x) Λ Q(x)}

For instance,
x is bird Λ x has wings → x can fly
= x is bird Λ x has wings

II. FUZZY TEMPORAL NON-MONOTONIC REASONING

The temporal logic is logic with time constraints and Time variables “t1-t0” like “before”, “meet”, “after”, where starting time t0 and ending time t1.

The time constraints are necessary to deal with data [1, 4]. Fuzzy temporal logic should deal with incomplete information of time constraints.

A temporal variable is “t_1-t_0”, where t0 is starting time and t1 ending time.
For instance “past”=t_1-t_0, t1<t_0
“Present”= t_1 approximately t_0
“feature”=t_1-t_0, t_1>t_0
A fuzzy temporal set is fuzzy set of temporal constraint.

Fuzzy Temporal non-monotonic reasoning May be formalized with fuzzy predicate for the proposition of type “ x is A”, when A is fuzzy set may be defined as
∀x (P(x) ∧ T(x) → R(x))
∀x (P(x) ∧ T(x) →┐ R(x)
For instance ,
∀x (bird(x) ∧ young(x) → fly(x))
∀x (bird(x) ∧young(x) →┐ fly(x)

x is bird Λ x is young Λ x is unknown to fly →x can fly
Suppose ,
x is bird Λ x is young Λ x is unknown to fly →x can ‘t fly
or
x is bird Λ x is young Λ x is unknown to fly →x can’t fly

For example,
Ozzie is bird Λ Ozzie is young Λ Ozzie is unknown to fly → Ozzie can fly
Ozzie is bird Λ Ozzie is young Λ Ozzie is known to fly → Ozzie can’t fly
Ozzie is bird Λ Ozzie is young Λ Ozzie is unknown to fly → Ozzie can fly

The fuzzy temporal non-monotonic logic may be defined as
∀x (P(x) Λ Q(x) Λ T(x) Λ R(x) → S(x))
∀x (P(x) Λ Q(x) Λ T(x) Λ R(x) →┐ S(x)
For instance ,
∀x (bird(x) Λ wings(x) Λ young(x) Λ unknown-of-fly(x) → fly(x))
∀x (bird(x) Λ wings(x) Λ young(x) Λ known-of-fly(x) →┐ fly(x)

III. GENERALIZED FUZZY TEMPORAL NON-MONOTONIC REASONING

Zadeh [13] Proposed fuzzy set with single membership function. The two fold fuzzy set [12] will give more evidence than single membership function.

The fuzzy non-monotonic set may defined with two fold membership function using unknown and known

Definition: Given some Universe of discourse X, the proposition “ x is P” is defined as its two fold fuzzy membership function as

$\mu_P(x)$ = {$\mu_P^{unknown}$ (x), μ^{known} (x)}

P = {$\mu_P^{unknown}(x)$, $\mu_P^{known}(x)$}
Where P is Generalized fuzzy set and x Є X ,

$\mu_P(x)^{(unknown,\ known)}$ Λ $\mu_Q(x)^{(unknown\text{-},\ known)}$ → $\mu_S(x)^{(unknown,\ known)}$

where P,Q and S are twofold fuzzy sets with [known, unknown}.

$\mu_{bird}(x)$ Λ $\mu_{wings}(x)$ → $\mu_{fly}(x)$
$\mu_{bird}(x)^{(unknown,known)}$ Λ $\mu_{wings}(x)^{(unknown\text{-},known)}$ → $\mu_{fly}(x)^{(unknown,\ known)}$

The conflict of the incomplete information may be defined by fuzzy certainty factor(FCF).
FCF =(unknown- known)
$\mu^{FCF}{}_P(x)$ → [0,1], xϵX

$\mu_{bird}{}^{FCF}$ (x) → [0,1]
Where known and unknown are the fuzzy membership functions.

The fuzzy non-monotonic reasoning will bring uncertain knowledge in to certain knowledge.
$\mu_{bird}(x)^{(unknown,\ known)}$ → $\mu_{fly}(x)$

$\mu_P(x)^{(unknown,,known)}$ → $\mu_S(x)$
S= $\mu_S{}^{FCF}$ (x) = 1 $\mu_P{}^{FCF}$ (x >=α,

$0 \quad \mu_P^{FCF}(x) < \alpha$

fly= $\mu_{fly}^{FCF}(x) = 1 \quad \mu_{bird}^{FCF}(x >= 056$
$0 \quad \mu_{bird}^{FCF}(x) < 0.5$

Bird = {0.2/penguin +0.3/Ozzie+ 0.8/parrot+ 0.9/waterfowl + 1.0/eagle , 0.1/penguin +0.2/Ozzie+ . 0.1/parrot+ 0.1/waterfowl + 0.1/eagle }

Wings= {0.1/penguin +0.3/Ozzie+ 0.7/parrot+ 0.8/waterfowl + 0.9/eagle , 0.0/penguin +0.1/Ozzie+ . 0.2/parrot+ 0.1/waterfowl + 0.1/eagle }

Definition: The two fold fuzzy set for the proposition of the type "x is P" is given by

$\mu_P(x) = \{\mu_P^{unknown}(x), \mu_P^{nunknown}(x)\}$

$\mu_{bird}^{FCF}(x) = \{\mu_{bird}^{unknown}(x), -\mu_{bird}^{known}(x)\}$
= {0.2/penguin +0.3/Ozzie+ 0.8/parrot+ 0.9/waterfowl + 1.0/eagle , 0.1/penguin +0.1/Ozzie+ . 0.1/parrot+ 0.1/waterfowl + 0.1/eagle }
= { 0.1/penguin +0.2/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }
$\mu_{wings}^{FCF}(x) = \{\mu_{wings}^{unknown}(x), -\mu_{wings}^{known}(x)\}$

wings= {0.1/penguin +0.3/Ozzie+ 0.7/parrot+ 0.8/waterfowl + 0.9/eagle , 0.0/penguin +0.1/Ozzie+ . 0.2/parrot+ 0.1/waterfowl + 0.1/eagle }
= { 0.1/penguin +0.2/Ozzie+ . 0.5/parrot+ 0.6/waterfowl + 0.8/eagle }
$\mu_{young}^{FCF}(x) = \{\mu_{wings}^{unknown}(x), -\mu_{wings}^{known}(x)\}$

young= {0.3/penguin +0.4/Ozzie+ 0.7/parrot+ 0.8/waterfowl + 0.9/eagle , 0.0/penguin +0.1/Ozzie+ . 0.2/parrot+ 0.3/waterfowl + 0.3/eagle }
= = { 0.3/penguin +0.4/Ozzie+ . 0.3/parrot+ 0.5/waterfowl + 0.6/eagle }

Using (2.3), fuzzy inference is given by
if x is P_1 and x is P_2 …. x is P_n then x is Q

if x is P_1 and x is P_2 …. x is P_n then x is Q_1
$Q_1 = P_1 \Lambda P_2 \Lambda …. \Lambda P_n$
The fuzzy temporal non-monotonic logic may be defined as
$\forall x\ (P(x) \Lambda Q(x) \Lambda T(x) \Lambda R(x) \rightarrow S(x))$
$\forall x\ (P(x) \Lambda Q(x) \Lambda T(x) \Lambda R(x) \rightarrow \neg S(x)$
For instance ,
∀x (bird(x) Λ wings(x) Λ young(x) Λ unknown-of-fly(x) →fly(x))
∀x (bird(x) Λ wings(x) Λ young(x) Λ known-of-fly(x) →┐fly(x)

The two statements combined with two fold fuzzy logic.

x is birdΛ x has wings Λ x is young → x can fly
x can fly= x is bird Λ x is young Λ x has wings

$\mu_{bird}(x) \Lambda \mu_{wings}(x) \Lambda \mu_{young}(x) \rightarrow \mu_{fly}(x)$
$\{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\} \Lambda \{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\} \Lambda \{\mu_{young}^{unknown}(x), \mu_{young}^{known}(x)\} = \mu_{fly}(x)$
$\mu_{bird}^{FCF}(x) \Lambda \mu_{wings}^{FCF}(x) \Lambda \mu_{youngs}^{FCF}(x) \rightarrow \mu_{fly}(x)$
if x is P_1 and P_2 …x is P_n then x is P_1 and P_2 …. x is P_n = $\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$
$\mu_{fly}(x) = \mu_{bird}^{FCF}(x) \Lambda \mu_{wings}^{FCF}(x)$

$\mu_{bird}^{FCF}(x)$ = {0.1/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }

$\mu_{wings}^{FCF}(x) \quad \mu_{wings}(x) = \{\mu_{wings}^{unknown}(x), \mu_{wings}^{unknown}(x)\}$
= { 0.1/penguin +0.2/Ozzie+ . 0.5/parrot+ 0.6/waterfowl + 0.8/eagle }
$\mu_{young}^{FCF}(x) = \{\mu_{young}^{unknown}(x), \mu_{youngs}^{unknown}(x)\}$
= { 0.3/penguin +0.4/Ozzie+ . 0.3/parrot+ 0.5/waterfowl + 0.6/eagle }

Fuzzy conditional inference "consequent part "may be derived from "precedent part".

x is bird Λ x has wings Λ x is young → x can fly
x can fly =min { x is bird , x has wings }
= { 0.1/penguin +0.2/Ozzie+ . 0.3/parrot+ 0.5/waterfowl + 0.6/eagle }

The inference of " x can fly" for $\alpha >= 0.5$ is given by
= 1/waterfowl + 1/eagle
Here fuzzy logic made incomplete information to precise information's. Some birds can fly and some birds can't fly.
The fuzzy decision sets or quasi fuzzy set is defined by

R= $\mu_A^R(x) = 1 \quad \mu_A^{FCF}(x) >= \alpha$,
$0 \quad \mu_A^{FCF}(x) < \alpha$

The parrot, waterfowl and eagle can fly.
The penguin and Ozzie can't fly
The inference of " x can't fly" for $\alpha < 0.5$ is given by
= 0/.1penguin +0.2/Ozzie
The inference of " x can fly" for $\alpha >= 0.5$ is given by
= 0.6/parrot+ 0.6/waterfowl + 0.8/eagle

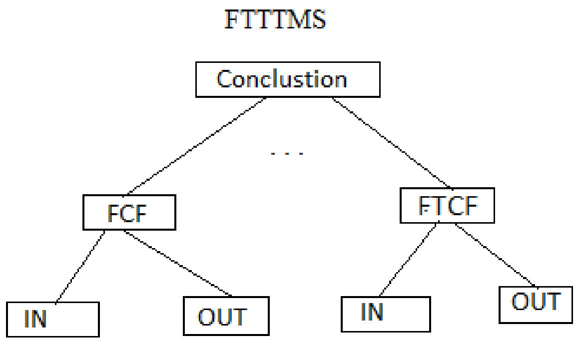
he parrot, waterfowl and eagle can fly and, penguin and Ozzie are can't fly.

IV. FUZZY TEMPORAL TRUTH MAITANACE SYSTEM

Doyel [4] studied truth maintenance system TMS for non-monotonic reasoning

The fuzzy truth maintenance system (FTTMS) for fuzzy non-monotonic reasoning using fuzzy conditional inference as

FTTMS is having There is list of justification and conditions.



if x is bird and x has wings and x is young x is fly-age then x can fly

List L(IN-node, OUT-node), FCF-node
IN-node is unknown
OUT-node is known
FCF is (belirf-disbilief)
Consider the proposition " if x is bird then x can fly)
x is bird
IN 0.9
OUT 0.1
FCF 0.8
Conclusion : x can fly if FCF>=0.5.
x is bird
IN 0.3
OUT 0.1
FCF 0.2
Conclusion :x can't fly if FCF<0.5.

Consider the proposition " if x has wings then x is fly)
x has wings
IN 0.8
OUT 0.1
FCF 0.7
Conclusion : x can fly if FCF>=0.5.
x has wings
IN 0.3
OUT 0.1
FCF 0.2
Conclusion :x can't fly if FCF<0.5.
Consider the proposition " if x is young then x is fly)
X is young
IN 0.6
OUT 0.1
FCF 0.5
Conclusion : x can fly if FCF>=0.5.
x is young
IN 0.5
OUT 0.1
FCF 0.4
Conclusion :x can't fly if FCF<0.5.

Consider the proposition " if x is fly-age then x is fly)
X is fly-age
IN 0.8
OUT 0.2
FCF 0.6
Conclusion : x can fly if FTCF>=0.5.
x is fly-age
IN 0.4
OUT 0.1
FCF 0.3
Conclusion :x can't fly if FTCF<0.5.
Where FTCF is Fuzzy Temporal Certainty Factor

Consider the proposition " if x is bird and x has wings and x is young and x is fly-age then x is fly)
x is bird
IN 0.9
OUT 0.1
FCF1 0.8
x has wings
IN 0.8
OUT 0.1
FCF2 0.7
X is young
IN 0.6
OUT 0.1
FCF3 0.5
X is fly-age
IN 0.8
OUT 0.2
FTCF 0.6

FCF=min{FCF1,FCF2, FCF3, FTCF4}=0.5

Conclusion : x can fly if FCF>=0.5.

Here is the fuzzy temporal no monotonic logic is making imprecise to precise.

V. Conclusion

Non-monotonic reasoning will give different concussions if some knowledge is added to the theory. Sometimes conclusion will be changed if time goes Fuzzy non-monotonic logic made imprecise to precise. A sometimes time constraint has to deal by fuzzy non-monotonic reasoning. A bird can't fly in the past but it can fly present and it can 'fly feature We discussed the difference between fuzzy temporal fuzzy non-monotonic reasoning and fuzzy monotonic reasoning..

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