# Stability Survey of 4WS Car with Double Lane Change Maneuver Test Track

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# STABILITY SURVEY OF 4WS CAR WITH DOUBLE LANE CHANGE MANEUVER TEST TRACK

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***Abstract*—When a vehicle travels along a roadway, its path of motion becomes highly intricate. The presence of lateral forces and tire deformations means the vehicle's actual trajectory deviates from the intended path, particularly at higher speeds. This discrepancy can compromise vehicle stability, potentially culminating in accidents. To mitigate these issues, the four-wheel steering system (4WS) or all-wheel steering system allows for the manipulation of all wheels , enhancing stability at elevated speeds and diminishing the steering radius at lower velocities. This research examined vehicular turning scenarios using the dual lane change test as outlined in ISO 3888-2:2011. In scenarios where vehicle control was implemented, cars adhered closer to the ideal trajectory and successfully navigated through the test corridor**

***Keywords— Four wheel active steering, PID controller, 4WS , 4 Wheel Steer, Active steering, ISO 3888-2:2011***

## I. INTRODUCTION (*HEADING 1*)

The Four-Wheel-Steering (4WS) system, an innovation with roots tracing back to early implementations[1][2][3], originally found its application within the realms of off-road and military vehicles. Its primary function aimed at minimizing the turning radius, thereby enhancing the vehicle's maneuverability. This was achieved through a configuration where the front and rear wheels steered in opposite directions, a mechanism dictated by the mechanical drive system interlinking them. In contemporary automotive engineering, the 4WS system has transcended its initial utility. Beyond merely reducing the turning radius at lower speeds, it plays a pivotal role in augmenting the vehicle's stability during high-speed directional shifts. This advancement sees the rear wheels, now capable of independent control, aligning their rotation with that of the front wheels to serve the dual objectives of improved maneuverability and enhanced directional stability.

### *A. Mechanically driven four-wheel steering.*

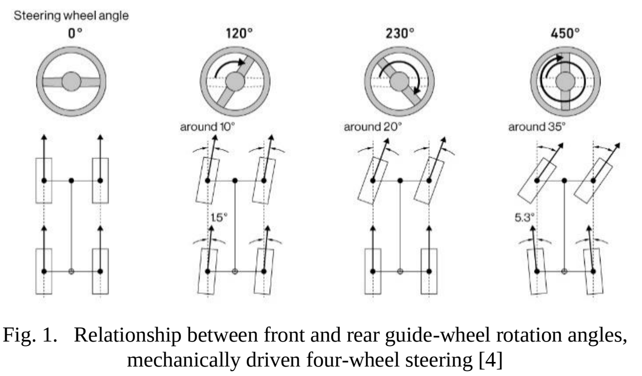


Fig. 1. Relationship between front and rear guide-wheel rotation angles, mechanically driven four-wheel steering [4]

Within the domain of mechanically actuated Four-Wheel-Steering (4WS) systems, the orientation of the rear wheels is intricately linked to the rotational angle of the front wheels through a sophisticated mechanical linkage. This correlation is depicted in Figure 1, illustrating a nuanced control strategy: at elevated velocities, characterized by minimal steering input (rotational angles ranging from 0° to 12°), the rear wheels are aligned to rotate congruently with the front wheels. Conversely, under low-speed conditions where the steering angle exceeds 45°, the rear wheels adopt an antiparallel alignment, rotating in a direction opposite to that of the front wheels. The mechanical architecture underpinning the 4WS system, despite its effectiveness in enhancing vehicular agility and stability, is noted for its structural complexity, significant mass, elevated cost, and a potential decrement in response sensitivity.

### *B. Hydraulically driven four-wheel steering.*

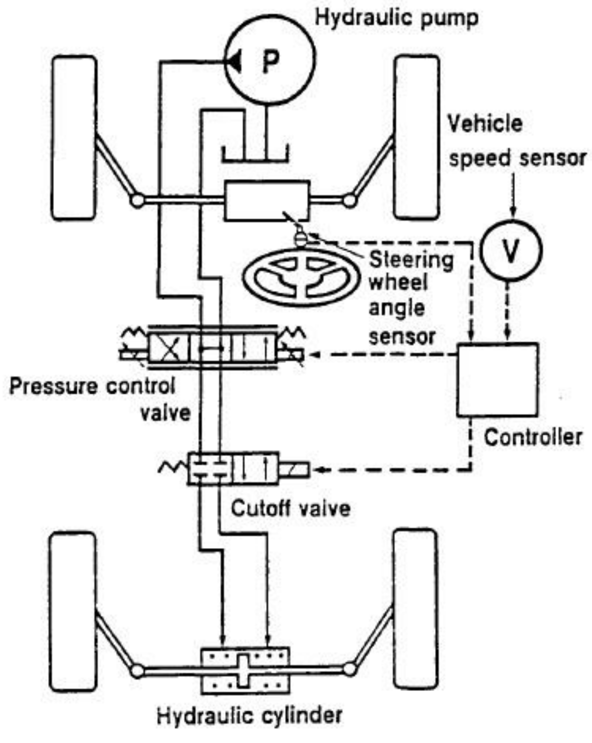


Fig. 2. Hydraulically driven four-wheel steering [5]

The hydraulic four-wheel steering (4WS) system, distinguished by its utilization of hydraulic pressure to modulate the rear wheels' rotational angle as depicted in Figure 2, has been incorporated into vehicles such as the Nissan Skyline and Mitsubishi Gallant. This system, akin to its mechanically driven counterparts, significantly diminishes the vehicle's turning radius at lower speeds while concurrently enhancing the stability of its trajectory at higher velocities [5]. Distinctively, the adjustment of the rear wheels' angle in this hydraulic variant is governed by an electronically controlled hydraulic mechanism. This sophisticated system dynamically calibrates the rear wheel orientation in response to a multitude of vehicular parameters, including velocity, steering wheel angle, lateral displacement, horizontal acceleration, and the angular velocity of the vehicle's chassis. Through this integration of electronic control and hydraulic actuation, the system achieves a nuanced and responsive adaptation to varying driving conditions, thus offering a marked improvement in both maneuverability and high-speed stability.

### C. *Electronically driven all-wheel steering*

The electronic four-wheel steering (4WS) system represents a pivotal advancement in vehicular dynamics, leveraging an electric actuator to meticulously adjust the orientation of the rear wheels, as illustrated in Figure 3. This system orchestrates the rear wheel steering through a sophisticated algorithm that integrates multiple vehicular dynamics and kinematic parameters. These parameters encompass the vehicle's velocity, the angle of the steering wheel rotation, and a suite of rotational kinematics including the lateral movement angle, the angular velocity of the vehicle's body, and its horizontal acceleration.

This integration allows for a highly responsive and adaptive steering mechanism, capable of optimizing the vehicle's handling characteristics across a diverse range of driving conditions. By dynamically adjusting the rear wheels' alignment in concert with the front wheels, the electronic 4WS system significantly enhances the vehicle's agility, stability, and cornering performance. This technological evolution marks a significant departure from traditional steering systems, offering a more nuanced and precise control over vehicle dynamics.

Fig. 3. Electronically driven 4WS steering system [6]

*Rear Wheel Control Status.*

The 4WS steering system has rear-wheel drive states (Fig. 4).

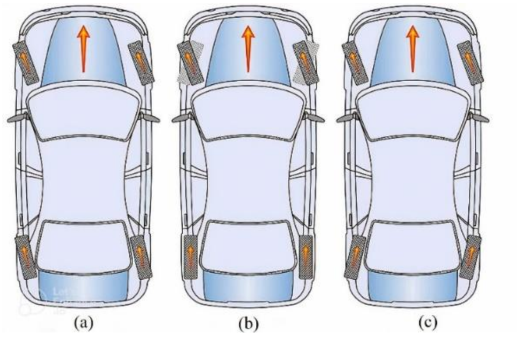


Fig. 4. 4WS steering system with 3 rear wheel control states

Figure 4 presents a comprehensive visual analysis of the operational dynamics within a four-wheel steering (4WS) system, categorized into distinct functional modes based on the orientation of wheel rotation and its implications on vehicle handling characteristics:

- Figure 4(a) illustrates a scenario where both the rear and front wheels are aligned to rotate in the same direction. This configuration is instrumental in diminishing the vehicle's centrifugal force during cornering, thereby enhancing lateral stability and reducing the likelihood of understeer or oversteer at higher speeds.
- Figure 4(b) delineates a condition in which the steering mechanism operates under a conventional two-wheel steering (2WS) paradigm, essentially guiding the vehicle through the front wheels alone. This mode is typical of standard vehicular operation, where the primary steering input is directed through the front axle, offering a balanced approach to handling under normal driving conditions.
- Figure 4(c) depicts a contrasting mode of operation, where the rear and front wheels are steered in opposite directions. This antiparallel wheel alignment significantly reduces the vehicle's turning radius, thereby substantially increasing its maneuverability at lower speeds. This mode is particularly advantageous in urban environments or tight spaces, where the ability to execute sharp turns is paramount.

These illustrations collectively underscore the versatility and adaptability of the 4WS system, showcasing its capacity to dynamically alter vehicle handling properties in response to varying driving conditions, thereby optimizing performance, safety, and driver control.

## II. CONTROL STRATEGIES

Vehicles equipped with all-wheel steering can be broadly categorized into two distinct types: load-dependent control systems, referred to as passive control, and systems that are contingent upon velocity and the steering wheel's rotation angle, known as active control. Scholarly research focusing on the optimization of the 4WS (four-wheel steering) system has been extensively documented, with particular emphasis on enhancing attributes that significantly contribute to vehicular motion stability [7, 8]. The primary objective of 4WS control is to augment mobility at lower velocities and ensure stability at higher speeds. It is often achieved through a synergistic application of Active Front Steering (AFS) and Active Rear Steering (ARS) controls, collectively termed as 4WAS (Four-Wheel Active Steering) [9, 10].

The vehicle's chassis incorporates various control systems including those for braking, steering, and actuation. The operational management of the body-rotation-angle-stability-control-system encompasses several strategies:

- Direct manipulation of body-rotation-angle torque;
- Implementation of Active Steering Control;
- A holistic approach that integrates direct control over vehicle body rotation torque with the active steering system.

Within the realm of active steering control, the system dynamically adjusts the angle of the steering wheel as determined by the driver, by incorporating a compensatory angle derived from the control system. This method of active steering control is particularly efficacious in enhancing vehicular stability, especially under conditions where the lateral tire forces remain within linear ranges. Numerous studies have been conducted on active steering control, underscoring its efficacy in bolstering motion stability [7, 13]. Active steering control can be further subdivided into three primary categories:

- Active Front Steering Control (AFS);
- Active Rear Steering Control (ARS);
- Four-Wheel Active Steering (4WAS).

For vehicles equipped with conventional front-drive systems, AFS control has emerged as a focal area of research due to its potential for integration with braking or suspension control systems, as outlined in Table I. This comprehensive approach to vehicle steering and stability control underscores the ongoing advancements in automotive engineering, aimed at enhancing the safety, performance, and maneuverability of modern vehicles.

TABLE I. OPTIONS FOR STABILIZING THE CAR'S MOTION TRAJECTORY WITH THE STEERING

| | **Stable control of the trajectory of movement of the car** | | |
|---|---|---|---|
| | ***AFS*** | ***ARS*** | ***4WAS*** |
| Advantage | -Effective for steady-state driving conditions;<br>-Easy Integration with Brake-Control-System<br>-Good for body rotation velocity control. | - Rear Steering Angle Control;<br>- Good for body rotation velocity control. | -Two different input-steering-signals;<br>- Good for body rotation velocity control. |
| Defect | Less effective in dangerous driving conditions | Less effective in dangerous driving conditions | Less effective in dangerous driving conditions |

Active Rear Steering (ARS) control plays a pivotal role in augmenting vehicular rotation dynamics at lower velocities, with the system primarily responding to the δr, which represents the angle of rear wheel steering. In the pursuit of achieving enhanced maneuverability at low speeds juxtaposed with heightened stability at elevated velocities, a harmonized integration of Active Front Steering (AFS) and ARS control, collectively termed as Four-Wheel Active Steering (4WAS), is routinely employed. The operational premise of 4WAS control encompasses an intricate analysis, wherein the body rotation angle signal and horizontal motion vectors are treated as discrete, independent inputs.

This methodological approach to steering control underlines a sophisticated paradigm in automotive engineering, focusing on the nuanced balance between agility and stability across the speed spectrum. By treating the body rotation angle and horizontal motion as separate entities, the 4WAS system facilitates a more responsive and adaptive steering feedback mechanism. This, in turn, significantly contributes to the vehicle's dynamic stability, ensuring a safer and more controllable driving experience.

The strategic amalgamation of AFS and ARS into a unified 4WAS framework epitomizes the cutting-edge advancements in vehicular steering systems. Through the precise modulation of steering angles at both the front and rear axles, 4WAS adeptly manages the complex interplay between maneuverability and stability, thereby setting a new benchmark in automotive handling performance.

## III. ANALYSIS OF STABILITY OF THE ORBIT WHILE ROTATION

In the study of vehicle dynamics, the classic Single-Trace-Model is shown in Fig. 2. The Single-Trace-2-DOF-Model is widely used to analyze and control controller-stability [11–16]. Dieses Modell ist linearized from a nonlinear Equation based on the following Assumptions: The force acting on the tire in the linear zone; Cars moving on flat surfaces/flat roads; The left and right wheels at the front and rear axles are replaced by an equivalent wheel at the centerline of the vehicle; Wheel speed remains constant; The steering angle and side-slide angle are considered very small (≈ 0); The Wheel has no longitudinal force; The Center of gravity does not change when the vehicle mass changes; Watch Cars move easily.

$\dot{x} = 0$ ; $\delta_f$ ; $\delta_r$ Small. Consider the One-Trace-Model: vertical shift x, horizontal shift y and rotation angle quang trục vertically passing through the center of gravity of the car. $\psi$ Following the XCY mobile coordinate-axis-system mounted at the center of gravity C of the car [17].

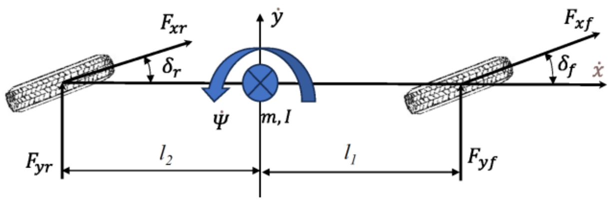


Fig. 5. Single trace model of the 4WAS steering system

$$\begin{bmatrix} \ddot{y} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -\dfrac{C_{\alpha_f}+C_{\alpha_r}}{mv_0} & -\left(v_0-\dfrac{l_1C_{\alpha_f}-l_2C_{\alpha_r}}{mv_0}\right) \\ \dfrac{l_1C_{\alpha_f}-l_2C_{\alpha_r}}{Jv_0} & -\dfrac{l_1^2C_{\alpha_f}+l_2^2C_{\alpha_r}}{Jv_0} \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dfrac{C_{\alpha_f}}{m} & \dfrac{C_{\alpha_r}}{m} \\ -\dfrac{l_1C_{\alpha_f}}{J} & \dfrac{l_2C_{\alpha_r}}{J} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}$$

Write briefly

$$\begin{bmatrix} \ddot{y} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix}$$

Where

$$a_{11} = -\frac{C_{\alpha_f}+C_{\alpha_r}}{mv_0}, a_{12} = -\left(v_0 - \frac{l_1C_{\alpha_f}-l_2C_{\alpha_r}}{mv_0}\right)$$

$$a_{21} = \frac{l_1C_{\alpha_f}-l_2C_{\alpha_r}}{Jv_0}, a_{22} = -\frac{l_1^2C_{\alpha_f}+l_2^2C_{\alpha_r}}{Jv_0}$$

$$b_{11} = \frac{C_{\alpha_f}}{m}, b_{12} = \frac{C_{\alpha_r}}{m}, b_{21} = -\frac{l_1 C_{\alpha_f}}{J}, b_{22} = \frac{l_2 C_{\alpha_r}}{J}.$$

The characteristic polynomial *D(s)*, the characteristic polynomial of the system, *D(s)*, is defined by the determinant of *sI–A*:

$$\begin{aligned} D(s) &= \det(sI - A) \\ &= \det\begin{bmatrix} s - a_{11} & -a_{12} \\ -a_{21} & s - a_{22} \end{bmatrix} \\ &= (s - a_{11})(s - a_{22}) - (-a_{12})(-a_{21}) \\ &= s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21} \end{aligned}$$

Replace *a11, a12, a21, a22* with specific values, we get *the last* D(s):

$$D(s) = s^2 + \left( \frac{C_{\alpha_f} + C_{\alpha_r}}{mv_0} + \frac{l_1^2 C_{\alpha_f} + l_2^2 C_{\alpha_r}}{Jv_0} \right) s + \frac{C_{\alpha_f} C_{\alpha_r} (l_1 + l_2)^2}{Jmv_0^2} + \frac{l_2 C_{\alpha_r} - l_1 C_{\alpha_f}}{J}$$

The transfer function from this transfer function expresses the relationship between the steering input and the responses from the system in the frequency domain $\delta f$ to $\dot{y}$ given by the first element of $(sI - A)^{-1B}$ :

$$\begin{aligned} \frac{\dot{y}}{\delta_f} &= \left[ (sI - A)^{-1} B \right]_{1,1} \\ &= \frac{1}{D(s)} \left[ (s + a_{22}) b_{11} - a_{21} b_{12} \right] \\ &= \frac{s\dfrac{C_{af}}{m} + a_{22}\dfrac{C_{af}}{m} - a_{21}\dfrac{C_{ar}}{m}}{D(s)} \end{aligned}$$

Similar calculation for the magnetic transfer function $\delta_r$ come $\dot{y}$

$$\frac{\dot{y}}{\delta_r} = \left[ (sI - A)^{-1} B \right]_{1,2}$$

How to find the transfer function from the front steering angle $\delta_f$ and rear $\delta_r$ to the same vehicle body angular velocity, using other appropriate elements in the matrix $(sI - A)^{-1}B$.

$$\frac{\dot{\psi}}{\delta_f} = \left[ (sI - A)^{-1} B \right]_{2,1}$$

$$\frac{\dot{\psi}}{\delta_r} = \left[ (sI - A)^{-1} B \right]_{2,2}$$

Following the XCY This transfer function represents the relationship between the steering input and the responses from the system in the frequency domain. The PID controller is designed to minimize the difference e(t) between the desired vehicle body rotation angular velocity and the actual vehicle body rotation angular velocity. Desired value is determined on the basis of the ideal rotation model of the 2WS steering system, based on the average rotation angle of the front guide wheels δf and the cars moving speed v0 [18].

$$\dot{\psi}_d = \frac{v_0 \delta_f}{l_1 + l_2 + \dfrac{mv_0^2 (l_2 C_{\alpha r} - l_1 C_{\alpha f})}{C_{\alpha f} C_{\alpha r} (l_1 + l_2)}}$$

In which: $l_1$ and $l_2$ – are the distance from the car's center of gravity to the front and rear axles, respectively; m – mass of the Car; $C_{\alpha f} C_{\alpha r}$ – are the lateral stiffness of the front and rear axle wheels, respectively.

$$e(t) = \dot{\psi}_d(t) - \dot{\psi}(t)$$

Choose the parameters Kp = 0.017; Ki = 0.111; Kd = -0.0002 for PID controller.

## IV. SURVEY RESULTS

On the test track according to ISO 3888-2:2011 standards, the driver steers from 0,5 second to 1 second to achieve the front steering wheel rotation angle $\delta_f = 6^0$, then keeps the steering wheel still for 1 second. From the 2nd to the 2.5th second, the driver turns the steering wheel to return the steering wheel's rotation angle to the straight moving position and continues to steer in the opposite direction with the above cycle. The speed when cornering is 45 [Km/h]. The survey results for the cases (Table II), with the front and rear guide wheel rotation angles (Figure 6) are shown in Figures 7 to 9.

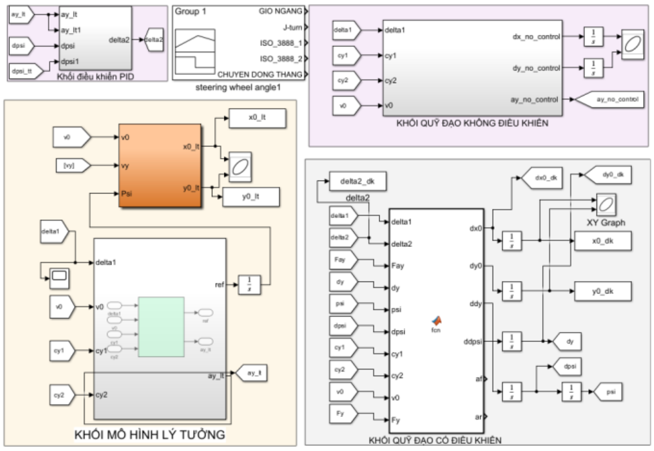


Fig. 6. Matlab Simulink Simulation Model

TABLE II. SYSTEM SURVEY CASES

| | **Stable control of the Car's Motion Trajectory** | | |
|---|---|---|---|
| | ***Turn around correctly*** | ***Turn over excess*** | ***Turn around, missing*** |
| $C_{\alpha_f}$ | 90000 | 85000 | 90000 |
| $C_{\alpha_r}$ | 90000 | 90000 | 60000 |

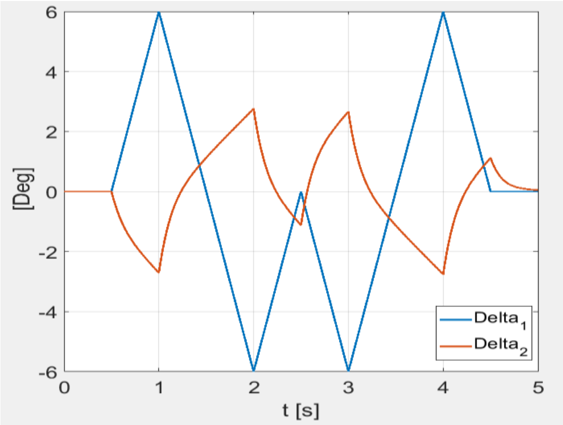


Fig. 7. Front and rear guide wheel rotation angle

Case 1:

$$C_{\alpha_f} = C_{\alpha_r} = 90000$$

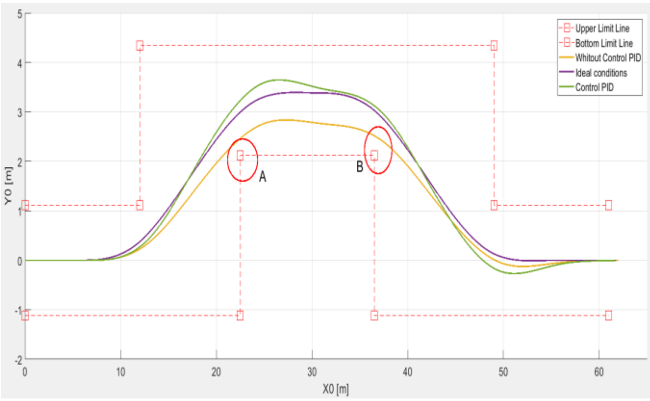


Fig. 8. Correct turnaround case

Case 2:

$$C_{\alpha_f} = 85500;\ C_{\alpha_r} = 90000$$

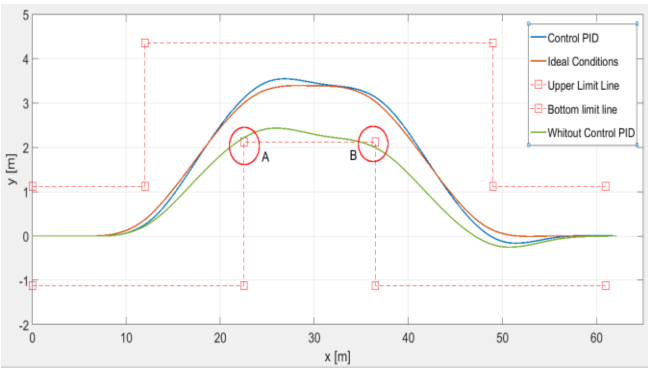


Fig. 9. Case of excess turnaround

Case 3:

$$C_{\alpha_f} = 90000;\ C_{\alpha_r} = 60000$$

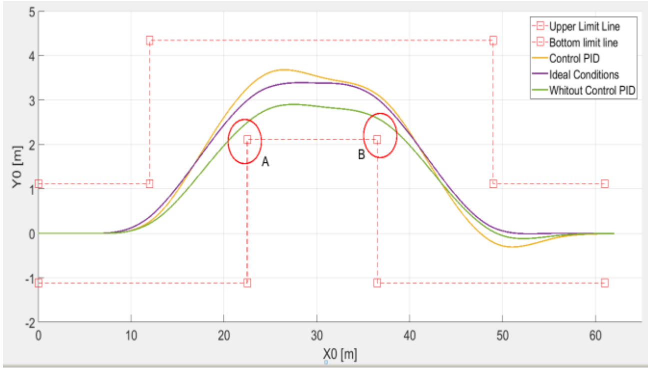


Fig. 10. Missing turnaround cases

## CONCLUSION

The trajectory of a vehicle navigating a roadway is subject to a complex array of factors, where lateral forces and tire deformation significantly influence the deviation of the vehicle's actual path from its intended course. Deviation ist especially pronounced at higher velocities, leading to potential stability-compromises that elevate the risk of vehicular accidents. In response to these challenges, the implementation of a four-wheel steering system (4WS), also known as an all-wheel steering system, emerges as a critical innovation.

System facilitates the manipulation of all four wheels, thereby enhancing vehicular stability at high speeds and reducing the turning radius at lower speeds. Studie explores the efficacy of 4WS in vehicular turning scenarios, employing the dual-lane-change-test-parameters, set forth in ISO 3888-2:2011. It was observed that, in instances where vehicle-control-mechanisms actively engaged, there was a marked improvement in adherence to the ideal trajectory, enabling successful navigation through the testcorridor.

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