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ABSTRACT

Key engineering properties of unsaturated soils such as volume change and shear strength can be defined using the effective stress principle. Several problems like prolonged drought, high-level radioactive waste, buried high voltage cables can subject surface and near-surface unsaturated soils to elevated temperatures. Such elevated temperatures can affect the hydraulic and mechanical behavior of unsaturated soils. It is very important to develop a closed-form model that can reasonably estimate the effective stresses under different elevated temperatures. For this purpose, the current study incorporates the temperature effect into a suction stress-based representation of Bishop's effective stress. The proposed model accounts for the effect of temperature on matric suction and degree of saturation. A temperature-dependent soil water retention curve is used to account for thermal effects on surface tension, contact angle, and enthalpy of immersion per unit area. The proposed effective stress model is then used to calculate the effective stress for two soils, Pachapa loam, and Seochang sandy clay, at various temperatures ranging from 25°C to 100°C. The validity of the model is examined by comparing the predicted effective degree of saturation and suction stress values against the measured data reported in the literature for GMZ01 bentonite. At a constant net normal stress, the results for both soils show that the impact of temperature on effective stress can be significant. The proposed model can be used for studying geotechnical and geoenvironmental engineering applications that involve elevated temperatures.

INTRODUCTION

Effective stress is recognized as one of the primary factors controlling the mechanical behavior of unsaturated soils, including shear strength and volume change. Mainly built upon Bishop (1959)'s effective stress principle, the existing models commonly represent the effective stress of unsaturated soils as a function of all or a combination of matric suction, degree of saturation and water retention properties of the soil.

There has been an increasing interest toward fundamental and applied research on the unsaturated soil behavior under elevated temperatures (e.g., Uchaipichat and Khalili, 2009; Alsherif and McCartney, 2015; Ng et al., 2017; Goodman and Vahedifard, 2019; Thota et al. 2019; Mortezaei et al., 2019). This growing interest is stimulated by the emergence of several temperature-dependent applications such as earthen structure-atmosphere interaction under a changing climate, radioactive barriers, nuclear waste disposal, ground-source heat pumps for geothermal heating/cooling systems, buried high voltage cables, thermal energy storage systems, and thermally active earthen structures (e.g., Gens and Olivella, 2001; Vahedifard et al., 2015; Leshchinsky et al., 2015; Robinson and Vahedifard, 2016; Vahedifard et al., 2017; Vahedifard et al., 2019). Some of the above applications can expose soil to elevated temperatures up to 100 ºC or even higher. This observation has led to the incorporation of effective stress into thermo-hydro mechanical (THM) constitutive models of unsaturated soils (e.g., Loret and Khalili, 2002; Laloui et al., 2003; Bolzon and Schrefler, 2005). Previous studies have demonstrated the promise in the use of the effective stress concept for describing the unsaturated soil behavior under elevated temperatures (e.g., Uchaipichat and Khalili, 2009; Alsherif and McCartney, 2015).

Despite advances in THM constitutive models, major gaps remain in the development of unified effective stress that can properly capture different aspects of the hydro-mechanical behavior of unsaturated soil under elevated temperatures. The main objective of this study is to develop a unified expression to describe the effective stress of unsaturated soils under elevated temperatures. Based on the model recently developed by Vahedifard et al. (2019), this paper employs a suction stress-based formulation for representing the effective stress of unsaturated soils subjected to elevated temperatures. The formulations incorporate temperature-dependent matric suction and an effective degree of saturation into effective stress expression. A parametric study for Pachapa loam and Seochang sandy clay is carried out using the proposed model. The validity of the model is examined by comparing the predicted effective degree of saturation and suction stress values against the experimental data of GMZ01 bentonite reported in the literature.

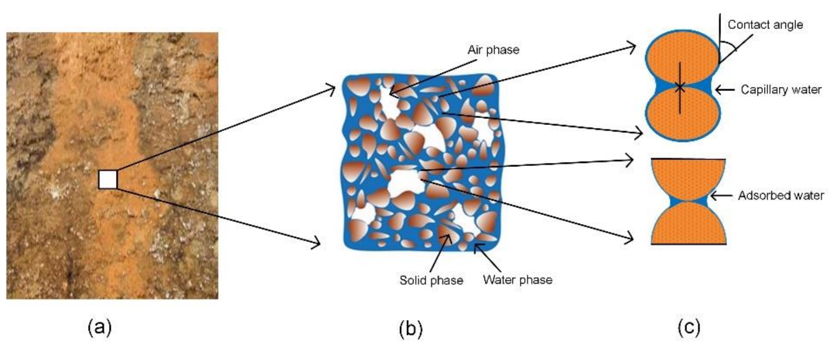

Fig. 1. Constituents of unsaturated soils: (a) macro scale, (b) representative element volume, and (c) capillary and adsorbed water

FORMULATION DEVELOPMENT

Fig. 1 schematically shows an unsaturated soil consisting of three constituents at any temperature. Fig. 1c shows the water stored in unsaturated soil by two distinct mechanisms: capillarity and adsorption. The forces of interaction between particles may lead to effective stresses in unsaturated soils. Elevated temperature changes in soil mass may alter the behavior of intergranular forces between the soil particles. The temperature can affect capillarity and adsorption differently depending on the range of matric suction and soil type.

Soil water retention model

Lu (2016) developed a SWRC model applicable for the entire range of matric suction including adsorption and capillary regions. Building upon van Genuchten (1980) SWRC model, the Lu (2016) SWRC model is rewritten in terms of the effective degree of saturation, S_e as:

$$S_e = 0.5\left[1 - erf\left(\sqrt{2}\,\frac{\psi - \psi_c}{\psi_c}\right)\right][1+(\alpha\psi)^n]^{1/n-1} \tag{1}$$

where erf () is the error function, ψ_c is the cavitation suction, ψ is the matric suction, α is a fitting parameter inversely related to the air-entry suction (1/kPa) and n is the pore-size distribution fitting parameter.

Closed-form solutions for suction stress and effective stress at elevated temperatures

Bishop's effective stress expression can be rewritten by assuming the term $-\chi(u_a - u_w)$ as suction stress (e.g. Lu and Likos, 2004, 2006):

$$\sigma' = (\sigma - u_a) - \sigma^s \tag{2}$$

where σ' is the effective stress, σ is the total stress, u_a is the pore air pressure assumed to be equivalent to atmospheric pressure, σ^s is the suction stress. Under ambient temperature, σ^s can be defined as follows (Lu et al., 2010):

$$\sigma^s = -(S_e\psi)_{T_r} \tag{3}$$

The suction stress equation can be extended for different temperature conditions as:

$$\sigma^s = -S_e\psi\left(\frac{\beta_{T_r} + T_r}{\beta + T}\right) \tag{4}$$

where β and β_{T_r} are regression parameters, defined in terms of contact angle, surface tension and enthalpy of immersion. More details can be found in Grant and Salehzadeh (1996) and Vahedifard et al. (2018, 2019).

Depending upon the availability of fitting parameters, any of the extended SWRCs introduced in the Vahedifard et al. (2018) can be used to define S_e with respect to elevated temperature conditions. Using the temperature-dependent matric suction, Eqs. 1, 2, and 4, a complete expression for temperature-dependent effective stress is developed. The equations for suction stress and effective stress at different temperatures can be defined as:

$$\sigma^{s} = -0.5\left[1-erf\left(\sqrt{2}\,\frac{\psi\left(\frac{\beta_{T_r}+T_r}{\beta+T}\right)-\psi_c}{\psi_c}\right)\right]\left[1+\left(\alpha\psi\left(\frac{\beta_{T_r}+T_r}{\beta+T}\right)\right)^{n}\right]^{1/n-1}\psi\left(\frac{\beta_{T_r}+T_r}{\beta+T}\right) \qquad (5)$$

$$\sigma' = \left(\sigma-u_a\right)+0.5\left[1-erf\left(\sqrt{2}\,\frac{\psi\left(\frac{\beta_{T_r}+T_r}{\beta+T}\right)-\psi_c}{\psi_c}\right)\right]\left[1+\left(\alpha\psi\left(\frac{\beta_{T_r}+T_r}{\beta+T}\right)\right)^{n}\right]^{1/n-1}\psi\left(\frac{\beta_{T_r}+T_r}{\beta+T}\right) \qquad (6)$$

PARAMETRIC STUDY

The proposed equations are employed to quantify the temperature effect on the effective degree of saturation and effective stress for Pachapa loam and Seochang sandy clay. Table 1 shows the parameters that are used for calculating the SWRCs and effective stress at temperature 25°C to 40°C, 60°C, 80°C and 100°C, respectively. All the parameters except Δh_{T_r} values, which are reported for similar soils by Grant and Salehzadeh (1996), are obtained from Lu (2016).

Table 1. Input parameters for the soils used in the parametric study.

Soil	θ_s	n	Δh_{T_r} (J/m^2)	α (1/kPa)	ψ_c (kPa)	T_r(K)
Pachapa loam	0.46	2.179	-0.516	0.062	1200000	298
Seochang sandy clay	0.42	1.511	-0.516	0.079	1200000	

Fig. 2 illustrates the effective degree of saturation versus matric suction for Pachapa loam and Seochang sandy clay subjected to temperatures ranging from 25 to 100 °C. It is seen that, for a given suction, higher temperature leads to lower S_e. For instance, the reduction of S_e for Pachapa loam (Fig. 2a) is approximately 14, 24, 33 and 48% by increasing the temperature from 25 °C to 40 °C, 60 °C, 80 °C, and 100 °C, at matric suction of 50 kPa. The reduction of S_e for Seochang sandy clay (Fig. 2b) is approximately 10, 17, 23 and 36% by increasing the temperature from 25 to 40, 60, 80, and 100 °C, at matric suction of 100 kPa. For Pachapa loam with suctions higher than 100 kPa and Seochang sandy clay with suctions higher than 1000 kPa, the significance of temperature effect becomes less as the effective degree of saturation of soil reaches residual state.

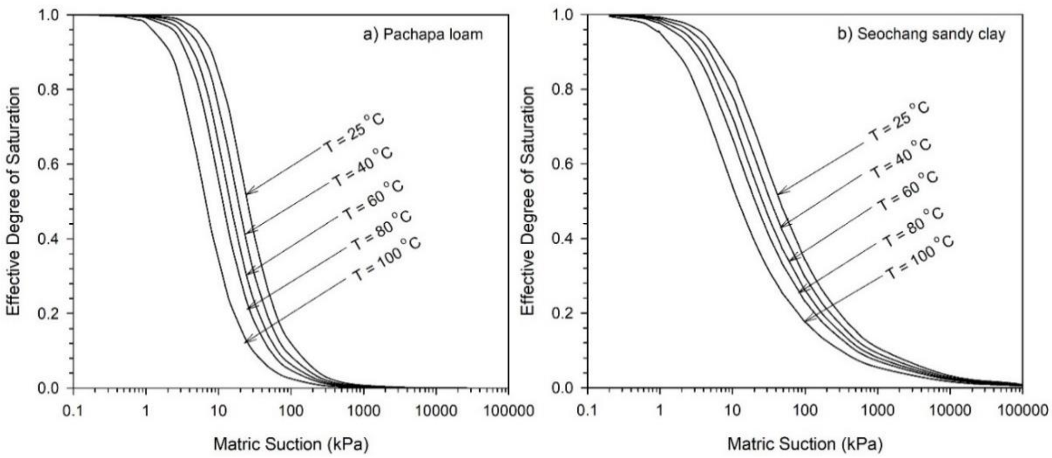

Fig. 2. Effective degree of saturation and matric suction at various temperatures for (a) Pachapa loam and (b) Seochang sandy clay

Fig. 3 shows the suction stress and matric suction for Pachapa loam and Seochang sandy clay at temperatures ranging from 25 to 100 °C. The trend of suction stress for both soils is generally affected by temperature-induced changes in matric suction and the effective degree of saturation. A non-monotonic trend is noticed for Pachapa loam. The absolute magnitude of suction stress first increases as matric suction increases. For each temperature, this increasing trend continues up to the magnitude of air entry suction (see Fig. 3a). Beyond the air entry suction where the soil desaturates and transitions to the capillary zone, the absolute magnitude of suction stress decreases as long as the suction increases. A monotonic behavior is observed in the case of Seochang sandy clay (see Fig. 3b). The absolute magnitude of suction stress increases with an increase in matric suction. Applying higher temperatures leads to a reduction in the air entry suction for both soils, implying that the unsaturated state starts earlier for higher temperatures. The temperature effect in the saturated zone (i.e. matric suction less than air entry suction) is consistent with the results reported by Campanella and Mitchell (1968).

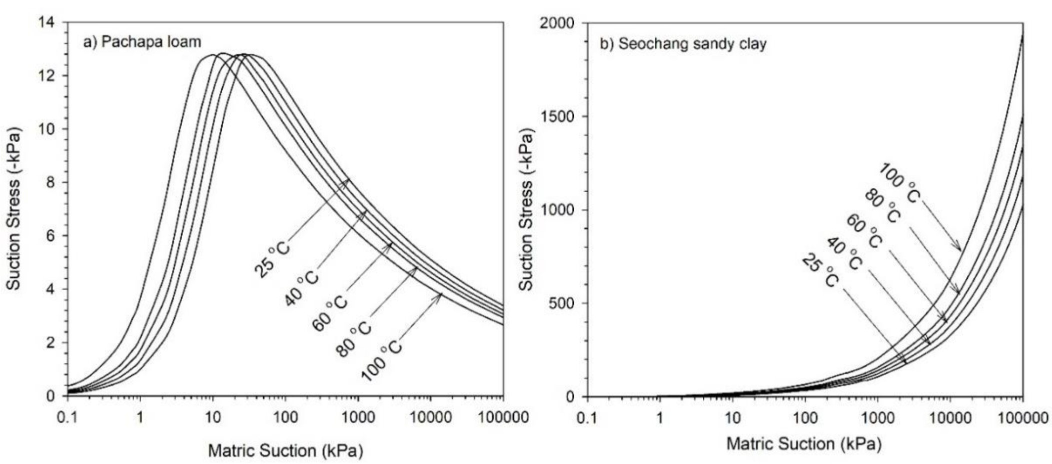

Fig. 3. Suction stress and matric suction at various temperatures for (a) Pachapa loam and (b) Seochang sandy clay

To explicitly show the effect of temperature on effective stress, parametric studies are performed at the total stress of 200 kPa, by accounting for the effect of temperature on the matric suction and effective degree of saturation. Fig. 4 illustrates the effective stress versus matric suction for Pachapa loam and Seochang sandy clay subjected to temperatures ranging from 25 to 100 °C. At each temperature, the variation in effective stress with matric suction of both Pachapa loam and Seochang sandy clay soils follows the similar trend of suction stress discussed in Fig. 3. Fig. 4(a) demonstrates the temperature dependency of effective stress for Pachapa loam. For each temperature the effective stress increases as matric suction increases reach a peak value at a certain magnitude of matric suction and then reduces with the matric suction. At a relatively lower matric suction, firstly the effective stress increases, then after attaining peak value decreases by increased temperatures.

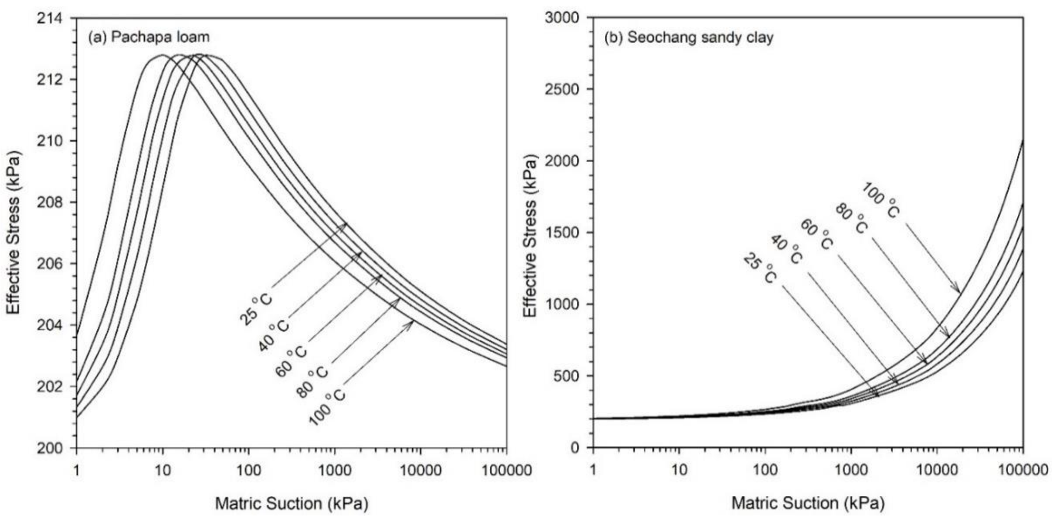

Fig. 4. Effective stress and matric suction at various temperatures for total stress = 200 kPa: (a) Pachapa loam and (b) Seochang sandy clay

A different trend is observed in Fig. 4(b) for Seochang sandy clay. The effects of temperature on capillarity become significant, thus the effective stress increases monotonically due to the increase of matric suction (while changes in the effective degree of saturation are minimal). The magnitude of matric suction that affects soil water retention properties can vary depending on the type of soil. For sandy soils, the low range of matric suction can cause significant reduction in the degree of saturation. However, relatively high matric suction is required to reduce the degree of saturation in clayey soils. The effective stress can vary differently with temperature depending on soil type and range of matric suction. For these two soils, the temperature-induced change in effective stress can be attributed to temperature-induced changes in surface tension, contact angle, and enthalpy of immersion (Vahedifard et al. 2018). This observation suggests that heating the soil can lead to a variation of effective stress, possibly explaining heat-induced hardening or softening depending on drainage conditions, soil type and suction as reported in the literature (e.g., Bolzon and Schrefler 2005; Cekerevac and Laloui 2004; Alsherif and McCartney 2015). It is important to note, imposing higher suctions at the boundary of the sample may not necessarily lead to an equal increase in the suction inside the disconnected trapped phases due to incompressibility of the wetting phase. Thermodynamic-based formulations, such as those employed in this study, may not properly describe suction evolution for the disconnected phases. Further studies are certainly needed to provide insight into the variation of nonisothermal effective stress for suction levels higher than the residual suction.

VALIDATION AGAINST EXPERIMENTAL DATA

Predicted values of temperature-dependent effective degree of saturation and suction stress from the proposed formulations are compared against the results of an experimental study conducted on Gaomiaozi bentonite (GMZ01) reported by Ye et al. (2012). The parameters used for predicting values: $\Delta h_{(T_r)}$ = -0.516 J/m^2, T_r = 293K, n = 1.31, α = 0.079 kPa^{-1} and ψ_c = 550 kPa.

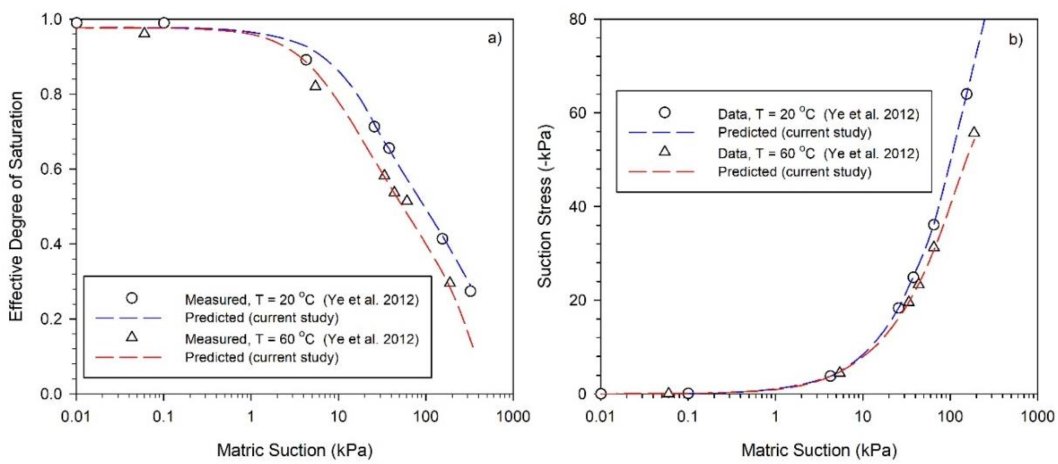

Fig. 5. Measured and predicted the effective degree of saturation and suction stress for GMZ01 bentonite at temperatures 20ºC and 60ºC

Fig. 5a shows a comparison between the experimentally measured effective degree of saturation data reported by Ye et al. (2012) and those predicted by the proposed model (Eq. 1) for GMZ01 bentonite at temperatures of 20ºC and 60ºC, respectively. The results show a very good agreement between the measured and predicted values. Similarly, Fig. 5b suggests that the suction stress values from the proposed model (Eq. 5) correspond very well with the data points that are interpreted from SWRC tests. It is to be noted that the experimental data points of suction stress are obtained based on the assumption that suction stress is equal to matric suction multiplied by the effective saturation. This assumption has been extensively tested and validated under ambient conditions (e.g., Lu et al. 2010). Further studies are needed to examine the validity of this assumption for temperature-dependent suction stress.

CONCLUSIONS

The current study presented a closed-form model to describe the effective stress of unsaturated soil under elevated temperatures. The nonisothermal effective stress expression is derived by considering the effects of temperature on matric suction and an effective degree of saturation. The temperature dependence of matric suction involves the effects of temperature on the surface tension, the soil-water contact angle and the enthalpy of immersion per unit area.

Employing the proposed formulation, parametric studies are performed to gain further insight into the effects of temperature on effective stress for two different soils. For Pachapa loam and Seochang sandy clay, applying higher temperatures leads to different trends in effective stress. In general, one can conclude that the effective stress trend versus temperature is heavily dominated by the temperature effect on the prevailing storage mechanism for each soil. The proposed model is also validated against the experimental of GMZ01 bentonite available in the literature. The results show a very good prediction of the model to laboratory results.

The proposed closed-form equation for the nonisothermal effective stress can be considered to be a unified framework for describing flow and stress in unsaturated soils under different temperatures. The findings of this study can be used to improve the representation of the SWRC and effective stress in applications that involve elevated temperatures.

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