The Discrete Element Method for Vehicle-Terrain Analysis

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John F. Peters[a,1], Farshid Vahedifard[b,2,*], Bohumir Jelinek[a,3], George L. Mason[a,4], Jody D. Priddy[c,5]

[a] *Center for Advanced Vehicular Systems (CAVS), Mississippi State University, Mississippi State, MS 39762, USA*
[b] *Mississippi State University, Mississippi State, MS 39762, USA*
[c] *US Army Engineer Research and Development Center (ERDC), Geotechnical and Structures Laboratory (GSL), Vicksburg, MS, 39180, USA*

Abstract

Numerical vehicle performance simulations can be used to better interpret field tests and can give preliminary data for vehicles still in the planning and design stage and no physical prototype yet exists. Although high-fidelity models exist for vehicle mechanics, vehicle-terrain interaction is still in its infancy. The discrete element method (DEM) is an increasingly popular method to model complex mechanics of wheel-terrain interaction on unpaved surfaces. This paper describes recent work with very large-scale DEM computations for subgrade modeling.

The DEM inherently captures significant phenomena such as localized shear deformation, friction-dependent resistance, and soil dilatancy, all items difficult to capture using continuum-based methods. The principal limitation of DEM is the large number of particles required to achieve a reasonably small ratio of discrete element radius to tire size within sufficiently large test bed dimensions. Parallel Computing technology has advanced to a point that minimal computing requirements can be met. Comparing simulated results to those in the extensive DROVE database demonstrates the accuracy of traditional drawbar-pull tests and allows evaluation of adequacy of particle size, test bed dimensions. Issues remain concerning calibration to achieve realistic engineering properties, placement density and use of cone penetration simulations to assess subgrade strength. An investigation is underway on simulations of transient traction development including transitions during reversal of slip.

Keywords: Vehicle simulation, Discrete Element Method, Particle Methods, Calibration

*Corresponding author

Email addresses: john@cavs.msstate.edu (John F. Peters), farshid@cee.msstate.edu (Farshid Vahedifard), bj48@cavs.msstate.edu (Bohumir Jelinek), mason@cavs.msstate.edu (George L. Mason), Jody.D.Priddy@erdc.dren.mil (Jody D. Priddy)

[1] Associate Research Professor, Center for Advanced Vehicular Systems

[2] Associate Professor, Dept. of Civil and Environmental Engineering, and Center for Advanced Vehicular Systems

[3] Assistant Research Professor, Center for Advanced Vehicular Systems

[4] Assistant Research Professor, Center for Advanced Vehicular Systems

[5] Research Civil Engineer, US Army Engineer Research and Development Center

1. Introduction

Numerical modeling of off road performance depends on accurately simulating large discontinuous soil deformation. Much progress has been made in the continuum approach to problems such as tool cutting and grinding (Özel and Altan, 2000; Brinksmeier et al., 2006), although applying these methods to realistic off-road mobility analysis is limited by the complexity of soil behavior. Frictional materials such as soil are subject to instabilities including as shear localization, pore water pumping, and spontaneous liquefaction. Constitutive models that capture such phenomena lead to ill-posed continuum formulations (Valanis and Peters, 1996), a problem that has shadowed geotechnical numerical analysis since the introduction of the finite element method.

The discrete element method (DEM) introduced by Cundall and Strack (1979) removes these problems because the computational idealization mirrors the actual mechanics of the granular media. Particle motion need not be continuous, a length scale is inherent in the method and instabilities such as shear localization are emergent properties of the particle interactions.

The DEM has been coupled to finite element method (FEM) to simulate very difficult mechanics problems such as soil tilling (Shmulevich et al., 2007; Shmulevich, 2010), and soil tire interaction (Nakashima and Oida, 2004; Nakashima et al., 2010; Zhao and Zang, 2017; Michael et al., 2015; Smith and Peng, 2013) including sophisticated coupling of rigid-body vehicle models and granular subgrade modeled (Recuero et al., 2017; Wasfy et al., 2016). The potential of these modeling techniques for unprecedented vehicle designs is emphasized by Li et al. (2010) and Knuth et al. (2012), where vehicles in the pre-prototype phase can be modeled in extra-planetary environments.

The principal limitation of DEM is the large computing requirements for practical problems. DEM implemented in parallel computers using the message passing interface (MPI) was recognized nearly two decades ago as a viable alternative to continuum methods for mobility problems (Carrillo et al., 1996, 1999; Horner et al., 1998, 2001). At that time the limitations in both computing capabilities and understanding of granular physics put the method at the boundary between academic investigations and practical applications. Cundall (2001) suggested 20 years would be needed to make discrete element methods viable for practical engineering analysis.

In the time since, both the computer hardware and the basic understanding of the granular mechanics (Kuhn, 2017) underlying the method has brought DEM into the realm of practical analysis (Steuben et al., 2016; Recuero et al., 2017). This assessment is certainly valid for many applications that can be solved for particles counts of 10^6 or less. Materials handling applications and basic granular media research dominate such problems (Shimizu and Cundall, 2001; Bharadwaj et al., 2012; Cleary, 2009). However, computer technology now puts problems with particle counts in the range $10^7 - 10^8$ possible. For example, the 93 million particle simulation shown in Figure 1 was performed in 84 cpu hours using 3384 processors. Although such an effort is formidable, the cost of obtaining data from physical tests using power trains not yet fabricated is also formidable. Moreover, the aforementioned advantages of being able to compare various designs using precisely

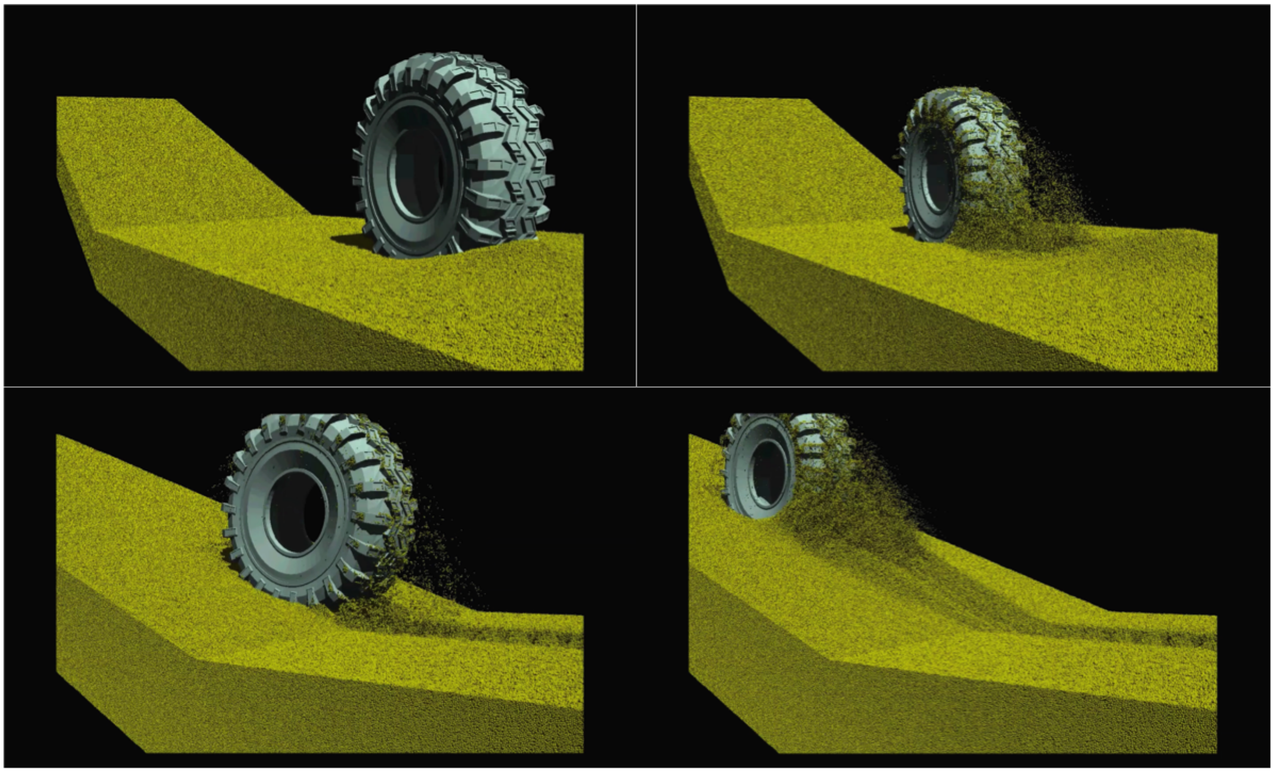


Figure 1: Scenes from simulation of tire traversing two-sloped terrain. The simulation consisted of 93 million particles computed using 3384 processors. Approximately 84 hours were required for 1 second of simulation. Particle diameters ranged from 0.08 to 0.24 inches. The tire diameter was 37.32 inches.

the same particle bed and the accessibility of detailed data on kinematics and associated traction development far exceeds anything possible from physical experiments. In addition to MPI implementations, Recuero et al. (2017) and Gan et al. (2016) have developed significant systems using graphic processing units (GPU).

This paper considers a relatively bare-bones MPI DEM implementation for modeling spherical particles with its principal attribute being the ability to use very large particle counts. The paper focuses on the technical challenges for making this seemingly limited scoped model into a practical tool for mobility analysis by presenting a systematic study of accurate calibration, initial particle placement including the initial stress and porosity state and their effect on results. For example, calibration of the particle interaction parameters using laboratory experiments is examined from a practical viewpoint including the role of rotation resistance in capturing shear localization. An important link between laboratory strength and field performance is made using mobility cone simulations.

2. The Test Bed

A numerical test bed is a collection of computer codes that can be used for simulations, problem development (pre-processing), and analysis of results (post-processing). The simulation part of the test bed can include multiple codes coupled to produce complex multi-physics simulations (Howington et al., 2012). The test bed components considered here, and shown in Fig 2, is a numerical system to simulate granular medium in various practical situations using the DEM. The test bed includes a simulated laboratory specimen to support material calibration, calibration chamber (Fig 2a) to simulate the mobility cone (or similar device), and a box of granular material on which to simulate vehicle-soil interaction. The state of the simulated particles can be probed (Fig 2b) using a mobility cone, thus establishing a link between laboratory and the field states. In each case the simulations involves objects that interact with particles and the particles themselves such as the wheel on a bed of particles shown in Fig 2c. A particular advantage of the DEM idealization versus FEM is illustrated in Fig 2d, which shows the cone tip and particle surface without the mathematical complication of a singularity. The objects are created by tessellating their surfaces with triangular finite elements that interact with the particles (see Horner et al. (1998)). In the work described here the objects are assumed rigid and non-interacting, although the scheme does not preclude more complex deformable interacting objects.

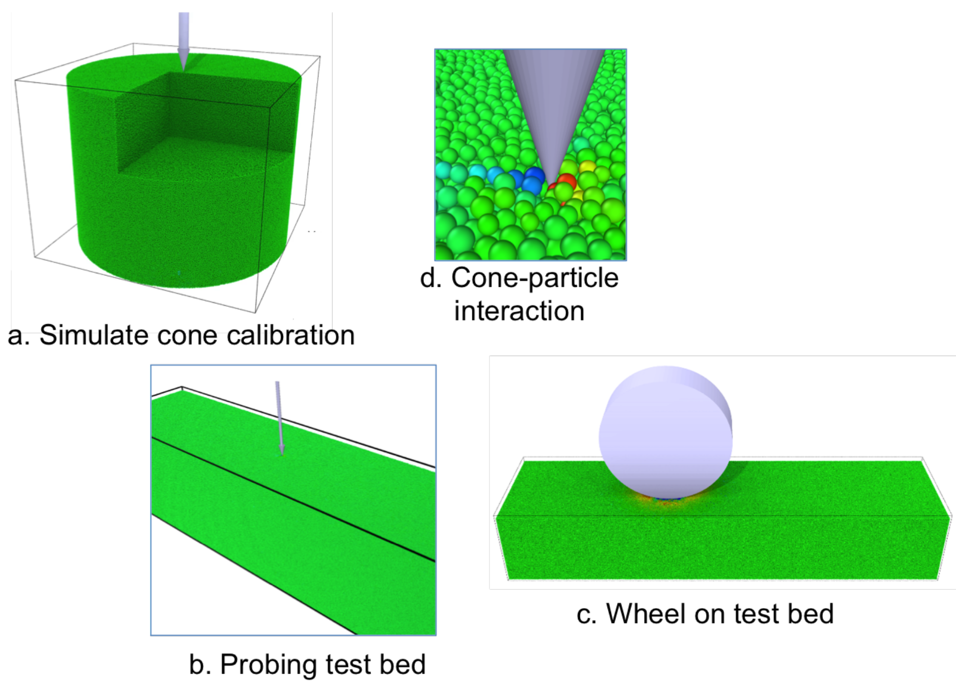


Figure 2: Simulation examples: a) Cone calibration, b) Probing test bed, c) Wheel simulation, d) Interaction of cone with particles

2.1. *Particle Attributes*

The DEM, as used for the study of micro-mechanics, is capable of modeling details such and grain sizes, shape and grain breakage. In principle, the particle attributes and material properties can be measured independently (Cole and Peters, 2007; Cole et al., 2012). In contrast, the DEM media used for prototype-scale analyses is not an exact representation of the physical soil media. The most important difference between the physical versus simulated particle is size. The number of simulated particles is generally limited to a few million, which precludes a one-to-one correspondence between real and simulated particle sizes. Moreover, there is a practical limit to the size difference between the largest and smallest particles making it difficult to match the shape of the grain size distribution curve. In practice, the largest particle used in the simulation is roughly twice the size of the smallest simulated particle, that difference selected by experience to limit "crystallization" of the particle into regular close-packed arrays. Therefore, the simulated particle-size distribution is effectively independent from that of the real soil.

Real soil particles are non-spherical, an important attribute for limiting the particle rolling that can lead to pathological instabilities in the macro-scale stress-strain response. However, there is great computational advantage to using spherical particles. The particle rolling is limited in simulations by introducing a resistance to relative rotation at the contacts. The simplification of both particle sizes and shapes requires an ability to reproduce the correct engineering behavior by adjusting the contact parameters and porosity. In effect, the relationship between the particle-scale details and the macro-scale engineering response must be non-unique such that many micro-scale idealizations can lead to the same macro-scale constitutive response.

2.2. *Contact parameters*

The contact parameters control the forces and moments between the particles in response to their relative motion. For non-bonded particles the principal contact modes are:

1. Normal mode response that relates the normal force acting between particles to the relative motion in the normal direction
2. Shear mode response that relates the tangential force to the relative motion in the tangential direction
3. Rotational mode response that relates a contact moment to the relative rotation between the particles

For each mode there are elastic and inelastic components.

2.3. *Energy Dissipation*

The force-displacement response in the normal mode is essentially hysteretic such that the unloading response is stiffer that the loading response, creating a rate-independent damping of energy. Cycles in the unload-reload portion of the response is elastic unless further "internal variables" are included to monitor load reversals. More complex hysteretic effects to dissipate energy during sub-cycle loading thus create additional complexity and computational time. For these sub-cycling cases, viscous damping is used. Limiting the use of viscous damping only for the sub-cycles removes high-frequency oscillations without adding excessively viscous response to the particle assemblage.

Both shear and rotational modes are characterized as simple elastic-plastic responses with the limiting contact force being proportional to the normal force. The frictional limit naturally imposes energy dissipation, although for the elastic portion of the response, viscous damping is applied similar to the normal load.

2.4. Porosity and Residual Stress

The engineering behavior depends strongly on the soil porosity. Porosity is a volume-averaged state that is determined by the particle initialization. This is in contrast to the contact parameters which are particle properties that can be simply assigned. Therefore, the calibration must be performed for a specific porosity, which must be achieved in the initialization process. Often, to get very low porosity some simulated compaction must by imposed, usually by surcharging the soil mass with a loading plate. Such loading can impose residual stress in the mass that increases the frictional strength of the soil mass.

3. Initialization

One of the most onerous part of the DEM simulation is specifying the initial particle placement. The particles must fill the domain such that the target porosity is met with the particle forces in an equilibrium state. For simple domains, particles can be settled into place from some initial positions similarly to sand pluviation of laboratory specimens. Particle settling is time consuming and can be impractical for creating sloping surfaces.

The method added for the DEM tool kit that overcomes some of these deficiencies is a combination lattice and growth method. The lattice is created by a tetrahedral mesh similar to that used for finite element analyses and can create any domain that is discretized using finite elements. This method is similar to a procedure described by Jerier et al. (2010).

The method is summarized in Fig 3 using the Open Source meshing tool *TetGen* (Si, 2015). The domain boundaries are created using a standard syntax for producing volumes using polyhedrons (Fig 3a). *TetGen* is then used to create a tetrahedral mesh of the domain volume (Fig 3b). Two sets of particles are placed: particles are inscribed within each tetrahedral and particles are placed at each node (Fig 3c). The particles are then moved and grown through a sequence of interactions to produce an assemblage such that particles just touch (Fig 3d). From this point, the domain can be subjected to boundary and gravity loads to create the initial particle state. A key advantage of this particle generation method is that the iterative steps to place particles in their ultimate locations employs a simple geometrically based logic that does not employ DEM equations of motion, thus obviating the limitation of small computational time steps. Note the initial "just touching" state is required to avoid excessive (and explosive) initial elastic energy created by arbitrary particle overlaps.

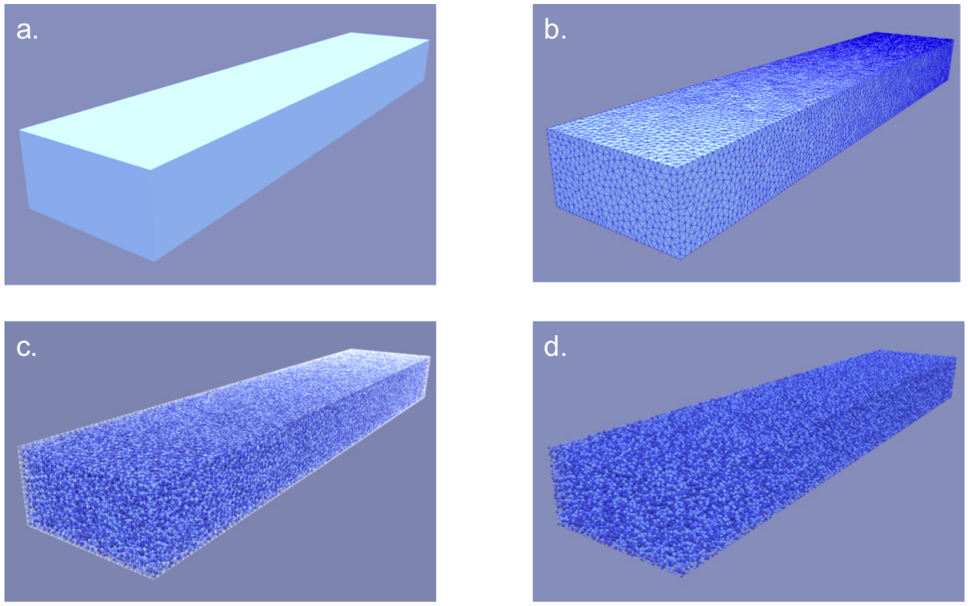
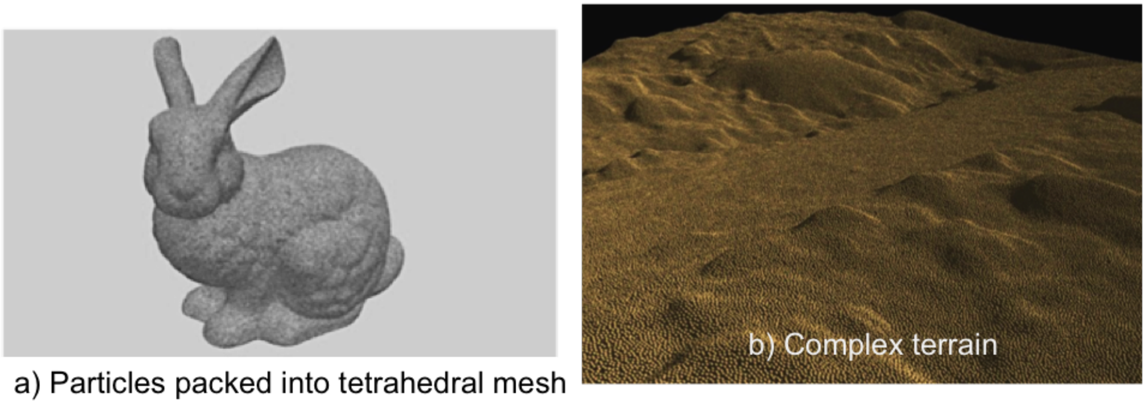


Figure 3: Particle domain generation: a. Create domain boundaries using format supported by mesh generator, b. Generated tetrahedral mesh, c. Generate inscribed spheres and spheres placed at nodes, and d. Move and grow spheres.

The advantage of the particle placement method is illustrated in Fig 4. Fig 4a is an example of the capability to model highly complex boundary geometries that could not be achieved using simple sedimentation methods. Fig 4b shows a natural undulating land surface created by combining LIDAR measurements with meshing software to create an initial grid. Even relatively smooth sloping landscapes, as in Figure 1, would be difficult to create using simple sedimentation-based placement.

4. Calibration

Figure 5 shows the elements of the calibration problem. The DEM computation involves the motion of particles and their mutual interactions as controlled by contact-force laws. In Fig 5a, calibration involves resolving behavior as seen from two scales. The engineering scale involves the observed behavior as averaged over a volume containing many particles; it is at this engineering scale that properties are prescribed. The calibration process requires finding the contact properties that lead to the prescribed engineering properties. This process is aided by the tendency for interacting particles to naturally reproduce the observed soil behavior such as non-linearity of the stress-strain response, shear-induced dilatancy, third-invariant dependency of yield and failure stresses, hysteretic response under load-unload-reload cycles, localization of strains at failure (shear banding) and dependence of behavior on confining stress and porosity. The goal of calibration is to obtain the prescribed numerical values of key engineering parameters such as initial



a) Particles packed into tetrahedral mesh

Figure 4: Example of ability create complex domains: a) Particles packed in complex shape, b) Natural landscape derived from LIDAR imagery.

stiffness, friction angle, and dilatancy rate. The initial stiffness is roughly proportional to the elastic components of the contact stiffness. The friction angle, ϕ, depends on the combination of sliding friction and rolling resistance. A parametric study provides data for a contour plot in which ϕ can be determined for a given *pair* of contact sliding and rolling resistance, as shown in Fig 5b. The rolling resistance plays a key role in localization (Fig 5c), which determines a relatively narrow band of rolling resistance values that give realistic localization behavior. Given a suitable value of rolling resistance, the contact sliding friction can be uniquely determined from the parameter contours.

4.1. Mobility Cone

The mobility cone index is used as a means to test the results of the laboratory-based calibration process. The cone index is the force applied to the cone divided by the cone area and is the principal field tool used to predict mobility performance. It therefore serves as a tie between the laboratory and field behaviors. Simulated cone experiments in a calibration chamber provides a direct means to compare soil behaviors at various stress and porosity states. The test bed used for mobility simulations can likewise be probed by a simulated cone for a direct evaluation of the initial soil state.

Typical results from cone simulation are shown in Fig 6, which includes results from both the calibration chamber and the test bed probes. Results from the calibration chamber (Fig 6a) includes the comparisons of simulations using various particle sizes versus laboratory data from Melzer (1971). Simulations were performed using three mean particle diameters. The notations 1x, 2x, 4x, refer to respectively the smallest mean particle size, twice the smallest mean particle size, and four times the smallest mean particle size. The purpose of multiple simulations is to systematically study the effect of particle size on computed result in a manner analogous to reducing the grid size in a finite element convergence study. The contact stiffness values were adjusted for particle size

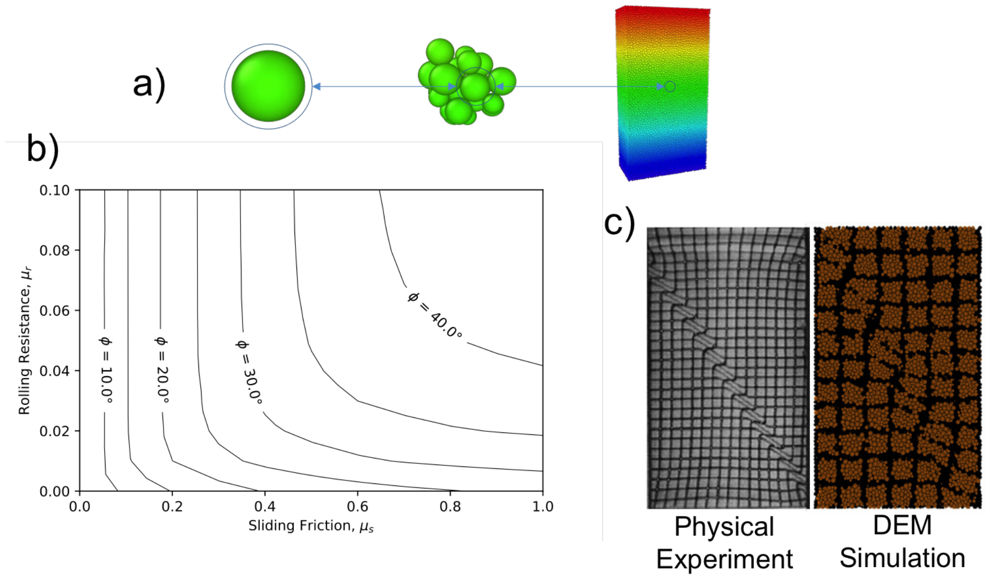
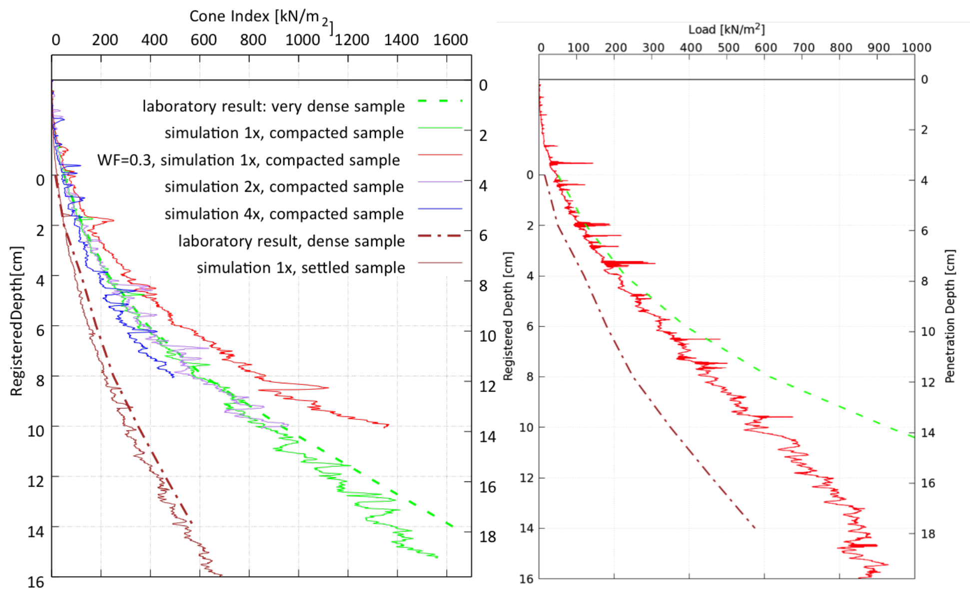


Figure 5: Key elements of calibration: a) Computation is performed at the particle scale (left) with macro-scale (right) behavior an emergent property; b) Micro-macro scale correlation built from parametric studies; c) Rolling resistance parameter primarily selected though ability to reproduce shear-localization.

based on dimensional analysis. These data show that these adjustments for the particle values were effective in reducing the contact stiffness to account for particles size.

Other particle size errors can arise from meso-scale effects such as force chain formation (*e.g.* Peters et al. (2005)) at rigid boundaries. These effects were not apparent in the cone penetration simulation. Also shown in the data is the variation caused by differences in soil porosity. In view of the arbitrary nature of the grain-size distribution, a fully validated measure of relative density does not yet exist for the simulated particles. Therefore, the simulated cone gives a direct measure of the strength of the particles at a particular porosity. Note also that there is uncertainty in the frictional resistance between the particles and cone surface. It was found that a friction parameter of $WF = 0.3$ gives a good correlation between data from simulated and physical experiments. Melzer (1971) did not give an independently measured value of WF nor is that properly generally available for field situations.

Figure 6b shows an apparent boundary effect on the simulated cone index in the test bed box that is not observed in the calibration chamber simulation based on comparing cone resistances at similar depths. The cause of the boundary effect and its possible effect on the soil tire interaction are still under investigation. There are two possibilities: the soil might not be fully compacted near the bottom of the box or the cone response might be affected near the bottom boundary in the rectangular box in a manner not observed in the circular calibration chamber.



a) Simulated Cone Calibration Chamber   b) Simulated Test Bed Box

Figure 6: Simulated mobility cone data: a) from calibration chamber including results of convergence study; b) from test bed probe showing effect of bottom boundary not shown in calibration chamber.

5. Terrain-Tire Interaction

Accuracy assessment of soil-tire interaction by direct comparison of simulations to experiments is another important component of test bed development. The assessment is based on comparisons of simulations to physical experiments documented in the DROVE database (Vahedifard et al., 2017). The parameters of interest include the torque T applied at the axle, the drawbar pull force F_s, and the slip ratio s_r, defined in fractional form as

$$s_r = 1 - \frac{v_s}{R_w \omega}, \tag{1}$$

where R_w is the wheel radius, ω is the angular velocity of the wheel (in radians/second) and v_s is the forward velocity of the axle. Pull force and torque are expressed in terms of dimensionless coefficients $p = F_s/W$ and $t = T/WR_w$, where W is the vertical load on the wheel.

The simulation is set up by placing a wheel on the prepared soil subgrade and allowing it to come in equilibrium under its weight with the degrees of freedom restricted to allow only vertical displacement. The wheel is then advanced such that motions are restricted to straight-ahead, vertical, and rotational degrees of freedom; side motions are restricted. The conjugate forces to these three degrees of freedom are either measured or prescribed as follows:

1. Prescribed slip: the wheel is advanced at specified velocity and rotation to produce a desired slip ratio.

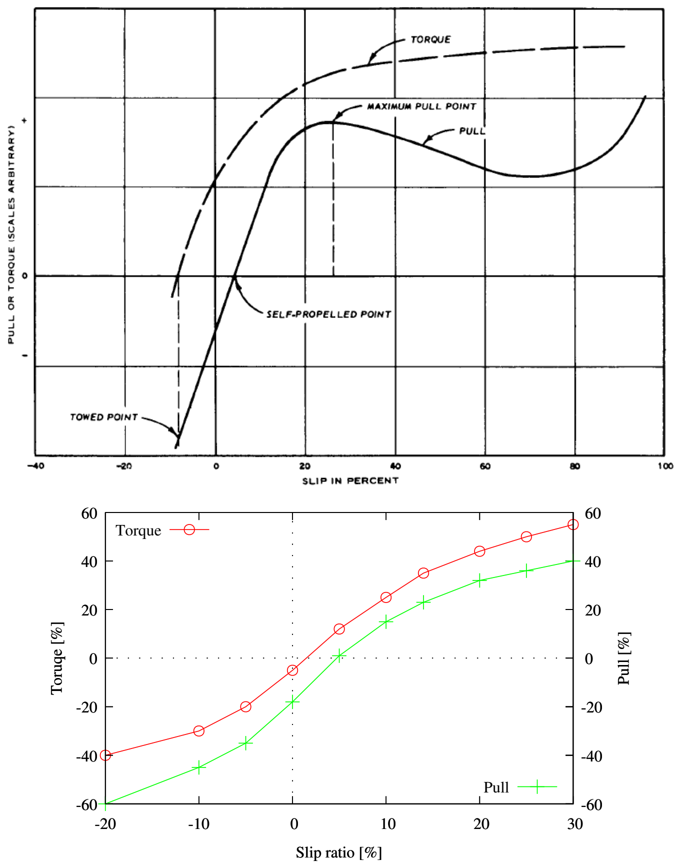


Figure 7: Characteristic shape of the traction relationship (Turnage, 1972) along with the simulated traction-slip curve.

2. Prescribed torque: the torque is applied to the wheel and its forward velocity and rotation are measured.

In both cases the wheel is free to move vertically whereby the sinkage is measured. The traction-slip measurements using the first method are in their initial phase. Results are shown in Fig 7, along with characteristic shape of the traction relationship reported by Turnage (1972). One of the most striking requirements of the wheel-soil parameter calibration is that relatively large soil-tire friction is needed in the simulated experiment to obtain the measured traction in the prescribed-slip experiments. One possible explanation is that in the physical test, soil grains can partially penetrate the wheel creating more resistance that the simple slipping mechanism of the simulated tire-particle interaction. The DROVE database does not contain independent measurements of the soil-wheel friction. The general slip-traction and slip-sinkage interactions seem to be reasonable.

6. Conclusion

Given continuing improvements in parallel computing technology the greatest challenge for analysis of off road mobility is the soil subgrade. The discrete element method is a popular modeling tool that captures the large discontinuous deformation associated with off-road soil-vehicle interaction. The DEM is mathematically simple compared to continuum method and adapts well to parallel-computing technology. For prototype-scale problems the method necessarily demands simplifying details of soil attributes including limited ranges in particle sizes and use of spherical particle shapes. The departure from full fidelity to particle-scale details does not limit the accuracy of macro-scale engineering behavior, but does require an effective calibration procedure. Calibration procedures based on parametric studies gives a practical approach to relating inter-particle contact properties to macro-scale engineering properties. Resistance to particle rolling is a key attribute for realistic behavior of spherical particles. The best indication of rotational resistance accuracy is the ability to generate realistic shear bands. In addition to contact properties, which can simply be assigned, the engineering behavior depends strongly on porosity, which is determined by the initial placement of particles. A particle placement procedure that uses a tetrahedral mesh to put particles in a "just touching" assemblage reduces the time required to reach an initial equilibrium position. The method can approximate complex boundary configurations. Placing the particles at a specified initial porosity and avoiding effects of compaction-induced residual stress are still challenges.

The accuracy of wheel modeling is being assessed using data from the DROVE database. The simulated mobility cone is used to evaluate the state of the granular subgrade. A simulated calibration chamber is used to relate laboratory-based material calibration to the cone resistance, thus creating a link among laboratory behavior, traditional mobility field evaluation methods and wheel performance. For the cone penetration test, the friction between the cone and soil particles is a source of uncertainty as is the friction between the soil particles and wheel for the tire traction tests. In both cases the particle-object friction is a parameter that is determined by adjusting to experimental data rather than predicted from independent measurements. These systematic studies will contribute to the value off-road vehicle modeling by improving accuracy in all steps of the analysis process.

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